出國報告（出國類別：參與國際會議）

第四屆工程與應用科學研討會

服務機關：國立暨南國際大學
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派赴國家：日本札幌
出國期間：2014 年 7 月 20 日至 2014 年 7 月 28 日
報告日期：2014 年 11 月 09 日

摘要

本次學生所參與的是第四屆工程與應用科學研討會（The 4th International Conference on Engineering and Applied Science），主要目的是將近幾個月的成果整理後，以「在 m, n 皆為偶數時，$T_{m,n}$ 之維度平均泛迴圈性質」（The Dimension-Balanced Pancyclicity on $T_{m,n}$ for Even m and n）為題發表。此研討會今年於 2014 年 7 月 22 日至 7 月 24 日在日本札幌所舉行，為期三天，是一個橫跨多領域的大型研討會。選擇這個研討會的原因除了領域契合外，也希望透過跨領域的交流可以研究領域較為偏向理論的學生能利用機會認識其他領域的研究者；恰巧大會將學生的報告與雲端運算的發表放在同一場次，也讓學生在此次的國際會議一行後，考量互連網路與雲端運算結合的可能性。同時，也因為研討會領域跨至其他工業領域，因此在共同演講和用餐時間皆能遇到其他領域如化學、生物之研究者，也因此得以和他們簡單的交流，是很好的體驗（無論在外語溝通和生活體驗交流上皆是）。

目錄

一、目的

隨著多核心(Multi-core)電腦與網路的發展，在核心之間或在各電腦間的傳輸問題已成為重要議題，因此，互連網路(interconnection networks)在資訊議題上的重要性也跟著水漲船高。因此學生主要的研究議題即為互連網路上的相關問題之討論與研究。由於資訊相關領域發展快速，研討會即成為一個交流與分享的重要場所，同時，國際研討會的舉辦更是肩負著跨國研究者互相交流的重要責任。因此，學生整理了過去數月的研究成果，將其發表至國際研討會：第四屆工程與應用科學研討會之上發表，除了將目前成果提出用以提升我們的國際能見度外，同時也希望相關的研究能得到研究先進的關注與提點，並且在會議上與相關研究者互相討論，期許能藉此激盪出更多的想法。從而得到未來研究的伸展以及將來把成果往期刊投稿時的幫助。

二、過程

會議舉行的日期為七月22至24，大會規劃中第一天主要是報到，演講和海報(Poster)的部分。學生抵達報到處時接近報到時間尾聲，一進到會場時演講已經準備開始，而大廳中的海報的發表人也已經抵達準備中，整個活動展開的氣氛非常熱烈。

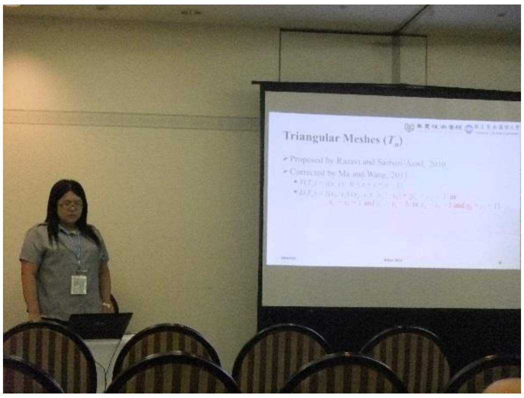
圖一：研討會會場大看板

圖二：研討會場地外

圖三：Day 2 大會主持人致詞

學生報告的時間為第二天（七月二十三日）上午場次，報告場次中有六篇文章，學生為其中兩篇文章的作者，且與共同作者分別擔任該場次的前二位報告者。在此一個報告場次中，同時包含了互連網路、影像資訊安全以及雲端傳輸與雲端安全等議題。除了自己的報告外，同場次中提到雲端傳輸的議題，是學生認為較有趣的議題，參與完報告場次之後，有和同場次的學者於會後簡單的交流；並因

為雲端上傳輸的性質，讓學生覺得或許可以將自己擅長的互連網路性質應用至雲端傳輸上。其後，第二天下午與第三天，則與同行者分開參與不同的場次，學生因為較喜歡能和海報發表者做仔細討論的形式，所以花費較多的時間在大廳和發表者討論。



圖四：實際發表現場

圖五：會後討論合影

三、心得

此會議是今年在日本北海道札幌市所舉行的大型研討會，其中包含的領域除了我所發表的資訊工程領域外，也包含有數學、化學、生物等領域。因為是這樣大型研討會，所以可以看到工作人員規劃與同時管控多場地多場次的忙碌，我覺得可以作為未來校內要舉辦研討會流程的參考。此外參與國際研討會時的熱烈提問與討論，也會讓人覺得我們國內研討會中的提問者比較少，也因此不會引發這麼熱烈的討論。讓我深感覺到，這是我們教育方式下還可以努力的方向：需要提升獨立思考和不怕提問的態度。

因此除了上述研討會經過所提到的預見許多相同領域的前輩之外，在開場演講和午、晚餐時刻更會遇到完全不同領域的人，互相交換一些對於大會的意見，或是某一場共同演講中所討論議題，更或是到這個都市裡的感想，都是此行的收穫。

對我而言，參與這樣的研討會除了練習向他人發表自己的研究成果外，得到對於該議題的討論或是提問可以幫助我回頭去思考是否在議題上有更多的發展空間，另外更是去看其他國家其他學校的研究者在研究上的成果的好機會。

四、建議

一直認為，参與國際研討會是博士班研究生與國外交流的重要管道，能以實際鼓勵博士生參加相關國際研討會是相當好的政策，這個政策除可以使研究生提升個人外語能力之外，更能透過會議中在相同議題上作相關交流學習，我想是非常能夠提升個人學術視野的，而這些研究生在回國後，一般都還會繼續將研究統整繼續努力或將後續研究交予實驗室學弟妹，我想長此以往，對於整個研究環境的提升是相當有幫助的。因此，我提出以下建議：

1. 廢除不得重複領取不同單位之補助規定

   現行規定下一次出國僅能接受一個單位之補助，然一般來說各單位都僅補助一個項目，如果能將規定改為單一項目僅能向一個單位補助，意即學生一次出國可向 A 單位申請機票費用；B 單位申請生活費，等。將可大幅提升研究生出席國際會議之意願。

2. 增加補助項目

   除持續保有對研究生的的支持，同時考量出差當地的物價，盡量給予除了機票金額以外的補助（例如住宿補助或生活費），或是在補助外能有其他實質獎勵，讓研究生更有出席發表之意願。

附錄

附錄一、研討會總議程

Conference Schedule

Tuesday, July 22, 2014		
Oral Session		
13:30-17:30	**Registration**	
14:00-15:30	Raffaeollo	Computer and Information Sciences I
	Michaelangelo	Life Sciences I
	Da Vinci	Material Science and Engineering I
15:45-17:30	Venezia	Environmental Sciences I
	Raffaeollo	Computer and Information Sciences II
	Michaelangelo	Civil Engineering I
	Da Vinci	Biomedical,Biological Engineering

Tuesday, July 22, 2014	
Poster Session	
B1, Renaissance Sapporo Hotel	
14:30-15:30	Electrical and Electronic Engineering
16:00-17:00	Life Sciences

Wednesday, July 23, 2014		
Oral Session		
Time	**Information**	
08:15-17:30	**Registration**	
08:30-10:00	Michaelangelo	Environmental Sciences II
	Da Vinci	Chemical Engineering I
10:00-10:15	**Tea Break**	
10:15-12:00	Michaelangelo	Material Science and Engineering II
	Da Vinci	Computer and Information Sciences III
10:30-11:45	Roma	Keynote Speech Keynote Speaker: Dr.Jun Mizuno
12:00-13:00	**Lunch Time**	
13:00-14:30	Michaelangelo	Life Sciences II
	Da Vinci	Fundamental and Applied Sciences
14:30-14:45	**Tea Break**	
14:45-16:15	Raffaeollo	Biological Engineering II
	Michaelangelo	Electrical and Electronic Engineering I
	Da Vinci	Mechanical Engineering I
16:15-16:45	**Tea Break**	
16:30-18:00	Michaelangelo	Civil Engineering II
	Da Vinci	Computer and Information Sciences IV

Wednesday, July 23, 2014	
Poster Session	
09:30-10:30	Psychology
	Computer and Information Sciences
	Electrical and Electronic Engineering
11:00-12:00	Life Sciences
	Fundamental and Applied Sciences
12:00-13:00	**Lunch Time**
14:00-15:00	Environmental Sciences
	Chemical Engineering
	Material Science and Engineering
	Fundamental and Applied Sciences
16:00-17:00	Biological Engineering
	Life Sciences
	Biomedical Engineering

Thursday, July 24, 2014		
Oral Session		
08:15-16:30	**Registration**	
08:30-10:00	Michaelangelo	Chemical Engineering II
	Da Vinci	Life Sciences III
10:00-10:15	**Tea Break**	
10:15-12:00	Michaelangelo	Computer and Information Sciences V
	Da Vinci	Material Engineering I
12:00-13:00	**Lunch Time**	
13:00-14:45	Raffaeollo	Civil Engineering III
	Michaelangelo	Computer and Information Sciences VI
	Da Vinci	Electrical and Electronic Engineering II
14:45-15:00	**Tea Break**	
15:00-16:45	Raffaeollo	Environmental Sciences III
	Michaelangelo	Electrical and Electronic Engineering III
	Da Vinci	Mechanical Engineering II

附錄二、論文發表場次論文列表

Computer and Information Sciences III

Da Vinci

2014/07/23 Wednesday 8:30-10:00

Session Chair: ***Prof.Justie Su-Tzu Juan***

ICEAS-2938

Shortest Path Routing Algorithms for Triangular Meshes and Triangular Pyramids

Justie Su-Tzu Juan | *National Chi Nan University*
Yi-Chun Wang | *National Chi Nan University*
Dyi-Rong Duh | *Hwa Hsia Institute of Technology*

ICEAS-2939

The Dimension-Balanced Pancyclicity on T_{m, n} for Even m and nJustie Su-Tzu Justie
Su-Tzu Juan | *National Chi Nan University*
Wen-Fang Peng | *National Chi Nan University*
Yi-Chun Wang | *National Chi Nan University*

ICEAS-2943

A Flexible Visual Multiple Secret Images Sharing Scheme by Shifting Random Grids

Justie Su-Tzu Juan | *National Chi Nan University*
Lu-Chung Chen | *National Chi Nan University*

ICEAS-2826

Cloud Computing Based Image Retrieval System

Yeong-Yuh Xu | *Hungkuang University*
Chi-Huang Shih | *Hungkuang University*

ICEAS-3029

Anomaly Detection of Unknown Internet Attacks Using Multivariate Normal Model Approach

Han-Wei Hsiao | *National University of Kaohsiung*
Yi-Chang Chen | *Institute for Information Industry*
Kun-Yu Chen | *Institute for Information Industry*
Bo-Jun Fang | *National University of Kaohsiung*

ICEAS-2950

Implementation of Module Replacement Mechanism for Partial Reconfigurable Architecture

Chin-Hsiung Wu | *Shih Chien University*
Chih-Tung Lin | *National Taiwan University of Science and Technology*
Shi-Jinn Horng | *National Taiwan University of Science and Technology*

ICEAS-2939
The Dimension-Balanced Pancyclicity on T_{m, n} for Even m and nJustie

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Abstract

The dimension-balanced cycle problem is a quiet new topic of graph theorem. Given a graph $G = (V, E)$, whose edge set can be partitioned into d dimensions for positive integer d. For any cycle C on G, the set of all i-dimensional edge of C, which is a subset of $E(G)$, is denoted as $E_i(C)$. If $||E_i(C)| - |E_j(C)|| \leq 1$ for each $1 \leq i < j \leq d$, C is called a dimension-balanced cycle. In this paper, we prove that when m and n are even, for toroidal mesh graph $T_{m, n}$: (i) $T_{m, n}$ does not contain any dimension-balanced cycle whose length l is odd or $l \equiv 2 \pmod 4$, (ii) $T_{m, n}$ contains a dimension-balanced cycle whose length is $4k$ for any integer $k \geq 1$.

Keyword: Toroidal mesh graph, Dimension-balanced cycle, Pancyclic, Hamiltonian cycle

1. Introduction

It is well-know that a topological structure of an interconnection network is usually modeled by a graph whose vertices represent processors/cores and edges represent communication links between processors. By this transformation, an interconnection network can be transformed to a graph. For a graph $G = (V, E)$, where $V(G)$ is vertex set and $E(G)$ is edge set, $|V(G)|$ denotes the number of vertex set and $|E(G)|$ denotes the number of edge set.

Hamiltonicity and Pancyclicity are both important in graph theory and widely discussed recently (see[1], [2], [3], [5], [6], [7], [8]). A *Hamiltonian path* of graph G is a path that contains all vertices. A *Hamiltonian cycle* of G is a cycle that contains every vertex of G

exactly once, except the origin vertex equal to the terminus one. A graph G is *pancyclic* if it embeds cycles of every length ranging from 3 to N, where N is the number of vertices in G. Since there exist no odd cycles in bipartite graph, a bipartite graph G is called *bipancyclic* if G contains cycles of every even length between 4 to N.

The definitions and notations follow those in ref. [8]. Given a graph $G = (V, E)$, and $\{E_1(G), E_2(G), \ldots, E_d(G)\}$ is a partition of E, d is called the *dimension* of G. Let C be a cycle on G and $E_i = E(C) \cap E_i(G)$ represent the set of all i-dimensional edge of C, which is a subset of $E(C)$, for any $1 \leq i \leq d$. If $||E_i(C)| - |E_j(C)|| \leq 1$ for all $1 \leq i < j \leq k$, C is called a *dimension-balanced cycle* (DBC, for short). By definition of DBC, given a graph $G = (V, E)$, let C be a Hamiltonian cycle on G, if C is also a DBC on G, C is called *dimension-balanced Hamiltonian cycle* (Hamiltonian DBC, for short). And if G contains a DBC of every length between 3 to $|V(G)|$, G is called *dimension-balanced pancyclic* (DB pancyclic, for short). If G embeds a DBC of every even length between 4 to $V(G)$, G is called a *dimension-balanced bipancyclic* (DB bipancyclic, for short).

The dimension-balanced cycle problem is a quite new topic of graph theory. The first research about dimension-balanced cycle has been proposed in the ref. [8]. Ref. [8] presented that G contains a Hamiltonian DBC when G is the hypercube Q_k, $T_{n, n}$, $C_n \times K_{m, n}$, $C_3 \times K_m$ and $C_4 \times C_m$ for k = 2, 4 and 8, m, $n \geq 3$. In 2012, Peng and Juan proposed a method for finding a Hamiltonian DBC on $T_{m, n}$ if these exist, where m, $n \geq 3$ [5]. In this paper, we study the problems about dimension-balanced pancyclicity on $T_{m, n}$ for even m and $n \geq 3$.

The rest of this paper is organized as follows. Section 2 introduces some basic definitions and theorems that will be used throughout this paper. Section 3 investigates the dimension-balanced pancyclicity on $T_{m, n}$, for even m and $n \geq 3$. Our conclusions are in Section 4.

2. Preliminaries

For defining of the toroidal mesh graph, we define the Cartesian product of graphs firstly as follows.

Definition 1. [9] Given two graph G_1, G_2, the Cartesian product $G_1 \times G_2$ of G_1 and G_2 is a graph with vertex set $V(G_1 \times G_2) = \{(x, y) \mid x \in V(G_1), y \in V(G_2)\}$ and the edge set $\{(u, v), (u', v') \mid u = u' \in V(G_1)$ and $(v, v') \in E(G_2)$, or $v = v' \in V(G_2)$ and $(u, u') \in E(G_1)\}$.
Let $C_m = \langle 0, 1, \ldots, m-1\rangle$ be a cycle with m vertices for any integer $m \geq 3$ and $P_n = \langle 0, 1, \ldots, n-1\rangle$ be a path with n vertices for any integer $n \geq 2$.

Definition 2. For m and $n \geq 3$, the *toroidal mesh graph*, $T_{m,n}$ is the graph $C_m \times C_n$.
That is, the dimension of $T_{m,n}$ is 2. Thus, let $V(T_{m,n}) = \{(x, y) \mid 0 \leq x \leq m-1, 0 \leq y \leq n-1\}$. Consider two adjacent vertices $u = (x_0, y_0)$ and $v = (x_1, y_1)$. If $y_0 = y_1$ and $\min\{|x_0 - x_1|, m - |x_0 - x_1|\} = 1$, $uv \in E_1(T_{m,n})$. Similarly, if $x_0 = x_1$ and $\min\{|y_0 - y_1|, n - |y_0 - y_1|\} = 1$, $uv \in E_2(T_{m,n})$. Fig. 3 shows an example of $T_{4,3}$.

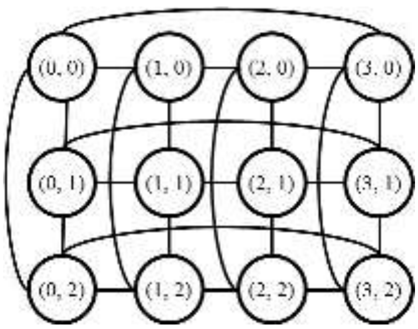

Fig. 3: The structure of $T_{4,3}$.

In this paper, we express path and cycle following [3]. Given two graphs G_1, G_2, vertex $x \in V(G_1)$ (or $y \in V(G_2)$, respectively) and a subgraph $H \subseteq G_2$ (or $H \subseteq G_1$, respectively), we use xH (or Hy, respectively) to denote the subgraph $\{x\} \times H$ (or $H \times \{y\}$, respectively) of $G_1 \times G_2$.
For a path $P = \langle x_1, x_2, \ldots, x_i, \ldots, x_{j-1}, x_j\rangle$, $P(x_i, x_j)$ represents the section $\langle x_i, \ldots, x_j\rangle$ of P, and $P^{-1}(x_j, x_i)$ represents the section of $P^{-1} = \langle x_j, x_{j-1}, \ldots, x_i, \ldots, x_2, x_1\rangle$. If $x_1 = x_n$, P is a cycle. That is, $P_m \subseteq C_m$ for integer $m \geq 3$.
Let the symbol k-cycle denote a cycle C of length k for $k \geq 3$ is an integer. Similarly, k-DBC denotes a DBC of length k for $k \geq 3$ is an integer.
The following theorems of $T_{m,n}$ are useful in our methods.
Theorem 1. [5] For mn mod 4 = 2, there is no Hamiltonian DBC on $T_{m,n}$.
Theorem 2. [5] For $m, n \geq 3$, there is a Hamiltonian DBC on $T_{m,n}$, except for the state on mn mod 4 = 2.

3. Main Result

In this section, we study the DB pancyclicity on $T_{m,n}$ with even m and n. According to length of cycle, we complete this section by three theorems. Firstly, we investigate the case of $(4k + 2)$-DBC by Theorem3. Secondly, we investigate the case of $4k$-DBC by Theorem 4 and discuss the case of $(2k + 1)$-DBC by Theorem 5.

Theorem 3. For $k \geq 1$, there is no $(4k + 2)$-DBC on $T_{m,n}$ when one of m and n is even.

Proof. Without loss of generality, we say n is even and m is any positive integer. Assume that there exists a $(4k + 2)$-DBC C^* on $T_{m,n}$ for some $k \in \{1, 2, \ldots, \lfloor mn - 2 / 4 \rfloor\}$. Since the length of C^* is $(4k + 2)$, $|E_1(C^*)| = |E_2(C^*)| = 2k + 1$ is an odd integer.

We say a vertex u in $V(C^*)$ is *black* if $u \in \{(x, y) \mid 0 \leq x \leq m - 1, 0 \leq y \leq n - 1$ and y is odd$\}$; *white* if $u \in \{(x, y) \mid 0 \leq x \leq m - 1, 0 \leq y \leq n - 1$ and y is even$\}$. According to the definition of $E_2(C^*)$, $E_2(C^*)$ should trace black vertex to white vertex or white vertex to black vertex. After tracing all edges of C^*, if the origin vertex of C^* is white, the terminate vertex of C^* is black due to $|E_2(C^*)|$ is odd. Vice versa, if the origin vertex of C^* is black, the terminate vertex of C^* is white due to $|E_2(C^*)|$ is odd. Obviously, in each case, the origin vertex and the terminate vertex of C^* are different. That is a contradiction. So, there is no $(4k + 2)$-DBC on $T_{m,n}$ when one of m and n is even.

□

Theorem 4. If both m and n are even, $T_{m,n}$ contains every $4k$-DBC for $1 \leq k \leq mn / 4$.

Proof. Without loss of generality, let $m \leq n$. Note that $T_{m,n} = C_m \times C_n$ and $C_l = \langle 0, 1, 2, \ldots, l - 1\rangle$, $P_l \subseteq C_l$ for $l = m, n$. We distinguish the following four cases according to k.

Case 1. $1 \leq k \leq m - 1$.

Construct a $4k$-cycle $C = \langle(0, 0), 0P_n(0, k), (0, k), P_m(0, k)k, (k, k), kP_n^{-1}(k, 0), (k, 0), P_m^{-1}(k, 0)0, (0, 0)\rangle$

Obviously, $|E_1(C)| = k + k = 2k$ and $|E_2(C)| = k + k = 2k$. Hence C is a $4k$-DBC.

Case 2. $m \leq k \leq n - 2$.

By the condition of k, we divide this case into two subcases.

Subcase 2.1. k is even.

Firstly, let $\alpha_1 = (2k - 2m + 2) \bmod (k - 2)$, $m_1 = (2k - 2m + 2 - \alpha_1) / (k - 2)$. Note that α_1 must be even because k is even.

Subcase 2.1.1. $\alpha_1 = 0$.

Now, we construct a $4k$-cycle $C_1 = \langle(0, 0), 0P_n(0, k), (0, k), P_m(0, m-1)k, (m-1, k), (m-1, k-1), (m-1, k-2), P_m^{-1}(m-1, m-m_1-1)(k-2), (m-m_1-1, k-2), (m-1, k-2), P_m^{-1}(m-1, m-m_1-1)(k-2), (m-m_1-1, k-2), (m-m_1-1, k-1), P_m(m-m_1-1, m-1)(k-3), \ldots, (m-1, 2), (m-1, 1), (0, m-1), P_m^{-1}(m-1, 0)0, (0, 0)\rangle$.

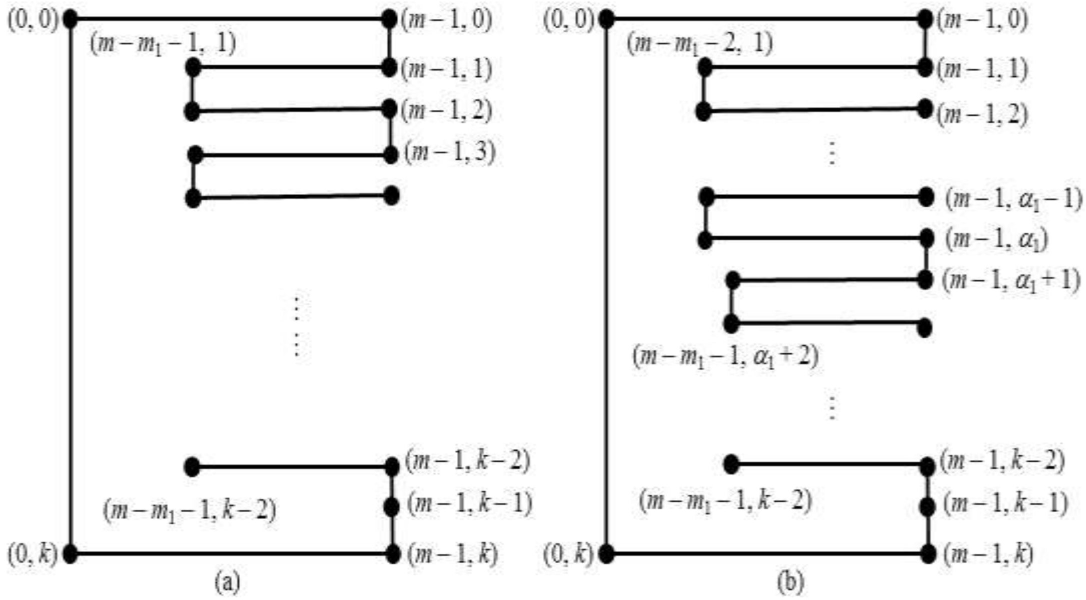

Fig. 2: The DBC of subcase 2.1.

Fig. 2(a) shows the structure of C_1. Thus $|E_1(C_1)| = 2(m-1) + (k-2)m_1 = 2k$, $|E_2(C_1)| = k + k = 2k$ and C_1 is a $4k$-DBC.

Subcase 2.1.2. $\alpha_1 > 0$.

By Subcase 2.1.1, there is a $4k$-cycle $C_2 = \langle(0, 0), C_1((0, 0), (m-1, \alpha_1)), (m-1, \alpha_1), P_m^{-1}((m-1, m-m_1-2))\alpha_1, (m-m_1-2, \alpha_1), (m-m_1-2, \alpha_1-1), P_m(m-m_1-2, m-1)(\alpha_1-1), (m-1, \alpha_1-1), (m-1, \alpha_1-2), P_m^{-1}(m-1, m-m_1-2), \ldots, (m-1, 0), P_m^{-1}(m-1, 0)0, (0, 0)\rangle$.

Fig. 2(b) illustrates the structure of C_2. Hence, $|E_1(C_2)| = m-1 + \alpha_1 + (k-2)m_1 + m-1 = 2m-2 + \alpha_1 + (k-2)m_1 = 2k$, $|E_2(C)| = k + k = 2k$ and C is a $4k$-DBC on $T_{m,n}$.

Subcase 2.2. k is odd.

Let $\alpha_2 = (2k - 2m + 2) \bmod (k-1)$, $m_2 = (2k - 2m + 2 - \alpha_2) / (k-1)$, $k' = k + 1$ $\alpha_1' = \alpha_2$ and $m_1' = m_2$. According to α_1', we use k', α_1' and m_1' to replace k, α_1 and m_1 in Subcase 2.1.1 or Subcase 2.1.2, then construct a cycle C_1'. Thus, $|E_1(C_1')| = 2m - 2 + (k'-2)m_1' + \alpha_1' = 2m - 2 + (k-1)m_1' + \alpha_1' = 2k$, $|E_2(C_1')| = 2k' = 2k + 2$. Since C_1' is a $(4k+2)$-cycle on $T_{m,k+1}$, we construct a $4k$-cycle $C = \langle(0, 0), C_1'((0, 0), (0, k)), (0, k), P_m(0, m-1)k, (m-1, k), C_1'((m-1, k), (0, 0)), (0, 0)\rangle$ with $|E_1(C)| = m-1 + \alpha_2 + (k-1)m_2 + m-1 = 2m-2 + \alpha_2 + (k-1)m_2 = 2k$, $|E_2(C)| = k + k = 2k$. Obviously, C is a $4k$-DBC on $T_{m,n}$.

Case 3. $n - 1 \le k \le mn / 4 - 1$.

Let $\beta_1 = ((2k - 2n + 2) \bmod (n - 2)) / 2$, $n_1 = (2k - 2n + 2 - 2\beta_1) / (n - 2)$, $\alpha_3 = (2k - 2m + 2) \bmod (n - 2)$ and $m_3 = (2k - 2m + 2 - \alpha_3) / (n - 2)$. Note that if $k = (mn / 4 - 1)$, $2\beta_1 = m - 4$, $n_1 = m / 2 - 2$, (i) $\alpha_3 = n - m - 2$ and $m_3 = m / 2 - 1$ for $n > m$, (ii) $\alpha_3 = n - 4$ and $m_3 = m/ 2 - 2$ for $n = m$. According to n_1, we separate this case into two subcases.

Subcase 3.1. n_1 is even.

In this case, according to the structure of cycle, we separate this case into two subcases.

Subcase 3.1.1. $m_3 > 0$ and $2\beta_1 + \alpha_3 + 4 \le n - 2$.

Let $\alpha_3' = \alpha_3 + (n - 2)$, $m_3' = m_3 - 1$, $\gamma_1 = (\alpha_3' \bmod 4) / 2$ and $\chi_1 = (\alpha_3 - 2\gamma_1) / 4$. If $\chi_1 = 0$, we construct a $4k$-cycle $C_3 = \langle(0, 0), 0P_n(0, n - 1), (0, n - 1), (1, n - 1), 1P_n^{-1}(n - 1), (1, 1), (2, 1), 2P_n(1, n - 1), \ldots, (n_1, n - 1), P_m(n_1, n_1 + m_3' + \gamma_1 + 1)(n - 1), (n_1 + m_3' + \gamma_1 + 1, n - 1), (n_1 + m_3' + \gamma_1 + 1, n - 2), P_m^{-1}(n_1 + m_3' + \gamma_1 + 1, n_1 + 1)(n - 2), (n_1 + 1, n - 2), (n_1 + 1, n - 3), P_m(n_1 + 1, n_1 + m_3' + 1)(n - 3), (n_1 + m_3' + 1, n - 3), (n_1 + m_3' + 1, n - 4), P_m^{-1}(n_1 + m_3' + 1, n_1 + 1)(n - 4), (n_1 + 1, n - 4), (n_1 + 1, n - 5), \ldots, (n_1 + 1, 1), P_m(n_1 + 1, m - 2)1, (m - 2, 1), (m - 2)P_n(1, 1 + \beta_1), (m - 2, 1 + \beta_1), (m - 1, 1 + \beta_1), (m - 1) P_n^{-1}(1 + \beta_1, 0), (m - 1, 0), P_m^{-1}(m - 1, 0)0, (0, 0)\rangle$. Since $|E_1(C_3)| = 2m - 2 + (n - 2)m_3' + 2\gamma_1 = 2k$ and $|E_2(C_3)| = 2n - 2 + (n - 2)n_1 + 2\beta_1 = 2k$, C_3 is a $4k$-DBC.

If $\chi_1 > 0$, we construct a $4k$-cycle $C_4 = \langle(0, 0), C_3((0, 0), (n_1, n - 1)), (n_1, n - 1), P_m(n_1, n_1 + m_3' + 3)(n - 1), (n_1 + m_3' + 3, n - 1), (n_1 + m_3' + 3, n - 2), P_m^{-1}(n_1 + m_3' + 3, n_1 + 1)(n - 2), (n_1 + 1, n - 2), (n_1 + 1, n - 3), P_m(n_1 + 1, n_1 + m_3' + 3)(n - 3), (n_1 + m_3' + 3, n - 3), (n_1 + m_3' + 3, n - 4), \ldots, (n_1 + 1, n - 2\chi_1 - 1), P_m(n_1 + 1, n_1 + m_3' + \gamma_1 + 1)(n - 2\chi_1 - 1), (n_1 + m_3' + \gamma_1 + 1, n - 2\chi_1 - 1), (n_1 + m_3' + \gamma_1 + 1, n - 2\chi_1 - 2), P_m^{-1}(n_1 + m_3' + \gamma_1 + 1, n_1 + 1)(n - 2\chi_1 - 2), (n_1 + 1, n - 2\chi_1 - 2), (n_1 + 1, n - 2\chi_1 - 3), P_m(n_1 + 1, n_1 + m_3' + 1)(n - 2\chi_1 - 3), (n_1 + m_3' + 1, n - 2\chi_1 - 3), (n_1 + m_3' + 1, n - 2\chi_1 - 4), P_m^{-1}(n_1 + m_3' + 1, n_1 + 1)(n - 2\chi_1 - 4), (n_1 + 1, n - 2\chi_1 - 4), (n_1 + 1, n - 2\chi_1 - 5), \ldots, (n_1 + 1, 1), C_3((n_1 + 1, 1), (0, 0)), (0, 0)\rangle$. Then $|E_1(C_4)| = 2m - 2 + (n - 2)m_3' + 2\gamma_1 + 4\chi_1 = 2k$ and $|E_2(C_4)| = 2n - 2 + (n - 2)n_1 + 2\beta_1 = 2k$ and C_4 is a $4k$-DBC.

Note that $(\beta_1 + 1) + 2\chi_1 + 2\gamma_1 = (\beta_1 + 1) + (\alpha_3' - 2\gamma_1) / 2 + 2\gamma_1 = (\beta_1 + 1) + (\alpha_3 + n - 2) / 2 - \gamma_1 + 2\gamma_1$, if $k = (mn / 4 - 1)$, (i) $(\beta_1 + 1) + (\alpha_3 + n - 2) / 2 + \gamma_1 = (m - 4) / 2 + 1 + (n - m - 2 + n - 2) / 2 + 1 = n - 2 < n - 1$, $n_1 + m_3' + 3 = n_1 + (m_3 - 1) + 3 = m / 2 - 2 + (m / 2 - 1 - 1) + 3 = m - 1$ for $n > m$ and (ii) $2\beta_1 + \alpha_3 + 4 = m + n - 4 > n - 2$ is invalid for $n = m$, thus, the condition (ii) does not exist in Subcase 3.1.1. Then C_3 and C_4 are well-defined cycle.

Subcase 3.1.2. $m_3 = 0$ or $2\beta_1 + \alpha_3 + 4 > n - 2$.

Let $\gamma_2 = (\alpha_3 \bmod 4) / 2$ and $\chi_2 = (\alpha_3 - 2\gamma_2) / 4$. If $\chi_2 = 0$, we make a $4k$-cycle $C_5 = \langle(0, 0), C_3((0, 0), (n_1, n - 1)), (n_1, n - 1), P_m(n_1, n_1 + m_3 + \gamma_2 + 1)(n - 1), (n_1 + m_3 + \gamma_2 + 1, n - 1), (n_1 + m_3 + \gamma_2 + 1, n - 2), P_m^{-1}(n_1 + m_3 + \gamma_2 + 1, n_1 + 1)(n - 2), (n_1 + 1, n - 2), (n_1 + 1, n - 3), P_m(n_1 + 1, n_1 + m_3 + 1)(n - 3), (n_1 + m_3 + 1, n - 3), (n_1 + m_3 + 1, n - 4), P_m^{-1}(n_1 + m_3 + 1, n_1 + 1)(n - 4), (n_1 + 1, n - 4), (n_1 + 1, n - 5), P_m(n_1 + 1, n_1 + m_3 + 1)(n - 5), \ldots, (n_1 + 1, 1), C_3((n_1 + 1, 1)(0, 0)), (0, 0)\rangle$.

Since $|E_1(C_5)| = 2m - 2 + (n - 2)m_3 + 2\gamma_2 = 2k$ and $|E_2(C_5)| = 2n - 2 + (n - 2)n_1 + 2\beta_1$, C_5 is a $4k$-DBC.

If $\chi_2 > 0$, we make a $4k$-cycle $C_6 = \langle(0, 0), C_3((0, 0), (n_1, n - 1)), (n_1, n - 1), P_m(n_1, n_1 + m_3 + 3)(n - 1), (n_1 + m_3 + 3, n - 1), (n_1 + m_3 + 3, n - 2), P_m^{-1}(n_1 + m_3 + 3, n_1 + 1)(n - 2), (n_1 + 1, n - 2), (n_1 + 1, n - 3), P_m(n_1 + 1, n_1 + m_3 + 3)(n - 3), \ldots, (n_1 + 1, n - 2\chi_2 - 1), P_m(n_1 + 1, n_1 + m_3 + \gamma_2 + 1)(n - 2\chi_2 - 1), (n_1 + m_3 + \gamma_2 + 1, n - 2\chi_2 - 1), (n_1 + m_3 + \gamma_2 + 1, n - 2\chi_2 - 2), P_m^{-1}(n_1 + m_3 + \gamma_2 + 1, n_1 + 1)(n - 2\chi_2 - 2), (n_1 + 1, n - 2\chi_2 - 2), (n_1 + 1, n - 2\chi_2 - 3), P_m(n_1 + 1, n_1 + m_3 + 1)(n - 2\chi_2 - 3), (n_1 + m_3 + 1, n - 2\chi_2 - 3), (n_1 + m_3 + 1, n - 2\chi_2 - 4), P_m^{-1}(n_1 + m_3 + 1, n_1 + 1)(n - 2\chi_2 - 4), (n_1 + 1, n - 2\chi_2 - 4), (n_1 + 1, n - 2\chi_2 - 5), \ldots, (n_1 + 1, 1), C_3((n_1 + 1, 1), (0, 0)), (0, 0)\rangle$.

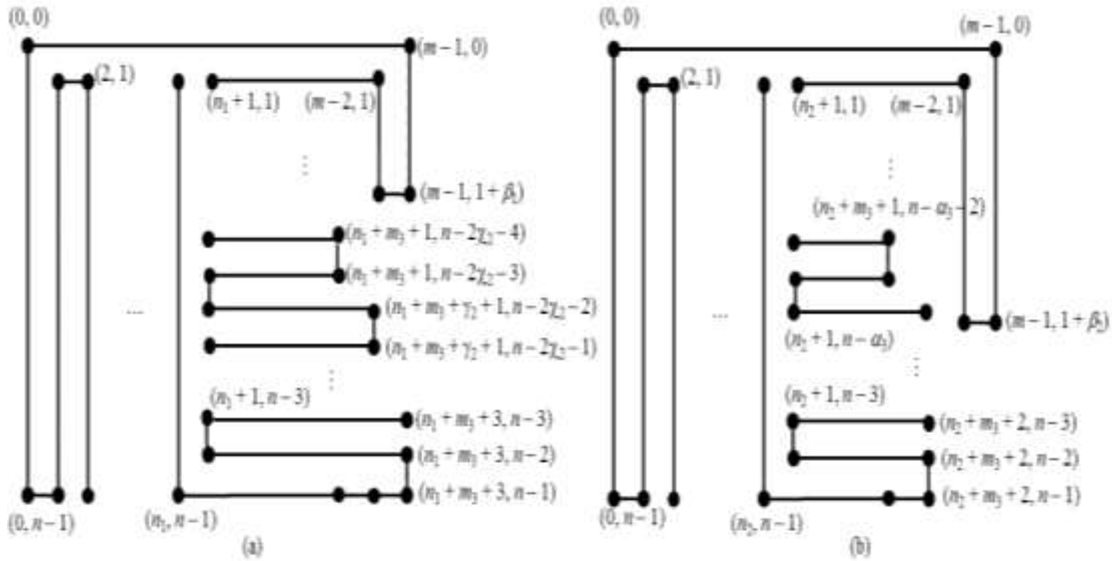

Fig. 3: The DBC of Subcase 3.1.2 and Subcase 3.2.2, respectively.

Fig. 3 (a) shows the structure of C_6. Then $|E_1(C_6)| = 2m - 2 + (n - 2)m_3 + 2\gamma_2 + 4\chi_2 = 2k$, $|E_2(C_6)| = 2n - 2 + (n - 2)n_1 + 2\beta_1 = 2k$, C_6 is a $4k$-DBC.

Note that $(\beta_1 + 1) + 2\chi_2 + 2\gamma_2 = (\beta_1 + 1) + (\alpha_3 - 2\gamma_2) / 2 + 2\gamma_2 = (\beta_1 + 1) + \alpha_3 / 2 + \gamma_2$, if $k = (mn / 4 - 1)$, (i) $2\beta_1 + \alpha_3 + 4 = n - 2$ is invalid for $n > m$, thus, the condition (i) does not exist in Subcase 3.1.2 and (ii) $(m - 4) / 2 + 1 + (n - m - 2) / 2 + 1 = n - 1$, $n_1 + m_3 + 3 = m / 2 - 2 + m / 2 - 2 + 3 = m - 1$ for $n = m$. Then C_5 and C_6 are well-defined cycles.

Subcase 3.2. n_1 is odd.

Let $n_2 = n_1 - 1$ and $\beta_2 = \beta_1 + ((n - 2) / 2)$. Note that if $k = (mn / 4 - 1)$, $2\beta_2 = m - 4 + n - 2 = n + m - 4$, $n_2 = m / 2 - 2 - 1 = m / 2 - 3$. We divide this case into two subcases.

Subcase 3.2.1. $2\beta_2 + \alpha_3 + 4 \leq 2n - 4$.

As Subcase 3.1.2, $\gamma_2 = (\alpha_3 \bmod 4) / 2$ and $\chi_2 = (\alpha_3 - 2\gamma_1) / 4$. According to χ_2, we use n_2 and β_2 to replace n_1 and β_1 in Subcase 3.1.2, then construct a cycle C_7. Hence, $|E_1(C_7)| = 2m - 2 +$

$(n-2)m_3 + 2\gamma_2 + 4\chi_2 = 2k$ and $|E_2(C_7)| = 2n - 2 + (n-2)n_2 + 2\beta_2 = 2n - 2 + (n-2)(n_1 - 1) + 2(\beta_1 + ((n-2)/2) = 2n - 2 + (n-2)n_1 - (n-2) + 2\beta_1 + (n-2) = 2n - 2 + (n-2) + 2\beta_1 = 2k$. Then C_7 is a $4k$-DBC.

Note that $(\beta_2 + 1) + 2\chi_2 + 2\gamma_2 = (\beta_2 + 1) + (\alpha_3 - 2\gamma_2)/2 + 2\gamma_2 = (\beta_2 + 1) + \alpha_3/2 + \gamma_2$, if $k = (mn/4 - 1)$, (i) $(\beta_2 + 1) + \alpha_3/2 + \gamma_2 = (n + m - 4)/2 + 1 + (n - m - 2)/2 + 1 = n - 1$, $n_2 + m_3 + 3 = m/2 - 3 + (m/2 - 1) + 3 = m - 1$ for $n > m$ and (ii) $2\beta_2 + \alpha_3 + 4 = 2n + m - 6 > 2n - 4$ is invalid for $n = m$, thus, the condition (ii) does not exist in Subcase 3.2.1. Then C_7 is a well-defined cycle.

Subcase 3.2.2. $2\beta_2 + \alpha_3 + 4 > 2n - 4$.

Now, we construct a $4k$-cycle $C_8 = \langle(0, 0), 0P_n(0, n-1), (0, n-1), (1, n-1), 1P_n^{-1}(n-1, 1), (1, 1), (2, 1), 2P_n(1, n-1), (2, n-1), (3, n-1), 3P_n^{-1}(n-1, 1), \ldots, (n_2, n-1), P_m(n_2, n_2 + m_3 + 2)(n-1), (n_2 + m_3 + 2, n-1), (n_2 + m_3 + 2, n-2), P_m^{-1}(n_2 + m_3 + 2, n_2 + 1)(n-2), (n_2 + 1, n-2), (n_2 + 1, n-3), P_m(n_2 + 1, n_2 + m_3 + 2)(n-3), (n_2 + m_3 + 2, n-3), (n_2 + m_3 + 2, n-4), \ldots, (n_2 + 1, n - \alpha_3 - 1), P_m(n_2 + 1, n_2 + m_3 + 1)(n - \alpha_3 - 1), (n_2 + m_3 + 1, n - \alpha_3 - 1), (n_2 + m_3 + 1, n - \alpha_3 - 2), P_m^{-1}(n_2 + m_3 + 1, n_2 + 1)(n - \alpha_3 - 2), (n_2 + 1, n - \alpha_3 - 2), (n_2 + 1, n - \alpha_3 - 3), P_m(n_2 + 1, n_2 + m_3 + 1)(n - \alpha_3 - 3), \ldots, (n_2 + 1, 1), P_m(n_1 + 1, m-2)1, (m-2, 1), (m-2)P_n(1, 1 + \beta_2), (m-2, 1 + \beta_2), (m-1, 1 + \beta_2), (m-1) P_n^{-1}(1 + \beta_2, 0), (m-1, 0), P_m^{-1}(m-1, 0)0, (0, 0)\rangle$.

Fig. 3 (b) illustrates the structure of C_8. Because $|E_1(C)| = 2m - 2 + (n-2)m_3 + \alpha_3 = 2k$ and $|E_2(C)| = 2n - 2 + (n-2)n_2 + 2\beta_2 = 2k$, C_8 is a $4k$-DBC.

Note that if $k = (mn/4 - 1)$, (i) $2\beta_2 + \alpha_3 + 4 = n + m - 6 + n - m - 2 + 4 = 2n - 4$ is invalid for $n > m$, thus, the condition (i) does not exist in Subcase 3.2.2 and (ii) $(n_2 + m_3 + 2) + 2 = ((m/2 - 2 - 1) + m/2 - 2 + 2) + 2 = m - 1$. Then C_8 is a well-defined cycle.

Case 4. $k = mn/4$.

$T_{m,n}$ contains a Hamiltonian DBC by Theorem 2, this case is hold. □

Theorem 5. There is no $(2k+1)$-DBC on $T_{m,n}$ for both m, n are even and $k \geq 1$.

Proof. Let the parity of a vertex (x, y) of $T_{m,n}$ be defined to be $((x + y) \bmod 2)$. Then $V(T_{m,n})$ can be partition into two subsets $V_{\text{even}}(T_{m,n})$ and $V_{\text{odd}}(T_{m,n})$ by setting $V_{\text{even}}(T_{m,n}) = \{u \in V(T_{m,n}) \mid \text{the parity of } u \text{ is even}\}$ and $V_{\text{odd}}(T_{m,n}) = \{u \in V(T_{m,n}) \mid \text{the parity of } u \text{ is odd}\}$, so that each edge has one end-vertex in V_{even} and another in V_{odd}. Obviously, $T_{m,n}$ is a bipartite graph when m and n both are even. That is, $T_{m,n}$ does not exist odd cycle. So, $T_{m,n}$ dose not exist $(2k + 1)$-DBC for m, n both are even and $k \geq 1$. □

4. Conclusions

In this paper, we prove that when *m* and *n* are even, (i) $T_{m,n}$ does not exist any DBC whose length *l* is odd by Theorem 5; or $l \equiv 2 \pmod 4$ by Theorem 3, (ii) $T_{m,n}$ exists a 4*k*-DBC for any integer $k \geq 1$ by Theorem 4. That is, $T_{m,n}$ is neither DB pancyclic nor DB bipancyclic for even *m* and *n*. For future work, we will study this property on $T_{m,n}$ for one of *m* and *n* is even; another is odd, and both *m* and *n* are odd.

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