# 5. Evaluation of Uncertainty of the Drying Method 

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## Measurement procedure of the drying method

- 1: Prepare the samples of adjusted moisture.
- 2: Grind the samples in a grinder.
- 3: Measure the constant weight of the weighing can to be used.

Repeat once.

- 4: Transfer the crushed samples into the weighing can and weigh them.

Repeat once.
Samples...Two samples are taken from the same lot.

- 4: Dry the samples.
- 5: Weigh the samples after drying.

Repeat once.

- 6: Calculate the moisture content.


## The source of uncertainty of the drying method

- Uncertainty caused by the distribution of temperature in the dryer.
- The uncertainty of this distribution is evaluated from the deviation between the samples' moisture content which are measured when the same samples are placed at different regions.


## The source of uncertainty of the drying method

- Uncertainties caused by the repeatability and the deviation between samples.
- This is calculated simultaneously from the above experiment.


## The source of uncertainty of the drying method

- Uncertainty caused by the reproducibility of the grinders.
- Prepare the grinders, which is the same type. And evaluate the deviation between grinders.


## The source of uncertainty of the drying method

- Uncertainty in mass measurement.
- Uncertainty of the mass of a weighing can.
- Uncertainty of the mass of a weighing can and the samples.
- Uncertainty in the calibration of a weighing machine.


## Model equation of the drying method

- Model equation

$$
M=\frac{m_{0}-m_{1}}{m_{0}-m_{c}} \times 100+e_{T}+e_{R}+e_{G}
$$

M: The moisture content in the sample.
$m_{0}$ : The mass of the sample before drying + the mass of the weighing can.
$m_{1}$ : The mass of the sample after drying + the mass of the weighing can.
$m_{c}$ : The mass of the weighing can.
$e_{T}$ : Moisture dispersion caused by the distribution of temperature in the dryer.
$e_{R}$ : Moisture dispersion caused by the repeatability and the deviation between
the samples.
$e_{G}$ : Moisture dispersion caused by the reproducibility of the grinders.

## AIST

## Model equation of the drying method

- The law of propagation of uncertainty is applied in the above equation

$$
\begin{gathered}
\frac{\partial M}{\partial m_{0}}=\frac{100\left(m_{1}-m_{c}\right)}{\left(m_{0}-m_{c}\right)^{2}} \quad \frac{\partial M}{\partial m_{1}}=-\frac{100}{m_{0}-m_{c}} \quad \frac{\partial M}{\partial m_{c}}=\frac{100\left(m_{0}-m_{1}\right)}{\left(m_{0}-m_{c}\right)^{2}} \\
u_{c}^{2}(M)=\left[\frac{100\left(m_{1}-m_{c}\right)}{\left(m_{0}-m_{c}\right)^{2}}\right]^{2} u_{m 0}^{2}+\left[-\frac{100}{m_{0}-m_{c}}\right]^{2} u_{m 1}^{2}+\left[\frac{100\left(m_{0}-m_{1}\right)}{\left(m_{0}-m_{c}\right)^{2}}\right]^{2} u_{m c}^{2}+u_{T}^{2}+u_{R}^{2}+u_{G}^{2}
\end{gathered}
$$

## Evaluation of several standard uncertainty

- Uncertainty caused by the distribution of temperature in the dryer.
- The interior of the dryer is divided into 25 regions. The moisture content of 25 samples that are sampled from the same lot and placed in several regions are measured. This measurement is repeated twice.

Uncertainty caused by the distribution of temperature in the dryer.


Weighing can with sample which derived from same lot.

## Uncertainty caused by the distribution of temperature in the dryer.

|  |  | n 1 |
| :--- | ---: | ---: |
| P1 | 13.97 | 13.99 |
| P2 | 13.89 | 13.98 |
| P 3 | 13.96 | 13.98 |
| P4 | 13.89 | 13.92 |
| P5 | 13.99 | 13.95 |
| P6 | 13.97 | 13.98 |
| P 7 | 13.94 | 13.94 |
| P8 | 13.97 | 13.96 |
| P9 | 13.96 | 13.99 |
| P10 | 13.91 | 13.97 |
| P11 | 13.90 | 13.92 |
| P12 | 13.98 | 13.99 |
| P13 | 13.95 | 13.98 |
| P14 | 13.98 | 13.93 |
| P15 | 13.96 | 13.99 |
| P16 | 13.97 | 13.98 |
| P17 | 13.89 | 13.94 |
| P18 | 13.97 | 13.97 |
| P19 | 13.94 | 13.96 |
| P20 | 13.90 | 13.96 |
| P21 | 13.96 | 13.93 |
| P22 | 13.99 | 13.97 |
| P23 | 13.88 | 13.92 |
| P24 | 13.92 | 13.97 |
| P25 | 13.97 | 13.95 |


| Factor | Sum of square, <br> $S$ | Deg. <br> free. $f$ | Variance, $V$ | Expectation of <br> variance, $E(V)$ |
| :---: | :---: | :---: | :---: | :---: |
| ANOVA** | 0.03071200 | 24 | 0.00127967 | $\sigma_{\mathrm{e}}{ }^{2}+2 \sigma_{\mathrm{P}}{ }^{2}$ |
| Error | 0.01665000 | 25 | 0.00066600 | $\sigma_{\mathrm{e}}{ }^{2}$ |
| Total ST | 0.04736200 | 49 |  |  |

*ANOVA: Analysis of variance developed by Sir Ronald Aylmer Fisher

## Uncertainty caused by the distribution of temperature in the dryer.

- Therefore, in actual measurement, two samples taken from the same lot are measured and the result is calculated from the mean of the values as follows:

$$
u_{T}=\frac{0.01752^{\sigma^{\circ}}}{\sqrt{2}}=0.01239 \%
$$

## Uncertainties caused by the repeatability and the deviation between samples．

－The variance of the error calculated in 1） represents the combination of repeatability and the deviation between the samples and the measurement result is calculated from the mean of the values as follows：

$$
u_{R}=\frac{0.025811^{\sigma_{e}}}{\sqrt{2}}=0.01825 \%
$$

# Uncertainty caused by the reproducibility of the grinders 

Example：Japanese Standard

Specification of the grinder is the one of the most influential factors to change the rice moisture in the drying method． Therefore，we recommend that specifications of the grinders are determined by several country standards．

## Uncertainty caused by the reproducibility of the grinders

Although, the specifications of the grinders are determined, there is an uncertainty caused by the reproducibility of the grinders.
The samples that are sampled from the same lot are crashed by the $\mathbf{1 3}$ grinders, and the moistures are measured. This measurement is repeated twice.

## AIST

Uncertainty caused by the reproducibility of

| No. | Moisture(\%) | Average(\%) |
| :---: | :---: | :---: |
| 21 | 13.906 |  |
|  | 13.912 | 13.91 |
| 16 | 13.783 |  |
|  | 13.789 | 13.79 |
| 20 | 13.820 |  |
|  | 13.827 | 13.82 |
| 17 | 13.977 |  |
|  | 13.973 | 13.98 |
| 22 | 13.911 |  |
|  | 13.907 | 13.91 |
| 14 | 13.895 |  |
|  | 13.907 | 13.90 |
| 11 | 13.951 |  |
|  | 13.903 | 13.93 |
| 12 | 13.895 |  |
|  | 13.867 | 13.88 |
| 18 | 13.788 |  |
|  | 13.810 | 13.80 |
| 15 | 13.780 |  |
|  | 13.806 | 13.79 |
| 13 | 13.947 |  |
|  | 13.918 | 13.93 |
| 23 | 14.011 |  |
|  | 13.925 | 13.97 |
| 19 | 13.816 |  |
|  | 13.860 | 13.84 | the grinders


| Factors | $S$ | $f$ | $V$ | $E(V)$ |
| :---: | :---: | :---: | :---: | :---: |
| ANOVA |  |  |  |  |
| Reproducibility <br> of the grinders | 0.10355988 | 12 | 0.00862999 | $\sigma_{e 2}{ }^{2}+2 \sigma_{G}{ }^{2}$ |
| Error | 0.00735825 | 13 | 0.00056602 | $\sigma_{e 2}{ }^{2}$ |
| Total | 0.11091813 | 25 |  |  |

# Uncertainty caused by the reproducibility of the grinders 

The following values are taken from the ANOVA table:

$$
\hat{\sigma}_{G}=0.06350 \%
$$

In actual measurement, only one grinder is used

$$
u_{G}=\hat{\sigma}_{G}=0.06350 \%
$$

Uncertainty caused by the mass of a sample before and after drying + the mass of a weighing can

- $u_{m 0}$ and $u_{m 1}$ are evaluated using a combination of $u_{S}$ and $u_{m r}$.

$$
u_{m 0}=u_{m 1}=\sqrt{u_{S}^{2}+u_{m r}^{2}}=\sqrt{0.000025^{2}+0.0001080^{2}}=0.0001109 \mathrm{~g}
$$

## Budget Sheet

| Symbol | Source | Standard Uncertainty | Sensitivity Coefficient | Standard <br> Uncertainty (\%) |
| :---: | :---: | :---: | :---: | :---: |
| $u_{\text {T }}$ | Uncertainty caused by the distribution of temperature in the dryer | 0.01239 (\%) | 1 | 0.01239 |
| $u_{R}$ | Uncertainties caused by the repeatability and the deviation between samples | 0.01825 (\%) | 1 | 0.01825 |
| $u_{G}$ | Uncertainty caused by the reproducibility of the grinders | 0.06350 (\%) | 1 | 0.06350 |
| $u_{m 0}$ | Uncertainty caused by the mass of a sample before drying + the mass of a weighing can | 0.0001109 (g) | 17.1715 (\%/g) | 0.001904 |
| $u_{s}$ | Uncertainty in the calibration of a weighing machine | 0.000025 (g) |  |  |
| $u_{m r}$ | Uncertainty of the weighing can and samples | 0.0001080 (g) |  |  |
| $\underline{u m 1}$ | Uncertainty caused by the mass of a sample after drying + the mass of a weighing can | 0.0001109 (g) | -19.9605 (\%/g) | 0.002214 |
| $u_{S}$ | Uncertainty in the calibration of a weighing machine | 0.000025 (g) |  |  |
| $u_{m r}$ | Uncertainty of the weighing can and samples | 0.0001080 (g) |  |  |
| $u_{\text {m }}$ | uncertainty of the weighing can | 0.0000629 (g) | 2.78894 (\%/g) | 0.0001754 |
| $u_{S}$ | Uncertainty in the calibration of a weighing machine | 0.000025 (g) |  |  |
| $U_{\text {CAN }}$ | Uncertainty of the weighing can | 0.0000577 (g) |  |  |
|  |  | Combined Standard | Uncertainty(\%) | 0.06729 |
|  |  | Expanded Uncerta | nty(\%) (k=2) | 0.13 |

## Measurement Results

- The mass of sample A before drying + the mass of weighing can A.......... 15.8234 g
- The mass of sample $B$ before drying + the mass of weighing can B..........15.9631 g
- The mass of weighing can A.......... 10.8135 g
- The mass of weighing can B..........11.2915 g
- The mass of sample A after drying + the mass of weighing can A..........15.1234 g
- The mass of sample B after drying + the mass of weighing can B..........15.3112 g
- The moisture content in sample A..........13.97\%
- The moisture content in sample B...........13.95\%
- Mean value of the samples..........13.96\%
- The moisture content in the lot of the rice is:

$$
M=\underline{13.96(\%) \pm 0.13(\%)} \quad(k=2)
$$

