出國報告（出國類別：國際會議）

自然語意學的邏輯和工程第十屆國際研討會
(The Tenth International Workshop of Logic and Engineering of Natural Language Semantics)

服務機關： 國立中正大學哲學系

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派赴國家：日本

出國期間：102 年 10 月 25 日至 102 年 10 月 29 日

報告日期：102 年 11 月 5 日

摘要：

本次出國目的為參加國際會議發表英文論文 From 3-valued semantics, Adams' Thesis rises (中文譯名：亞當斯論題在三值語意論上成立)，本人此次為王一奇副教授 (本論文的第二作者) 的隨行人員，是本論文的第一作者及通訊作者。此次的會議名稱為: 自然語意學的邏輯和工程國際研討會 (The Tenth International Workshop of Logic and Engineering of Natural Language Semantics)，會議地點在日本橫濱市(Japan, Yokohama)慶應大學日吉校區，主辦單位為日本人工智能協會 (JSAI)，會議全程使用英文。本會議為此國際會議所舉辦的第十屆，依循往例，本次會議邀請了自然語意學領域中重要的專家及學者與會，本人在會議中和王一奇副教授一同以英文口頭發表論文，獲得許多重要的建議及指教。

目次：

本文：

目的

本次出國目的為參加國際會議發表論文，此次的會議名稱為：自然語意學的邏輯和工程國際研討會 (The Tenth International Workshop of Logic and Engineering of Natural Language Semantics)，會議地點在日本橫濱市(Japan, Yokohama)慶應大學日吉校區，主辦單位為日本人工智能協會 (JSAI)。發表的英文論文名稱 From 3-valued semantics, Adams' Thesis rises (中文譯名：亞當斯論題在三值語意論上成立)，本人擔任王一奇副教授的隨行人員，是本論文的第一作者及通訊作者，王一奇副教授則是第二作者。本論文的內容主要分為三部分，第一部份探討亞當斯論題可能遭遇到的最大挑戰 - 貧乏性結果，說明這是由於對亞當斯論題採取錯誤的解讀所導致，第二部分從打賭的概念出發，提供一個對亞當斯論題恰當的解讀，第三部份提出一個對指示條件句一般性的機率與可斷說性理論，進一步證明亞當斯論題是我們理論下的一個特例，並說明為何這樣的理論不會有貧乏性結果的產生。基於本論文內容與自然語意學的邏輯有關，因此參加此國際會議以獲得機會與國際學者交流，希望獲得建議及指教，發現論文中可能的問題，以便有機會能加強論文中不足的部分。

過程

102 年 10 月 25 日:出發前往橫濱市參加『自然語意學的邏輯和工程國際研討會』。

102 年 10 月 26 日：學術參訪，與各國學者(包括本次會議籌辦人之一艾瑞克 馬克瑞迪 (Eric McCready) 教授)見面及交流。

102 年 10 月 27 日至 102 年 10 月 28 日：在橫濱市慶應大學日吉校區內參加『自然語意學的邏輯和工程國際研討會』，包括發表論文、參與其他學者的演講並進行討論。

102 年 10 月 27 日：下午於會議發表論文 From 3-valued semantics, Adams' Thesis rises(中文譯名：亞當斯論題在三值語意論上成立)，英文口頭報告 25 分鐘；在發表論文後，針對論文中的重要議題，和與會學者進行細緻的討論，取得很好的進展。

102 年 10 月 29 日：搭機返台。

以下將從會議議程及議場主題、與會內容重點及心得、個人報告內容及交流三方面說明與會過程。

會議議程及議場主題

本次會議主題為自然語意學，內容相當的廣泛，會議中一共報告了二十三篇論文

(詳細議程請見附錄1)，其中包含兩位特邀講者 (keynote speaker) 兩篇關於模糊性的報告，數量相當豐富。在二十三篇會議論文中，內容涵蓋自然語意學的不同面向，包含（1）條件句在初階邏輯及模態上的分析、（2）自然語言中模糊性所具有的問題、（3）用動態語意學來分析自然語言的某些特性、以及（4）分析不同的自然語言所具有的獨特性質。

與會內容重點及心得

在二十三篇會議論文報告中，有許多學者的報告內容令人印象深刻，以下僅對幾篇最令本人印象深刻的內容及本人的學習收穫提供大概的說明：

(1) 英國劍橋大學（Kobe University）的資工所博士生丸山（Yoshihiro Maruyama）先生發表的論文，題目為「量子語言學：從語言哲學到量子模糊性的邏輯」(Quantum Linguistics: From Philosophy of Language to Logic of Quantum Vagueness)。論文中想要說明量子語言學的在語言哲學上有什麼後果，並提出一個一統的方法，把不同的邏輯連接詞用量子語言學來說明。作者首先說明意義如何在量子語言學上被呈現，並認為意義和同義性本身都有可能是模糊的，並論證意義理論必須是經驗導向且順從經驗上的測試，最後試圖用量子語言學來重新分析羅素的確定描述詞。是一篇相當創新而有趣的論文。

(2) 德國慕尼黑大學 (LMU Munich) 的博士後艾特林 (David Etlin) 發表的論文，題目為「條件句，模態，和重複」(Conditionals, Modals, and Iteration)。論文中指出如果我們要以條件句邏輯來定義什麼是必然性和可能性的話，會有很多違反直覺的情況產生。論文中比較幾個條件句邏輯公理，說明不同的公理會定義出不同的模態概念，但很難找到一個合適的公理來定義模態中的必然性和可能性，這樣的結果似乎告訴我們想要用條件句邏輯來說明模態概念是錯誤的進路。

(3) 澳洲雪梨大學史密斯 (Nicholas J. J. Smith) 教授發表的論文，論文題目為「模糊性，計數，和數目」(Vagueness, Counting and Cardinality)。在論文中，史密斯說明在有些例子裡，計算數量的方式可以有很多種，而由於這樣的方式會有模糊性，因此，數目的計算也會有模糊性。這對我而言，算是很新奇的想法，模糊性的存在範圍原來比我想像中的廣泛。

(4) 日本大板大學 (Osaka University) 的中山 (Yasuo Nakayama) 教授發表論文，論文題目為「基於動態規範邏輯分析言行」(Analyzing Speech Acts based on Dynamic Normative Logic)。在論文中，中山教授基於他2010年發展的規範系統邏輯，試圖用動態語意學去擴充這個系統，以便能夠分析不同的言行。中山教授首先介紹他的規範系統邏輯，然後說明如何用動態語意學去擴充，然後用這樣架構去描述言行，最後說明如何應用這個理論去分析言行。中山教授論證動態規範邏輯可以明白地表達社會行為的條件，以及詳細地描述社

會行動之間的互動和規範推論，我覺得這是一個很廣泛的理論，非常值得注意。

(5) 德國法蘭克福大學 (University of Frankfurt) 研究助理古茲曼 (Daniel Gutzmann) 的論文，論文題目是：「組合性的多面性和語彙－語意的接口」(Compositional multidimensionality and the lexicon-semantics interface)。論文中提出一個多面向的語意學，用較少的組合規則，試圖解決自然語言中的組合性議題。論文中提出三個和組合性相關的議題，然後作者論證自己的三面向語意論可以對這些議題有良好的說明，而且使用更少的組合性規則，使得其理論更為簡潔有力。這樣的形式語意論對我而言是相當新鮮的，畢竟國內從事這類的研究似乎很少。

(6) 日本青山學院大學 (Aoyama Gakuin University) 麥克瑞迪 (Eric McCready) 教授的論文，論文題目是：「交談層面的禮貌和蘊含」(Discourse-Level Politeness and Implicature)。論文中想要探要在談話中禮貌這樣的現象如何從形式語言學來捕捉，作者基於之前的研究，進一步擴展到其它的語言行為。作者的策略是區分正式的(formal)和禮貌的(polite)的語言行為，並用賽局的策略來分析談話中交談者的這些行為，是個非常有趣的研究。

個人報告及交流

本人會議報告論文題目為 From 3-valued semantics, Adams' Thesis rises（亞當斯論題在三值語意論上成立），論文中文摘要如下(論文全文請見附錄 2)：

摘要：亞當斯論題一直被視為是指示條件句相當重要的看法，可是貧乏性結果卻說它是不可信的。我們論證，貧乏性結果實際攻擊到的是史東內克假說，而那是對亞當斯論題錯誤的解讀，因此，我們提出一個指示條件句的三值語意論，證明在這樣的語意論下，我們可以有一個一致的指示條件句機率理論，然後可以給出一個去打賭指示條件句的方式，證明亞當斯論題是我們理論下的一個特例。因此，我們論證亞當斯論題是一個正確的看法，只是它只限於簡單條件句下會成立，最後，我們進一步說明為何我們的理論不會有貧乏性結果的產生。

在本次論文的口頭報告中，與會人士對論文提出許多重要意見，其中兩個特別重要的疑問如下。

第一個疑問是來自於這次的特邀講者之一迪茲(Richard Dietz) 教授，他本身在 2012 年提出了一個貧乏性結果，於是想問我們的理論會不會有他論文中導致貧乏性結果的前提。由於他的問題相當具有技術性，由於時間的關係，我們無法在當下回答，於是在報告結束後，跟他進行了不少的討論。針對他的疑問，我們有詳細向他計算我們的理論不會有迪茲教授用來導出貧乏性結果的前提，他也覺得非常有趣，但又提出一個潛在的問題，那就是「在給定 A 的狀況下，若 A 則

B 的機率 P(A→B|A)」在我們的理論下會等於 1，這似乎是不可信的。這是個很難回答的問題，我也預計會在之後的論文中加入一些新段落來回應這類的問題。

第二個疑問針對論文未來的發展，詢問我們的理論是否可以說明一些語用學上的問題。事實上，我們並沒有注意到這個面向，而這是一個非常值得發展的方向，畢竟語言學家們非常在意這方面的問題。

總之，在口頭報告後，王一奇副教授和我與學者們對論文內容有一些進一步的討論，這對我們文章內容的改進有許多實質的幫助。

心得

本會議為此國際會議第十次舉辦，按照往例，本次會議邀請了自然語意學領域重要的專家及學者與會，如來自澳洲雪梨大學（The University of Sydney）著名模糊性研究學者史密斯（Nicholas J. J. Smith），及日本東京大學 (The University of Tokyo）的著名教授迪茲（Richard Dietz），及數位日本當地的優秀學者。有這些學者的參與，讓本會議在討論時有很精闢的問題與精彩的討論。

本會議含口頭報告及討論部分，皆全程使用英文，如前所述，本人和

王一奇副教授在會議中發表論文，獲得許多重要的建議及指教。本會議也有來自其它不同國家（日本、德國、法國、新加坡）的學者，其中也有日本當地的博士生，是個非常有國際觀的會議。

建議事項

經由這個國際會議可以看出，日本所主辦的國際會議水準相當高，而且非常歡迎世界各地的學者來參加會議，畢竟日本已是個相當國際化的國家，使得國際學者對於來日本參加會議有強烈的興趣，參加這樣的學術會議不僅能精進個人的研究能力，而且可以接觸到世界各地目前感興趣的研究方向，也能夠勉勵自己要能和國際的哲學研究接軌，從國際化的眼光來看待學術研究的發展。

從與會日本學者的論文報告，可以看出他們對學術的發展比台灣學者有更強的企圖心，他們樂於邀請世界各地的學者一同來發表論文，這不但能得到他們研究上實質的建議，也能造成學術之間良好的競爭。而從日本研究生的報告可以看出他們的英文口說能力比較不足，可是仍勇於在這樣的會議上發表，是個非常難能可貴的經驗。我個人的想法是，台灣的消費遠低於日本，如果有好的機會，世界各地的學者應樂於來台灣參加類似的會議，如果能促成台、日，甚至中、韓之間的交流，我們可以把社群進一步擴展，也許能吸引更多優秀的學者一起舉辦類似的研究會。

附錄 1：會議議程。

附錄 2：會議論文 From 3-valued semantics, Adams' Thesis rises（亞當斯論題在三值語意論上成立）全文。

Logic and Engineering of Natural Language Semantics 10 (LENLS 10)

Program

October 27th (Sun), 2013

9:00-9:20 Reception and Coffee Break
9:20-9:30 Opening Remarks
9:30-11:30 Session 1

- Takeshi Yamada

  "On Dummett's Critique of Davidsonian Theory of Meaning"

- Hidenori Kurokawa

  "Hypersequent calculi for modal logics extending S4"

- Yoshihiro Maruyama

  "Quantum Linguistics: From Philosophy of Language to Logic of Quantum Vagueness"

- Jiri Marsik and Maxime Amblard

  "Integration of Multiple Constraints in ACG"

11:30-13:00 Lunch
13:00-14:30 Session 2

- Daisuke Bekki

  "A Type-theoretic Approach to Double Negation Elimination in Anaphora"

- Ribeka Tanaka, Yuki Nakano and Daisuke Bekki

  "Constructive Generalized Quantifiers Revisited"

- Laurence Danlos, Philippe De Groote and Sylvain Pogodalla

  "A Type-Theoretic Account of Neg-Raising Predicates in Tree Adjoining Grammars"

14:30-14:45 Coffee break
14:45-16:15 Session 3

- David Etlin

  "Conditionals, Modals and Iteration"

- Hanako Yamamoto and Daisuke Bekki

  "First-order conditional logic and neighborhood-sheaf semantics for analysis of conditional sentences"

- Chi-Yen Liu and Linton Wang

  "From 3-valued semantics, Adams' thesis rises"

16:15-16:45 Coffee break
16:45-17:45 Invited Talk 1

- Nicholas J.J. Smith

  "Vagueness, Counting and Cardinality"

17:45-18:15 Questions and Comments
19:00- Banquet

October 28th (Mon), 2013

9:00-9:30 Reception
9:30-11:30 Session 1

- Ai Kawazoe, Yusuke Miyao, Takuya Matsuzaki, Hikaru Yokono and Noriko Arai

  "World history ontology for reasoning truth/falsehood of sentences: Event classification to fill in the gaps be-tween knowledge resources and natural language texts"

- Marc Vincent and Gre'goire Winterstein

  "Argumentative insights from an opinion classification task on a French corpus"

- Bruno Mery, Richard Moot and Christian Retore

"Plurals: individuals and sets in a richly typed semantics"

- Yasuo Nakayama

  "Analyzing Speech Acts based on Dynamic Normative Logic"

11:30-13:00 Lunch
13:00-15:00 Session 2

- Yoshiki Mori and Hitomi Hirayama

  "Bare plurals in the left periphery in German and Italian"

- Christopher Davis

  "Building Rhetorical Questions in Japanese"

- Wataru Uegaki

  "Predicting the distribution of exhaustive inference in a QUD model"

- Matthijs Westera

  "Exhaustivity through the maxim of relation"

15:00-15:15 Coffee break
15:15-16:45 Session 3

- Daniel Gutzmann

  "Compositional multidimensionality and the lexicon-semantics interface"

- Nicholas Asher, Cedric Degremont and Antoine Venant

  "Semantic Similarity -- Extended Abstract"

- Eric Mccready and Nicholas Asher

  "Discourse-level Politeness and Implicature"

16:45-17:00 Coffee break
17:00-18:00 Invited Talk 2

- Richard Dietz

"The Possibility of Vagueness"

18:00-18:30 Questions and Comments

Alternates

- Satoru Suzuki

  "Averages, Comparisons, Contextual Definitions and Meaningfulness"

From 3-Valued Semantics, Adams' Thesis Rises

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Abstract. The triviality results challenge Adams' thesis from two aspects: (a) if interpreted as Stalnaker's Hypothesis on the probability of the truth of conditionals, the triviality results directly reject it, and (b) if interpreted as the assertability, the acceptability, and the like, the triviality results make it dubious that the corresponding notion is substantive to a theory of conditionals. Mostly, one holding truth-conditional semantics of conditionals gives up Adams' thesis because of (a), and one holding a non-truth-conditional semantics of conditionals gives up the Adams' thesis because of (b). In this paper, we propose a way to reconcile the truth-conditional semantics of conditionals and Adams' thesis, based on a 3-valued truth-conditional semantics of conditionals and a notion of assertability based on fair betting quotient.

Keywords: Indicative Conditional; Adams' Thesis; Conditional Probability; Probability of Conditional; Triviality Results; Wallflower Argument

1 Introduction

Concerning the right semantics of indicative conditionals (in short, *conditionals*), there are two main camps in this campaign. One camp claims that conditionals have truth values, but different truth-conditions for conditionals are proposed. The other camp claims that conditionals do not have truth values, and it is argued that truth-conditions are unnecessary for conditionals. In this battle, *Adams' thesis*, that the "probability" of a conditional is the conditional probability of the consequent given the antecedent [1], plays a significant role on demarcating the boundary: triviality results in the literature make Adams' thesis easily disfavored by the truth-conditional approach. Our objective in this paper is to propose an interpretation of Adam's thesis in a 3-valued truth conditional semantics of conditionals, and show that it bypasses triviality results.

Since Adams himself does not mean that the "probability" of a conditional is the probability of its being true, we use $P^*(A \to B)$ to represent it, where P^* is left unspecified.

Adams' Thesis (AT) $P^*(A \to B) = P(B|A)$, provided $P(A) \neq 0$.

Many scholars believe that Adams' thesis is intuitively correct, but they disagree on its exact meaning and why it is correct. One popular interpretation of Adams' thesis is Stalnaker's Hypothesis, that the probability of a conditional's being true is the conditional probability of the consequent given the antecedent.

Stalnaker's Hypothesis (SH) $P(A \to B) = P(B|A)$, provided $P(A) \neq 0$.

However, it has been argued that (SH) encounters the notorious "triviality results", and thus should be rejected. Unless one is ready to give up (AT), some alternative interpretation of (AT) to bypass triviality results should be given. In section 2, we briefly explain how triviality results challenge (SH).

Because of triviality results, people who believe that conditionals have truth values jettison (SH), and, unfortunately, many of them jettison (AT) as well. Comparatively, people who hold on to (AT) tend to reject a truth-conditional account of conditionals. We take the route in between. This route is inspired by Michael McDermott's "fair betting quotient" interpretation for Adams' thesis (cf. [14]), that the assertability of $A \to B$ is defined by the fair betting quotient of $A \to B$. In section 3, we will show that in [11] Richard Jeffrey gives us a hint about how this can be done. Then we provide a general account of probability and assertability for conditionals in section 4 and section 5. We conclude that, given the generalized probability theory and assertability calculation, a certain interpretation of Adams' thesis is not only a hypothesis but also one that can be properly explained by and derived from a semantics of conditionals.

2 Triviality results

Robert Stalnaker once believed that (SH) is true [16]. But David Lewis triggers a chain of triviality results to show the opposite (e.g. [12], [13], [17], [3], [6], [7], and [8]). As Dorothy Edgington indicates, the idea of triviality results is that there are two different ways to calculate the probability of $A \to C$, one is that $P(A \to C) = P((A \to C) \wedge C) + P((A \to C) \wedge \neg C)$, the other is that $P(A \to C) = P(C|A)$, but they do *not* always have the same values (cf. [5]: 274). Consider that, for any proposition $A \to C$ and C such that $P(A \wedge C) > 0$,

$$P(A \to C) = P((A \to C) \wedge C) + P((A \to C) \wedge \neg C)$$
$$= P((A \to C)|C)P(C) + P((A \to C)|\neg C)P(\neg C).$$

In [12], Lewis shows that if (SH) were true, $P((A \to C)|C)$ would be equal to $P(C|AC)$, and $P((A \to C)|\neg C)$ would be equal to $P(C|A\neg C)$.[1] Then:

The First Trivial Result (TR 1) $P(A \to C) = P(C|AC)P(C) + P(C|A\neg C)P(\neg C) = P(C)$.

[1] AC means $A \wedge C$, and $A\neg C$ means $A \wedge \neg C$.

Any theory of conditionals should not tell us that (TR1) holds.

Many believe that the culprit of (TR 1) is (SH), but why (SH) is wrong divides them. Some believe that the conditional probability plays a significant role in conditionals is just an illusion, so they reject (AT) as well. Many others believe that conditional probability does play a significant role in conditionals, and it should not be interpreted as probabilities of conditionals since they do not have truth values. Nonetheless, [10] and [14] try to reconcile the truth-conditional theory of conditionals with (AT).

First, in [10], Jackson interprets (AT) by that the assertibility of a conditional is the conditional probability of its consequent given its antecedent.

(JAT) Assert$\underline{\text{i}}$bility $(A \to B) = P(B|A)$, provided $P(A) \neq 0$. ([10])

Though in [9] Hájek claims that (JAT) is the best case of (AT), it is not hard to see that if one assumes that (JAT) is correct for conditionals in any complex form, then it is under the attack of (TR 1^+), which can be derived in a reasoning similar to deriving (TR 1).

(TR 1^+) Assert$\underline{\text{i}}$bility$(A \to C) = P(C|AC)P(C) + P(C|A\neg C)P(\neg C) = P(C)$.

A way for (JAT) to escape the threat from (TR 1^+) is to confine its application on only simple conditionals, i.e. antecedents and consequents not in the form of conditionals. By confining the application, we obtain (JAT*), a special case of (AT*).[2]

(JAT*) Assert$\underline{\text{i}}$bility $(A \to B) = P(B|A)$, provided $P(A) \neq 0$ and $A \to B$ is a simple conditional.

(AT*) $P^*(A \to B) = P(B|A)$, provided $P(A) \neq 0$ and $A \to B$ is a simple conditional.

However, Hájek argues that (JAT*) specifically, and (AT*) in general, would be attacked by his wallflower argument (cf. [9]: 151), a specific trivial result.

The Wallflower Argument (WA) Any non-trivial finite-ranged probability function has more distinct conditional probability values than distinct unconditional probability values. ([9]: 156)

In [9], Hájek doubts that any *substantive* (rather than stipulative) notion of assertibility for (JAT) or (JAT*), especially, to be applied only on simple conditionals, can *outrun* unconditional probabilities, and at the same time *keep pace* of conditional probabilities.

As Hájek indicates, the wallflower argument against (JAT) or (JAT*) is not a decisive argument. Maybe there is an interpretation of (AT) or (AT*) immune to the wallflower argument, but Hájek would like to hear more about the rules of the game ([9]: 158). We believe that McDermott's proposal, if properly generalized, is one to serve the purpose. In [14], McDermott interprets (AT) as follows:

[2] The constraint on simple conditionals can block the derivation of (TR 1*). We skip the proof here.

(MAT) Assert<u>a</u>bility $(A \to B)$ = the fair betting quotient of $(A \to B)$ $=P(B|A)$, provided $P(A) \neq 0$. ([14])

It is fairly clear that (MAT) cannot resist the challenge from triviality results, and constraining the application of (MAT) cannot resist the challenge from the wallflower argument. Nonetheless, we shall defend that a version of (MAT), if properly construed, is an interpretation of (AT) that can avoid the challenge from triviality results, including the wallflower argument.

3 Betting on Conditionals

A dice is rolled, can we have a bet on "if it's even, it will be 4"? It seems fine that we can bet on this. Actually, we can bet on many other conditionals. We can bet on "if Hilary runs the next president election, she will lose," or bet on "if Yankees win the first game of 2013 World Series, they will be the 2013 World Series champion." How do we decide who wins or loses these conditional bets? We can use table 1 to represent how we bet on $A \to B$:

Table 1.

A	B	bet on $A \to B$
T	T	win
T	F	lose
F	T	neither win nor lose
F	F	neither win nor lose

Table 1 only applies to simple conditionals, i.e. A and B are conditional-free sentences. We will provide a more general table for betting on conditionals in section 5.

As we know, everyone agrees with table 1 on betting conditionals (c.f. [4], [16], [14], [15]). We can see that betting on $A \to B$ is different from betting on bivalent propositions, in that sometimes one neither wins nor loses a bet on $A \to B$. In [11], Jeffrey gives us a hint about how a bet such as table 1 can be proceeded (cf. [11]: 12-13). Let the price you can win a bet is $\$n$, and the "maximal" stake you "rationally" would put on it is $\$x$. For betting on $A \to B$, it is worth of $\$n$ when you win, it is worthless when you lose, and it is worth of $\$x$ when the bet is off. Ticket 1 summarizes this.

Ticket 1 Worth $\$n$ if $A \wedge B$ is true, worth $\$x$ if A is false.

The rational maximal stake $\$x$ you would put on the ticket 1 depends how you evaluate Ticket 1. The fair betting quotient of ticket 1 for you is x/n. So, if we know the valuation of Ticket 1, we can know its fair betting quotient.

In [11], Jeffrey indicates that the valuation of Ticket 1 should be the sum of Ticket 2 and Ticket 3, otherwise one will be Dutch-booked.

Ticket 2 Worth $\$n$ if $A \wedge B$ is true.

Ticket 3 Worth $\$x$ if A is false.

The valuation of Ticket 2 should be $\$n \times P(AB)$, and the valuation of Ticket 3 should be $\$x \times P(\neg A)$. So, provided $n > 0$,

$$\textbf{(BC)}\ x = n \times P(AB) + x \times P(\neg A) \Leftrightarrow x = n \times P(B|A) \Leftrightarrow x/n = P(B|A)$$

From the point of view of betting, the conditional probability in (BC) guides us to the right evaluation of betting on conditionals. In this sense, the conditional probability may be understood as representing one's "confidence" of beting on a simple conditional.

But as the triviality results show, (BC) will not take us too far. What the conditional probability stands for is the fair betting quotient of "simple conditionals." McDermott ties fair betting quotient with the assertability, and that gives us a good reason why we should believe (MAT) is on the right track. But (MAT) is not general enough. In a fully general approach, two questions need to be answered. First, can we give a general account of the assertability of conditionals based on fair betting quotient, which can avoid the triviality result (TR1)? Second, we have to face Hájek's objection from the wallflower argument: why do the fair betting quotients of simple conditionals, in a manner substantively related to conditionals, outrun unconditional probabilities and keep pace of conditional probabilities?

To answer the first question, we provide an appropriate probability theory of conditionals in section 4 on top of a 3-valued semantics of conditionals. We then define the assertability of $A \to B$ as:

$$\textbf{(MAT}^{+}\textbf{)}\ Assa(A \to B) = \text{the fair betting quotient of } A \to B.$$

Based on (MAT^{+}) and the 3-valued semantics for conditionals given in section 4, a general account of the assertability of sentences involving conditionals follows in section 5. An interpretation of (AT) naturally arises in the case of simple conditionals. And we will illuminate why the assertability in our interpretation of (AT) outruns the unconditional probability values in a manner substantive to conditionals in section 5.

4 The Probabilities of 3-Valued Conditionals

For a bivalent sentence S, since S is either true or false, betting on it either wins or loses. So, the probability of winning the bet on S is the probability of S being true. But as we see in section 3, betting on $A \to B$ is not that simple, we also need to consider cases in which no one wins or loses. One way to model these situations for betting is to make use of 3-valued semantics.[3] Assume that (i) all sentences which are conditionals-free sentences are bivalent, (ii) $\neg A$ means 'A is false' and $\backsim A$ means 'A is neither true nor false', and (iii) $A, \neg A$, and $\backsim A$ are mutually exclusive and exhaustive sentences of the sample space. We claim conditionals are 3-valued sentences of which the truth-table is as follows.

Table 2.

A	B	$\neg A$	$A \to B$	$A \vee B$	$\neg A \vee \neg B$	$\neg(A \wedge B)$	$A \wedge B$	$\backsim A$
T	T	F	T	T	F	F	T	F
T	F	F	F	T	T	T	F	F
T	X	F	X	T	X	X	X	F
F	T	T	X	T	T	T	F	F
F	F	T	X	F	T	T	F	F
F	X	T	X	X	T	T	F	F
X	T	X	X	T	X	X	X	T
X	F	X	X	X	T	T	F	T
X	X	X	X	X	X	X	X	T

In table 2, first, De Morgan's laws holds for '$\neg$' but not for '$\backsim$'. Second, the laws of distribution still hold, e.g. $(A \wedge (B \vee C))$ is equivalent to $(A \wedge B) \vee (A \wedge C)$. Third, '$\backsim$' is truth-functional. This may seem odd in the first place. But consider the predicate 'meaningless'. For any meaningful sentence S, to say 'S is meaningless' is to say a false sentence. On the contrary, for a meaningless sentence (e.g., colorless green ideas sleep furiously), to say 'colorless green ideas sleep furiously is meaningless' it to say a true sentence. Likewise, for a bivalent sentence S, to say 'S is neither-true-nor-false' is to say a false sentence. Nonetheless, for a sentence S that is neither true nor false, to say 'S is neither-true-nor-false' is to say a true sentence.

As we mentioned in section 3, one only wins a bet on $A \to B$ when both A and B are true, only loses it when A is true and B is false, and neither wins

[3] There is psychologic evidence to show that the classification of true, false, and void for conditionals parallels that of win, lose, and void for conditional bets [15].

nor loses it in other cases. It it is natural in the 3-valued semantics that the probability of $A \to B$'s being true is the probability of $A \wedge B$, the probability of $A \to B$'s being false is the probability of $A \wedge \neg B$, and the probability of $A \to B$'s being neither-true-nor-false is the probability of other cases. Applying probability theory to table 2 can reflect these ideas. We give a general probability theory of conditionals as follows:

Definition 1. $P(A \to B) = P(AB)$

Definition 2. $P(\neg(A \to B)) = P(A\neg B)$

Definition 3. $P(\backsim (A \to B)) = P(\neg A \vee \backsim A \vee \backsim B)$

If A and B are conditionals-free sentences, $P(A \to B) + P(\neg(A \to B)) + P(\backsim (A \to B)) = P(AB) + P(A\neg B) + P(\neg A) = 1$. However, we have to make sure that when it comes to more complicated sentences, it still holds. We assume the following laws of probability:

(1) For any sentence $A, 0 \leq P(A) \leq 1$.
(2) If A and B are equivalent, then $P(A) = P(B), P(\neg A) = P(\neg B)$, and $P(\backsim A) = P(\backsim B)$.
(3) If A and B are incompatible, then $P(A \vee B) = P(A) + P(B)$.
(4) $P(S) = 1$, where S is the sample space.

Then we can obtain the following theorems, whose proofs, as well as the proofs of other theorems in this paper, can be found in appendix.

Theorem 1. $P(A) = P(AB) + P(A\neg B) + P(A \backsim B)$.

If A, B are both conditional-free sentences, $P(A) = P(AB) + P(A\neg B)$, which matches the standard law of probability.

Theorem 2. $P(A \to B) + P(\neg(A \to B)) + P(\backsim (A \to B)) = 1$.

Theorem 2 shows that we can have a consistent probability distribution for $P(A \to B), P(\neg(A \to B))$, and $P(\backsim (A \to B))$.

These results show that we can assign probabilities to 3-valued conditionals without violating the laws of probability. That is because we can reduce the probabilities of conditionals to probabilities of conditional-free sentences, that is, bivalent sentences. When we want to calculate the probability of $A \to B$ being neither-true-nor-false, all we need to do is to calculate the probability of the conditions that makes $A \to B$ neither-true-nor-false.

5 The Assertability of Conditionals

To give a general account for the assertability of conditionals, we need to extend table 1 to table 1*.

Table 1*:

A	B	bet on $A \to B$
T	T	win
T	F	lose
T	X	neither win nor lose
F	T	neither win nor lose
F	F	neither win nor lose
F	X	neither win nor lose
X	T	neither win nor lose
X	F	neither win nor lose
X	X	neither win nor lose

Combining our probability theory of conditionals in 3-valued semantics and the fair-betting-quotient interpretation of assertability, we have a general result on the assertability of conditionals.

Theorem 3. *$Assa(A \to B) = P(AB)/[P(AB) + P(A\neg B)]$, provided $P(AB) + P(A\neg B) > 0$.*

Theorem 3 is our general account of the assertability of conditionals. One can see that Adams' thesis is just a special case of theorem 3 when $A \to B$ is a simple indicative conditional, i.e. when A and B do not embed conditionals, because $P(AB) + P(A\neg B) = P(A)$ then.

> **(MAT*)** $Assa(A \to B) = P(AB)/P(A)$, provided $P(A) > 0$ and $A \to B$ is a simple conditional.

While Adams' thesis fits into our assertability when conditionals are simple, it does not in general fit our assertability in theorem 3. In this manner, (TR 1^+) does not threat (MAT$^+$). It is clear that *Assa* is neither a unconditional probability function nor a conditional probability function, though, in cases of simple conditionals, it outruns unconditional probability values and keeps pace with conditional probability values.

Furthermore, we can extend the fair-betting-quotient idea of assertability to more complicated sentences. Then we have theorem 4-6:

Theorem 4. *$Assa(A \to (B \to C)) = P(ABC)/[P(ABC) + P(AB\neg C)]$, provided $P(ABC) + P(AB\neg C) > 0$.*

Theorem 5. *$Assa((A \to B) \wedge C) = P(ABC)/[P(ABC) + P((A \wedge \neg B) \vee \neg C)]$, provided $P(ABC) + P((A \wedge \neg B) \vee \neg C) > 0$.*

Theorem 6. *$Assa((A \to B) \vee C) = P((A \wedge B) \vee C)/[P((A \wedge B) \vee C) + P(A\neg B\neg C)]$, provided $P((A \wedge B) \vee C) + P(A\neg B\neg C) > 0$.*

Theorems 4-6 tell us how to calculate the assertabilities of nested indicative conditionals and compound sentences involving conditionals.

Finally, we can extend it to a general account of assertability of all sentences.

Theorem 7. *For any sentence A, $Assa(A) = P(A)/[P(A) + P(\neg A)]$, provided $P(A) + P(\neg A) > 0$.*

Since $P(A)$ is equal to the probability that the bet on A will be won, $P(\neg A)$ is equal to the probability that the bet on A will be lost, theorem 7 reflects McDermott's claim about the fair betting quotient:

> The fair betting quotient for a bet on a proposition ϕ is the probability that the bet will be won, given that it will be won or lost (= not called off). ([14]: 4)

Let W_A mean winning the bet on A, L_A mean losing the bet on A. The fair betting quotient of betting on $A = P(W_A|(W_A \vee L_A)) = P(W_A \wedge (W_A \vee L_A))/P(W_A \vee L_A) = P(W_A)/P(W_A \vee L_A) = P(A)/[P(A) + P(\neg A)]$.

Moreover generally, theorem 7 can be read as follows:

> **(Assertability Thesis)** For any sentence A, the assertability of A is ratio (degree) of the probability of A's being true divided by the probability of that A is either true or false.

(Assertability Thesis) is a claim general enough to play a significant role in a theory of conditional: that assertability reflect our confidence on the truth of a conditional, given the situation that the conditional is either true or false.

In our proposal of the assertability, Adams' thesis cannot hold across the board, which in turn respects the triviality results. However, as Adams insists, in the form of (AT*), it still holds for simple conditionals. When Hájek doubts that, he says: "I would find it surprising if assertibilities of simple indicative conditionals could keep up!" ([9]: 159) Hájek's question, in terms of our formulation of the assertability, is that if the function of assertability is to guide us how to bet on conditionals, why are they *not* a kind of unconditional probability functions? Our reply is that conditionals are 3-valued sentences. For bivalent sentences, the assertability goes with unconditional probability function, because you either won or lost the bets on them. In these cases, we only get any money back when we win. But for conditionals, we also need to concern the cases in which you neither win nor lose the bets on them. Besides winning the bet, we also get money back when it is called off. When this happens, we do not "win" the bet, we just get our stakes back. In that sense, we should not count this as part of the probability of winning a conditional bet, and thus should not count it as part of the probability of a conditional being true.

6 Conclusion

We show the assertability driven from (MAT^+) and the 3-valued semantics represent an interpretation of Adams' thesis that does not hold across the board, for it only holds in simple conditionals but falls in compounds sentences involving conditionals. As Adams suggests in [2], this can escape Lewis' triviality results. As for the other triviality results, our strategy is to distinguish the assertability of conditionals from unconditional probability of conditionals. [5] shows us that triviality results arise from admitting two different ways to calculate the probabilities of conditionals, and they are not always in tune with each other. In our proposal, there is only one way to calculate the probability values of conditionals based on the 3-valued semantics which we show is consistent. And we also have another way to calculate the assertability of conditionals, which is the fair betting quotient of betting on conditionals.

We propose that conditionals are 3-valued sentences, so they are not always either true or false. As Hájek says, many people believe Adams' thesis is a touchstone for conditionals (cf. [9]). But why? We show that because the conditional probability does play a significant part in simple conditionals. But it is not the probability of a conditional but the assertability of a simple conditional. So, we argue that Adams' thesis is not only a hypothesis, but a certain interpretation of it is also one that can be properly explained and derived by an appropriate semantics of conditionals. So we conclude that from the point of view of betting, Adams' thesis rises in simple conditionals.

Appendix

Theorem 1. $P(A) = P(AB) + P(A\neg B) + P(A \backsim B)$

Proof.
$P(A) = P(A \cap S) = P(A \cap (B \cup \neg B \cup \backsim B))$
$= P(A \wedge (B \vee \neg B \vee \backsim B)) = P((A \wedge B) \vee (A \wedge \neg B) \vee (A \wedge \backsim B))$
$= P(AB) + P(A\neg B) + P(A \backsim B)$

Theorem 2. $P(A \to B) + P(\neg(A \to B)) + P(\backsim (A \to B)) = 1.$

Proof.
$P(A \to B) + P(\neg(A \to B)) + P(\backsim (A \to B))$
$= P(AB) + P(A\neg B) + P(\neg A \vee \backsim A \vee \backsim B)$
$= P(A) - P(A \backsim B) + P(\neg A) + P(\backsim A) + P(\backsim B) - P(\neg A \backsim A)$
$\quad - P(\backsim A \backsim B) - P(\neg A \backsim B) + P(\neg A \backsim A \backsim B)$
$= 1 - P(A \backsim B) + P(\backsim B) - P(\backsim A \backsim B) - P(\neg A \backsim B)$
$= 1 + P(\backsim B) - P(\backsim B) = 1$

Theorem 3. $Assa(A \to B) = P(AB)/[P(AB) + P(A\neg B)].$

Proof.
$x = P(AB)n + P(\neg A \vee \backsim A \vee (A \wedge \backsim B))x$

$\Leftrightarrow x = P(AB)n + [P(\neg A) + P(\backsim A) + P(A \backsim B)]x$
$\Leftrightarrow x = P(AB)n + [1 - P(A) + P(A \backsim B)]x$
$\Leftrightarrow x = P(AB)n + \{1 - [P(A) - P(A \backsim B)]\}x$
$\Leftrightarrow x = P(AB)n + \{1 - [P(AB) + P(A\neg B)]\}x$
$\Leftrightarrow x = P(AB)n + x - [P(AB) + P(A\neg B)]x$
$\Leftrightarrow x[P(AB) + P(A\neg B)] = P(AB)n$
$\Leftrightarrow x/n = P(AB)/[P(AB) + P(A\neg B)]$

Theorem 4. $Assa(A \to (B \to C)) = P(ABC)/[P(ABC) + P(AB\neg C)]$.

Proof.
Because $P(A \to (B \to C)) = P(ABC), P(\neg(A \to (B \to C))) = P(AB\neg C)$, $P(\backsim (A \to (B \to C))) = 1 - [P(ABC) + P(AB\neg C)]$,
$x = P(ABC)n + P(\backsim (A \to (B \to C)))x$
$\Leftrightarrow x = P(ABC)n + \{1 - [P(ABC) + P(AB\neg C)]\}x$
$\Leftrightarrow x[1 - 1 + P(ABC) + P(AB\neg C)] = P(ABC)n$
$\Leftrightarrow x/n = P(ABC)/[P(ABC) + P(AB\neg C)]$.

Theorem 5. $Assa((A \to B) \wedge C) = P(ABC)/[P(ABC) + P((A \wedge \neg B) \vee \neg C)]$.

Proof.
Because $P((A \to B) \wedge C) = P(ABC), P(\neg((A \to B) \wedge C)) = P((A \wedge \neg B) \vee \neg C)$, $P(\backsim ((A \to B) \wedge C)) = 1 - [P(ABC) + P((A \wedge \neg B) \vee \neg C)]$,
$x = P(ABC)n + P(\backsim ((A \to B) \wedge C))x$
$\Leftrightarrow x = P(ABC)n + \{1 - [P(ABC) + P((A \wedge \neg B) \vee \neg C)]\}x$
$\Leftrightarrow x[P(ABC) + P((A \wedge \neg B) \vee \neg C)] = P(ABC)n$
$\Leftrightarrow x/n = P(ABC)/[P(ABC) + P((A \wedge \neg B) \vee \neg C)]$

Theorem 6. $Assa((A \to B) \vee C) = P((A \wedge B) \vee C)/[P((A \wedge B) \vee C) + P(A\neg B\neg C)]$.

Proof.
Because $P((A \to B) \vee C) = P((A \wedge B) \vee C), P(\neg((A \to B) \vee C)) = P(A\neg B\neg C)$, $P(\backsim ((A \to B) \vee C)) = 1 - [P((A \wedge B) \vee C) + P(A\neg B\neg C)]$.
$x = P((A \wedge B) \vee C)n + P(\backsim ((A \to B) \vee C))x$
$\Leftrightarrow x = P((A \wedge B) \vee C)n + \{1 - [P((A \wedge B) \vee C) + P(A\neg B\neg C)]\}x$
$\Leftrightarrow x[P((A \wedge B) \vee C) + P(A\neg B\neg C)] = P((A \wedge B) \vee C)n$
$\Leftrightarrow x/n = P((A \wedge B) \vee C)/[P((A \wedge B) \vee C) + P(A\neg B\neg C)]$

Theorem 7. $Assa(A) = P(A)/[P(A) + P(\neg A)]$.

Proof.
$x = P(A)n + P(\backsim A)x$
$\Leftrightarrow x[1 - P(\backsim A)] = P(A)n$
$\Leftrightarrow x/n = P(A)/[1 - P(\backsim A)]$
$\Leftrightarrow x/n = P(A)/[P(A) + P(\neg A)]$

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