



國立屏東科技大學
National PingTung University of Science & Technology

越南碩士在職專班授課
出國報告書

報告人: 葉 憲 弘
出國地點: 越南河內

出國期間: 101 年 12 月 14 日至 101 年 12 月 17 日
國立屏東科技大學企業管理系

中 華 民 國 102 年 01 月 25 日

目 錄

壹. 目的	2
貳. 過程	3
參. 心得與建議	4
肆. 附件（授課投影片）	6
附件一：An Overview of Financial Management（共 4 頁）	
附件二：Financial Statements, Cash Flow and Taxes（共 6 頁）	
附件三：Discounted Cash Flow Valuation（共 11 頁）	
附件四：Bonds and Their Valuation（共 15 頁）	

壹. 目的

依據本校開辦越南境外專班原定計劃書，主要目的在促進國際學術合作，提升多元文化交流，除可提供越南太原大學學生企業管理領域方面的學習外，對本校、本系亦助益良多。首先，對本校、本系的預期效益有：

- (1) 增加學校招生機會：穩定的境外專班招生能增加學生總人數，多元招生管道能在少子化的衝擊中開闢新契機。
- (2) 營造國際化校園環境氣氛：外籍生來校就讀期間能提供更多在校師生接觸與學習外國文化之機會。
- (3) 拓展學校知名度：藉由境外專班海外教學，學校的辦學名聲將有機會直接傳播至友好地區或國家。
- (4) 提供師生多元文化學習環境：藉由境外專班的開設，可提供本國師生到他國交換教師或學生之學習機會。
- (5) 提供本國師生到他國交換教師或學生的學習機會：將學校的資源及專業知識分享到境外學校，強化學術外交。

其次，對太原大學的預期效益有：

- (1) 交流台灣技職教育經驗：結合海外台商產業資源，建立技職教育跨國訓練課程，培訓專業管理人力，深耕技職教育交流深度，擴大國與國技職教育體系之接觸。
- (2) 吸取台灣高科技發展、教育輸出、中小企業成長之成功經驗。
- (3) 促進與東南亞國家教育體系的合作及交流。
- (4) 專業實務之整合與運用：促進產業與學術交流，強化各項專業管理實務之整合及運用能力，進而改善產業體質，提昇競爭力。
- (5) 提供越南在職生進修機會，增進其企業管理相關知能，並提昇其學術研究能力。
- (6) 提供當地華人或台商進修及學習高階管理理論之機會，以提升華人或台商經營企業之知識及技能。

本人此行出差至越南河內太原大學國際學院，是為了支援此境外在職碩士專班之「公司理財專題」授課，該專班是本校第一個境外碩士專班。藉由支援碩士專班英語授課，有助於增加本校教師國際化「全英語授課」教學經驗與能力。

貳. 過程

一、出國期間行程

1. 在 101 年 12 月 14 日週五早上從高雄搭乘高鐵至桃園高鐵站，轉搭接駁車至桃園國際機場，搭乘越南航空班機直飛至越南河內，並於當地時間下午三點半左右抵達。
2. 12 月 15 日與 12 月 16 日兩天有十四小時授課。
3. 回國前，12 月 17 日早上與太原大學國際學院院長 Dr. Anh 與其行政人員等五人，針對本專班相關問題交換意見，也交由本人帶回相關文件轉交國際事務處處長陳和賢教授。並於 12 月 17 日週一下午搭乘班機直飛回國至高雄，可以順利接續隔日國內授課。

二、授課主題綱要（授課投影片請參見附件）

1. An Overview of Financial Management 財務管理概論
 - a) The role of finance 財務功能之角色
 - b) Different types of jobs in finance 不同類型的財務工作機會
 - c) Different forms of business organization 不同形態的商業組織
 - d) Corporate management's goal: Shareholder wealth maximization
公司經營目標：股東財富極大化
 - e) The importance of business ethics 企業倫理的重要性
 - f) Potential conflicts Arising from Agency Problem 代理問題引起的潛在衝突
2. Financial Statements, Cash Flow and Taxes
 - a) Financial statements and reports 財務報表與報導
 - b) Balance sheet 資產負債表
 - c) Income statement 損益表
 - d) Statement of cash flows 現金流量表
 - e) Statement of stockholders' equity 股東權益表
 - f) Free cash flow 自由現金流量
 - g) Tax effect 課稅效果
3. Discounted Cash Flow Valuation 折現現金流量評價
 - a) Future values 未來值 and Present values 現值
 - b) Finding the interest rate, i 計算出利率
 - c) Finding the number of years, N 計算出期別數
 - d) Ordinary annuity and Annuity Due 普通年金與到期年金

- e) Perpetuities 永續年金
 - f) Uneven cash flows 不均等現金流量
 - g) Loan amortization 貸款攤還
4. Bonds and Their Valuation 債券與其評價
- a) Key Features of Bonds 債券主要特性
 - b) Bond Valuation 債券評價
 - c) Measuring Bond Yield 衡量債券收益
 - d) Assessing Bond Risk 衡量債券風險

三、授課情形

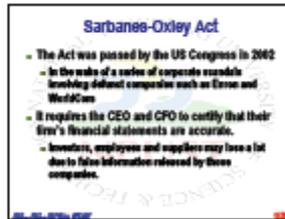
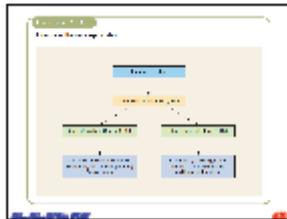
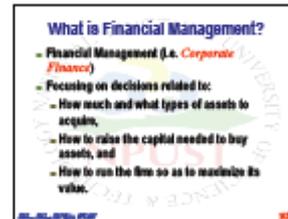
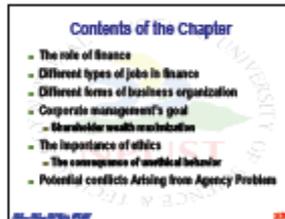
1. 第一次對越南學生上課，授課老師與學生雙方都需要適應與調整，因此此在進度上有稍為落後一些，但不會有太大影響。
2. 教學即教與學需要教師與學生雙方能有良好的溝通，才能有優異的教學成果。授課老師與越南學生雙方需要適當的溝通，以利雙方相互的瞭解，破除文化差異的隔閡，進而提升就學成效。

參. 心得與建議

1. 一般而言，與亞洲學生類似，在課堂學習上越南學生較內向、謙恭有禮，學習態度佳。該國經濟尚屬低度發展或開發中國家之型態，國民所得水準較低，但未來高等教育之需求，將會隨其經濟發展成長而持續成長。
2. 境外專班之開設，增加本校師生與對方學校師生之間相互了解，有助於雙方進一步的學術交流與研究合作。
3. 國內高等教育正值國際化轉型之際，境外專班的招生開設可以提高本校在越南教育產業之知名度，亦可藉機吸引更多當地國學生來本校進修。

肆. 附件(授課投影片)

附件一：An Overview of Financial Management (共 4 頁)



Corporation

- Advantages
 - Unlimited life
 - Easy transfer of ownership (股权转让)
 - Limited liability
 - Easy to raise large amount of capital
- Disadvantages
 - Costs of set-up and reporting
 - Double taxation

Special Forms of Business Organization in the U.S.

- S Corporation (small business corporation)
- Limited Liability Partnership (LLP)
- Limited Liability Company (LLC)

Special Forms of Business Organization in the U.S.

S corporation in the USA

- A relatively small, privately owned firm
- Subsection S of the Tax Code
- Number of Stockholders ≤ 75
- No double taxation

Special Forms of Business Organization in the U.S.

Limited Liability Company (LLC)

- A hybrid between a partnership and a corporation
- It is used by non-professional firms.
- It is like corporations but is taxed like partnerships.

Special Forms of Business Organization in the U.S.

Limited Liability Partnership (LLP)

- A hybrid between a partnership and a corporation
- It is used by professional firms in the fields of
 - Accounting, law and architecture
- It is like corporations but is taxed like partnerships.

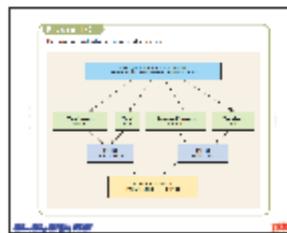
Share prices and Shareholder Wealth

- Shareholder wealth maximization
 - Management's primary goal
- Residual value of a firm
 - Total Assets - Total Debts
 - Available to be distributed to shareholders.
- Share (or stock) price maximization
 - The way to maximize shareholders' wealth

Intrinsic Values, Share Prices, and Compensation Plans

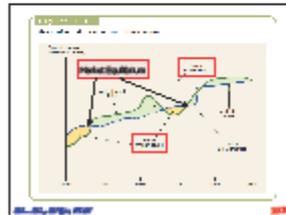
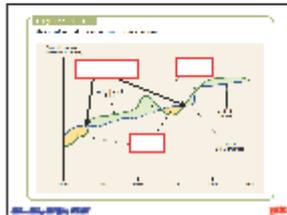
- The value of an asset
 - The sum of the present value of future cash flows
- The intrinsic value of a firm's share
 - Managerial actions
 - Economic environment
 - Political climate or environment

continued ...



Intrinsic Values, Share Prices, and Compensation Plans

- Intrinsic value = Market price \rightarrow Market equilibrium 市场均衡
- Intrinsic value > Market price \rightarrow stock is undervalued or underpriced.
- Intrinsic value < Market price \rightarrow stock is overvalued or overpriced.



- ### Some Important Business Trends
- More accuracy of published statements
 - To prohibit providing inaccurate information to shareholders and investors (Sarbanes-Oxley Act)
 - Increased globalization of business
 - Development in communication technology for the use of real-time data on business operations
 - Profound effect of information technology (IT)
 - Improvements in IT are sparking globalization.
 - Corporate governance
 - The way the top managers operate and interface with shareholders

- ### Business Ethics 企業倫理
- Definition of Ethics in Webster's dictionary
 - Standards of conduct or moral behavior
 - Can be thought of as a company's attitude and conduct toward its employees, customers, community, and shareholders.
 - New York Attorney General Eliot Spitzer and others sued companies for improper acts.
 - Congress has passed legislation imposing sanctions on executives who do bad things.
- continued ...

- ### Business Ethics 企業倫理
- A firm's commitment to business ethics can be measured by the tendency of its employees, from the top down, to adhere to laws, regulations, and moral standards relating to:
 - Product safety and quality,
 - fair employment, marketing and selling practices,
 - the use of confidential information for personal gain,
 - community involvement, and
 - illegal payments to obtain business.

- ### What Companies Are Doing
- Strong written codes of ethical behavior
 - Training programs to ensure that employees understand proper behavior
 - When conflicts arise involving profits and ethics, ethical considerations sometimes are so obviously important that they dominate.
 - In some cases, however, the right choices are not clear.

- ### Consequences of Unethical Behavior
- Unethical behavior can lead to a firm's decline.
 - Enron in USA
 - Ponds (pond.com) in Taiwan
 - These frauds (弊案) also contributed to fatal wounds to shareholders, other firms and even whole industries.
 - Arthur Andersen accounting firm collapsed.
 - Unethical actions can have consequences far beyond the firms that commit them.

- ### How should employees deal with unethical behavior?
- What would you do if you were the CEO or high ranking managers of Enron or Ponds (pond.com)?
 - If an employee goes ahead and sounds an alarm or reports the situation to a higher authority, he or she might be in trouble regardless of the merits of the case.
 - Disloyal and troublesome employees?
 - This places employees in a dilemma (地境/困境).

- ### Business Ethics Issues in Indonesia and Vietnam
- Unethical Behavior in Indonesia?
 - Unethical Behavior in Vietnam?

Conflicts between Managers, Stockholders, and Creditors

- An agency relationship exists whenever a **principal** hires an **agent** to act on their behalf.
- Within a corporation, agency relationships exist between:
 - Shareholders and managers
 - Shareholders and creditors

BS-BU, BPH, PGP ©20

Conflicts Between Managers and Shareholders

- Managers are naturally inclined to act in **their own best interests**.
- The following factors or actions affect managerial behavior:
 - Managerial compensation plans
 - Direct intervention by shareholders
 - The threat of firing and takeovers

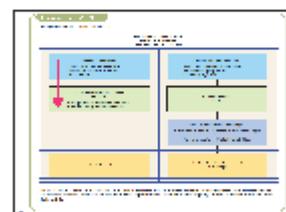
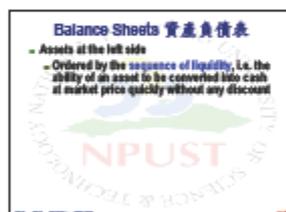
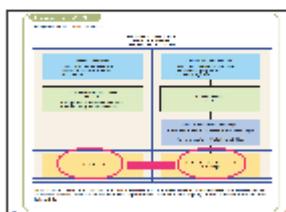
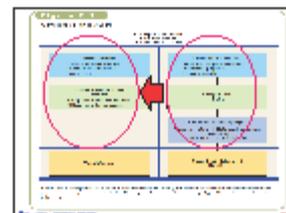
BS-BU, BPH, PGP ©20

Shareholders versus Creditors

- Shareholders (through managers) could take actions to **maximize share price** that are **damaging to creditors**.
- In the long run, such actions will raise the **cost of debt** and ultimately lower stock price.

BS-BU, BPH, PGP ©20

附件二：Financial Statements, Cash Flows and Taxes (共 6 頁)



Balance Sheets 資產負債表

- Assets at the left side
 - Ordered by the sequence of liquidity, i.e. the ability of an asset to be converted into cash at market price quickly without any discount
- Liabilities and equity at the right side
 - Claims against firm's assets
 - Liabilities are ordered in the time sequence in which they must be paid.

Balance Sheet

Assets		Liabilities and Equity	
Cash	100	Accounts payable	100
Marketable securities	100	Long-term debt	100
Accounts receivable	100	Common equity	100
Inventory	100		
Prepaid expenses	100		
Other current assets	100		
Fixed assets	100		
Total	500	Total	500

Balance Sheets 資產負債表
Current Assets

- Cash and cash equivalent,
- Marketable securities
- Accounts receivable (AR) and
- Inventory
- Prepaid expenses and
- Other current assets

Balance Sheet

Assets		Liabilities and Equity	
Cash	100	Accounts payable	100
Marketable securities	100	Long-term debt	100
Accounts receivable	100	Common equity	100
Inventory	100		
Prepaid expenses	100		
Other current assets	100		
Fixed assets	100		
Total	500	Total	500

Balance Sheets 資產負債表
Fixed Assets

- Tangible assets
 - Buildings
 - Real estate
 - Equipment and fixtures
- Depreciation
 - A deduction from the original value of the
 - No cash flow effect
- Intangible assets
 - Do not physically exist.
 - Intellectual property (IP): franchises, trademarks, patents, and copyrights

Balance Sheet

Assets		Liabilities and Equity	
Cash	100	Accounts payable	100
Marketable securities	100	Long-term debt	100
Accounts receivable	100	Common equity	100
Inventory	100		
Prepaid expenses	100		
Other current assets	100		
Fixed assets	100		
Total	500	Total	500

Balance Sheets 資產負債表
Current Liabilities

- Current liabilities are short-term obligations that must be paid in one year.
- Current liabilities include:
 - Short-term loans
 - Trade credits
 - Dividends and interest payable
 - Consumer deposits
 - Unpaid taxes and
 - Long-term debt that are due within one year

Balance Sheet

Assets		Liabilities and Equity	
Cash	100	Accounts payable	100
Marketable securities	100	Long-term debt	100
Accounts receivable	100	Common equity	100
Inventory	100		
Prepaid expenses	100		
Other current assets	100		
Fixed assets	100		
Total	500	Total	500

Balance Sheets 資產負債表
Long-term Liabilities

- Long-term liabilities are obligations that must be paid in more than a year.
- Long-term liabilities include long-term debts and long-term accrued liabilities.
- Long-term liabilities management concerns the issue of capital structure in finance.
 - Capital structure is one of the most important topics in corporate finance.

Account	2010	2009
Current assets	1,000	1,000
Property, plant, and equipment	1,000	1,000
Current liabilities	1,000	1,000
Equity	1,000	1,000
Retained earnings	1,000	1,000
Common stock	1,000	1,000
Preferred stock	1,000	1,000
Accumulated other comprehensive income	1,000	1,000
Other equity	1,000	1,000
Total	4,000	4,000

Balance Sheets 資產負債表

Owners' Equity

- Owners' equity represents a firm's net worth.
- The difference between the value of all assets and the value of all liabilities.
- Residual value
 - Contributed capital
 - Retained earnings

Account	2010	2009
Current assets	1,000	1,000
Property, plant, and equipment	1,000	1,000
Current liabilities	1,000	1,000
Equity	1,000	1,000
Retained earnings	1,000	1,000
Common stock	1,000	1,000
Preferred stock	1,000	1,000
Accumulated other comprehensive income	1,000	1,000
Other equity	1,000	1,000
Total	4,000	4,000

Additional Points about the Balance Sheet

- Inventory accounting
 - FIFO and LIFO methods
- Other sources of funds
 - A hybrid between equity and debt
- Depreciation
 - Accelerated and straight-line depreciation
- Market value versus book value
- Time dimension

Income Statement 損益表

- Summarizing the operation performance over a reporting period
- Operating income (i.e. EBIT)
 - Earnings from operations before interest and taxes
- Earnings before Taxes (EBT)
- The Bottom line (i.e. Earnings per share, EPS)
 - Dividend per share (DPS)
 - Book value per share (BVPS)
- EBITDA: Earnings from operations before interest, taxes, depreciation and amortization

Account	2010	2009
Operating income	1,000	1,000
Interest expense	1,000	1,000
Income before taxes	1,000	1,000
Taxes	1,000	1,000
Income after taxes	1,000	1,000
Dividend payment	1,000	1,000
Retained earnings	1,000	1,000
Net income	1,000	1,000
Total	4,000	4,000

Statement of Cash Flows 現金流量表

- OPERATING ACTIVITIES 營運活動
 - Net income
 - Adjustments:
 - non-cash charges (+)
 - non-cash revenues (-)
 - Changes in working capital other than cash
 - increase in non-cash CA (-)
 - increase in CL (+)

Statement of Cash Flows 現金流量表

- INVESTING ACTIVITIES 投資活動
 - increase in FA (-)
 - Decrease in FA (+)
- FINANCING ACTIVITIES 融資活動
 - increase (decrease) in Debt (+) (-)
 - Dividend payment (-)

Account	2010	2009
Operating activities	1,000	1,000
Investing activities	1,000	1,000
Financing activities	1,000	1,000
Net change in cash	1,000	1,000
Free cash flow	1,000	1,000
Total	4,000	4,000

Statement of Cash Flows: Summary

The statement of cash flows is used to help answer such questions:

- Is the firm generating enough cash to purchase the additional assets required for growth?
- Is the firm generating any extra cash that can be used to repay debt or to invest in new products?

Statement of Stockholders' Equity
股東權益表

	2011	2010	2009	2008	2007
Common stock	1,000	1,000	1,000	1,000	1,000
Retained earnings	1,000	1,000	1,000	1,000	1,000
Total	2,000	2,000	2,000	2,000	2,000

Statement of Stockholders' Equity
股東權益表

- Retained earnings represent a claim against assets, not assets per se. 保留盈餘是公司資產有求償權，但並非資產。
- Retained earnings reported on the balance sheet does not represent cash available for dividend payments or anything else. 保留盈餘本身並非現金，無法做為股利發放與其他資金用途之運用。

Free Cash Flow 自由現金流量

- FCF: the amount of cash that could be withdrawn from a firm without harming its ability to operate and to produce future CFs
- Operating cash flow = (1 - Tax rate) * (Net Inv.)
- EBIT * (1 - Tax rate) + Depreciation - (Gross Inv.)
- EBIT * (1 - Tax rate) + Depreciation - (Capital expenditure) + (increase in WC)
- EBIT * (1 - Tax rate) + Depreciation -

Example: Calculating Free Cash Flow

- FCF for TOE, 2008 = ?

Example: Calculating Free Cash Flow

FCF for TOE, 2008

- = EBIT * (1 - Tax rate) + Depreciation - (Gross Investment)
- = EBIT * (1 - Tax rate) + Depreciation - [(Capital expenditure) + (increase in WC)]
- = EBIT * (1 - Tax rate) + Depreciation - (FA+ΔCA - ΔNon-Interest-bearing CL)

Table 21. TOE's operating performance in 2008

Item	2008	2007
Revenue	1,000	1,000
Cost of goods sold	400	400
Operating expenses	100	100
Depreciation	100	100
EBIT	100	100
Interest expense	0	0
EBT	100	100
Taxes	30	30
Net income	70	70

Table 22. TOE's investment items in 2008

Item	2008	2007
Capital expenditure	100	100
Change in working capital	0	0
Change in non-interest-bearing current liabilities	0	0
Total investment	100	100

Table 23. TOE's cash flows in 2008

Item	2008	2007
Operating cash flow	70	70
Capital expenditure	(100)	(100)
Change in working capital	0	0
Change in non-interest-bearing current liabilities	0	0
Free cash flow	(30)	(30)

Account	Debit	Credit
Accounts receivable	1,000	400
Accounts payable	500	200
Inventory	1,000	200
Prepaid expenses	1,000	200
Depreciation expense	1,000	200
Accumulated depreciation		200
Retained earnings		1,000
Common stock		1,000
Dividends	1,000	
Net income		1,000
Net loss		

Example: Calculating Free Cash Flow

FCF for TGE, 2000

- = EBIT (1 - Tax rate) + Depreciation - (Gross Investment)
- = EBIT (1 - Tax rate) + Depreciation - (GFA+ΔCA - debt-interest-bearing CL)
- = \$4,000 (1 - 0.4) + \$2,000 - [\$2,000 + (1,570 - 6,300) - (1,400 - 1,200)]
- = \$2,130

Income Taxes

Individual income taxes 個人綜合所得稅

- Progressive tax system 累進稅制
- High income ⇨ High tax
- Regressive tax system 累退稅制
- High income ⇨ low tax
- Proportional tax system 比例稅制
- High-income and low-income taxpayers pay the same fraction of income.
- Marginal tax rate vs. Average tax rate

Income Tax Rate, Taiwan

Taxable Income	Rate
\$0 - \$410,000	0%
\$410,000 - \$1,000,000	13%
\$1,000,000 - \$2,100,000	21%
\$2,100,000 - \$4,000,000	30%
\$4,000,000 and above	40%

If taxable income = \$800,000, then
Income Tax = ?

Income Tax Rate, Taiwan

Taxable Income	Rate
\$0 - \$410,000	0%
\$410,000 - \$1,000,000	13%
\$1,000,000 - \$2,100,000	21%
\$2,100,000 - \$4,000,000	30%
\$4,000,000 and above	40%

If taxable income = \$800,000, then
Income Tax = \$410,000 × 0% + \$390,000 × 13%
= \$24,600 + \$50,700 = \$75,300

Income Tax Rate, Taiwan

Taxable Income	Rate
\$0 - \$410,000	0%
\$410,000 - \$1,000,000	13%
\$1,000,000 - \$2,100,000	21%
\$2,100,000 - \$4,000,000	30%
\$4,000,000 and above	40%

If taxable income = \$800,000, then
Average tax rate = \$75,300 / \$800,000
= 9.41%

Income Taxes

Individual income taxes 個人綜合所得稅

- Progressive tax system 累進稅制
- Marginal tax rate vs. Average tax rate
- Capital gain or loss 資本利得或損失
- Alternative minimum tax (AMT)
- Created by the US Congress in 1969 because 153 millionaires paid no taxes.

Income Taxes

Corporate taxes (USA)

- Interest and dividends received by a corporation
- Interest income received is taxed as ordinary income at the ordinary tax rate.
- 75% of dividends received is excluded from taxable income.
- Interest and dividends paid by a corporation
- Interest income paid can be deducted from operating income to reduce taxable income.
- However, dividends paid cannot be deducted.

Income Taxes

Tax Effect 稅收效果

- After-Tax Income
- Before-Tax Income × (1 - T)
- Inter-corporate dividend exclusion
- To avoid triple taxation (75% in the US)

國立屏東科技大學
National Pingtung University of Science & Technology

Special Topics on
Corporate Finance

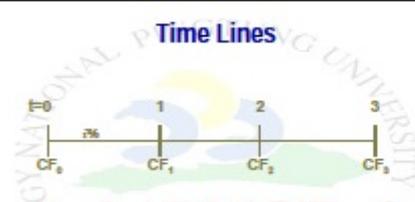
Discounted Cash Flow Valuation
(Time Value of Money)

NPUST

Topics of the chapter

- Future values 未來值 and Present values 現值
 - Finding the interest rate, i
 - Finding the number of years, N
- Annuities 年金
 - Ordinary annuity 普通年金 & annuity due 到期年金
- Perpetuities 永續年金
- Uneven cash flows 不均等現金流量
- Semiannual and other compounding
- Comparing interest rates
- Loan amortization 貸款攤還

Time Lines



- Each cash flow occurs at both the end of one period and the beginning of the next one.

Types of Cash Flows

- A lump-sum cash flow 單一整筆現金流量
- Even cash flows 均等現金流量
 - Ordinary annuity 普通年金
 - Annuity due 到期年金
- Uneven cash flows 不均等現金流量

Future Value and Compounding

- Compounding 複利計息
 - The process of going from present values to future values
- Compound interest 複利
 - Interest can be earned on prior period's interest
$$FV_n = PV (1 + i)^n$$
- Simple interest 單利
 -
$$FV_n = PV (1 + n \times i)$$

Approaches to solve time value problems 計算時間價值的方法

- Step-by-step approach 按部就班法
- Formula approach 公式法
- Financial calculator 財務計算機
 - hp calculator
 - Texas Instruments calculator
- Spreadsheets 試算表

To find the FV of \$100 compounded for 3 years at 5%

Timeline: $t=0$ (5% interest) $t=1$ $t=2$ $t=3$
 PV: \$100.00 → FV: \$???

7.88

Step-by-step Approach

Timeline: $t=0$ (5% interest) $t=1$ $t=2$ $t=3$
 \$100.00 → \$105.00 → \$110.25 → \$115.76

To find the FV of \$100 compounded for 3 years at 5%.

- \$100.00(1+5%)=\$105.00
- \$105.00(1+5%)=\$110.25
- \$110.25(1+5%)=\$115.76

8.88

Formula Approach

Timeline: $t=0$ (5% interest) $t=1$ $t=2$ $t=3$
 \$100.00 → \$115.76

$$FV_n = PV (1 + i)^n = \$100.00(1+5\%)^3 = \$115.76$$

9.88

Spreadsheets Approach: Excel Spreadsheet Software

- Formula = FV (rate, n, pmt, PV)

`=FV(0.05,3,0,-100)`
 "\$-100" denotes \$100 paid.
 \$115.76

10.88

Present Value and Discounting

- $PV = FV_n / (1 + i)^n$
 - Opportunity cost (i)
- Discounting
 - the reverse of compounding

11.88

Finding the interest rate, i

- $FV_n = PV (1 + i)^n$
- $\$150 = \$100(1 + i)^{10}$
- $1.5 = (1 + i)^{10}$
- $i = 4.14\%$

12.88

Finding the number of years, N

- $FV_n = PV (1 + i)^n$
- $\$1,000,000 = \$500,000(1+4.5\%)^N$
- $N = 15.7473 \approx 16$ years

The Rule of 70

- To become double with i (%) in about N years ($N \approx 70 / i$)
- Example:
 - \$500,000 deposit with 4.5% interest rate
 - $N \approx 70 / 4.5 = 16$ years

Annuities

Ordinary Annuity 普通年金

Annuity Due 到期年金

Future value of an ordinary annuity

$$FVA_{\text{ordinary},n} = \sum PMT (1 + i)^n$$

$$= PMT(1+i\%)^2 + PMT(1+i\%)^1 + PMT(1+i\%)^0$$

Example: Future Value of an Ordinary Annuity

N	3		
i	5%	PMT =	\$100.00
n	Payment(n)	FVIF	FV
1	100.00		
2	100.00		
3	100.00		

Example: Future Value of an Ordinary Annuity

N	3		
i	5%	PMT =	\$100.00
n	Payment(n)	FVIF	FV
1	100.00	110.25%	\$ 110.25
2	100.00	105.00%	\$ 105.00
3	100.00	100.00%	\$ 100.00
			\$ 315.25

Future value of an annuity due

$$FVA_{due,n} = \sum PMT (1+i)^n$$

$$= PMT(1+i\%)^3 + PMT(1+i\%)^2 + PMT(1+i\%)^1$$
 Thus, given an amount of PMT,

$$FVA_{due} = FVA_{ordinary}(1+i).$$

Example: Future Value of an annuity due

N	3		
i	5%	PMT =	\$100.00
N	Payment(N)	FVIF	FV
1	100.00	115.76%	\$ 115.76
2	100.00	110.25%	\$ 110.25
3	100.00	105.00%	\$ 105.00
	315.25	1.05	\$ 331.01

Present value of an ordinary annuity

$$PVA_{ordinary,n} = \sum PMT (1+i)^{-n}$$

$$= PMT(1+i\%)^{-1} + PMT(1+i\%)^{-2} + PMT(1+i\%)^{-3}$$

$$= PMT \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

Example: Present value of an ordinary annuity

N	3		
i	5%	PMT =	\$100.00
N	Payment	PVIF	PV
1	100.00	95.24%	\$ 95.24
2	100.00	90.70%	\$ 90.70
3	100.00	86.38%	\$ 86.38
			\$ 272.32

Present value of an annuity due

$$PVA_{due,n} = \sum PMT (1+i)^{-n}$$

$$= PMT(1+i\%)^0 + PMT(1+i\%)^{-1} + PMT(1+i\%)^{-2}$$

$$= [PMT(1+i\%)^{-1} + PMT(1+i\%)^{-2} + PMT(1+i\%)^{-3}] (1+i)$$

$$= PMT \left[\frac{1 - (1+i)^{-n}}{i} \right] (1+i)$$

Example: Present value of an annuity due

N	3		
i	5%	PMT =	\$100.00
N	Payment	PVIF	PV
1	100.00	100.00%	\$ 100.00
2	100.00	95.24%	\$ 95.24
3	100.00	90.70%	\$ 90.70
	272.32	1.05	\$ 285.94

Finding annuity payments, periods, and interest rates

- Finding annuity payments, PMT
- Finding periods, N
- Finding interest rates, i

25.88

Finding annuity payments, PMT

We need to accumulate \$10,000 and have it available 5 years from now. ($i=6%$)

- Ordinary annuity
- Annuity due

26.88

Finding annuity payments, PMT (Ordinary Annuity)

$$FV_{\text{ordinary},n} = \sum PMT(1+i)^n$$

$$\$10,000 = PMT(1+6\%)^4 + PMT(1+6\%)^3 + PMT(1+6\%)^2 + PMT(1+6\%)^1 + PMT(1+6\%)^0$$

$$PMT = \$???$$

27.88

Finding annuity payments, PMT (Ordinary Annuity)

N	5	future value	\$ 10,000.00
i	6%	PMT =	\$ 1,773.9640
N	Payment	FVIF	FV
1	1,773.9640	126%	\$ 2,239.5887
2	1,773.9640	119%	\$ 2,112.8195
3	1,773.9640	112%	\$ 1,993.2260
4	1,773.9640	106%	\$ 1,880.4018
5	1,773.9640	100%	\$ 1,773.9640
			\$ 10,000.0000

28.88

Finding annuity payments, PMT

We need to accumulate \$315.25 and have it available 3 years from now. ($i=5%$)

- Ordinary annuity
- Annuity due

29.88

Finding annuity payments, PMT (Ordinary Annuity)

$$FV_{\text{ordinary},n} = \sum PMT(1+i)^n$$

$$\$315.25 = PMT(1+5\%)^2 + PMT(1+5\%)^1 + PMT(1+5\%)^0$$

$$PMT = \$???$$

30.88

**Example: Finding annuity payments
Ordinary Annuity**

N	3		
i	5%	PMT =	\$100.00
n	Payment(n)	FVIF	FV
1	100.00	110.25%	\$ 110.25
2	100.00	105.00%	\$ 105.00
3	100.00	100.00%	\$ 100.00
			\$ 315.25

31.88

**Example: Finding annuity payments
Annuity Due**

N	3		
i	5%	PMT =	\$100.00
n	Payment(n)	FVIF	FV
1	100.00	115.76%	\$ 115.76
2	100.00	110.25%	\$ 110.25
3	100.00	105.00%	\$ 105.00
			\$ 331.01

32.88

**Example: Finding annuity payments
Ordinary Annuity**

N	3		
i	5%	PMT =	\$98.23
n	Payment(n)	FVIF	FV
1	98.23	115.76%	\$ 113.71
2	98.23	110.25%	\$ 108.30
3	98.23	105.00%	\$ 103.14
			\$ 325.15

33.88

**Finding annuity payments, PMT
(Annuity Due)**

$FV_{\text{ordinary},n} = \sum PMT(1+i)^n$
 $\$10,000 = PMT(1+6\%)^5 + PMT(1+6\%)^4 + PMT(1+6\%)^3 + PMT(1+6\%)^2 + PMT(1+6\%)^1$
PMT = \$???

34.88

**Finding annuity payments, PMT
(Annuity Due)**

N	5	future value	\$ 10,000.00
i	6%	PMT =	\$1,673.5509
N	Payment	FVIF	FV
1	1,673.5509	134%	\$ 2,239.589
2	1,673.5509	126%	\$ 2,112.819
3	1,673.5509	119%	\$ 1,993.226
4	1,673.5509	112%	\$ 1,880.402
5	1,673.5509	106%	\$ 1,773.964
			\$ 10,000.000

35.88

Finding Periods, N: Ordinary Annuity

N	5	future value	\$ 6,764.5116
i	6%	PMT =	\$1,200
N	Payment	FVIF	FV
1	1,200.0000	126%	\$ 1,514.9724
2	1,200.0000	119%	\$ 1,429.2192
3	1,200.0000	112%	\$ 1,348.3200
4	1,200.0000	106%	\$ 1,272.0000
5	1,200.0000	100%	\$ 1,200.0000
			\$ 6,764.5116

36.88

Finding Periods, N : Ordinary Annuity

N	6	future value	\$ 8,370.3822
i	6%	PMT =	\$1,200
N	Payment	FVIF	FV
1	1,200.0000	134%	\$ 1,605.8707
2	1,200.0000	126%	\$ 1,514.9724
3	1,200.0000	119%	\$ 1,429.2192
4	1,200.0000	112%	\$ 1,348.3200
5	1,200.0000	106%	\$ 1,272.0000
6	1,200.0000	100%	\$ 1,200.0000
			\$ 8,370.3822

37.88

Finding Periods, N : Ordinary Annuity

N	7	future value	\$ 10,072.6052
i	6%	PMT =	\$1,200
N	Payment	FVIF	FV
1	1,200.0000	142%	\$ 1,702.2229
2	1,200.0000	134%	\$ 1,605.8707
3	1,200.0000	126%	\$ 1,514.9724
4	1,200.0000	119%	\$ 1,429.2192
5	1,200.0000	112%	\$ 1,348.3200
6	1,200.0000	106%	\$ 1,272.0000
7	1,200.0000	100%	\$ 1,200.0000
			\$ 10,072.6052

38.88

Finding Periods, N : Annuity Due

N	6	future value	\$ 8,872.6052
i	6%	PMT =	\$1,200
N	Payment	FVIF	FV
1	1,200.0000	142%	\$ 1,702.2229
2	1,200.0000	134%	\$ 1,605.8707
3	1,200.0000	126%	\$ 1,514.9724
4	1,200.0000	119%	\$ 1,429.2192
5	1,200.0000	112%	\$ 1,348.3200
6	1,200.0000	106%	\$ 1,272.0000
			\$ 8,872.6052

39.88

Finding Periods, N : Annuity Due

N	7	future value	\$ 10,676.9615
i	6%	PMT =	\$1,200
N	Payment	FVIF	FV
1	1,200.0000	150%	\$ 1,804.3563
2	1,200.0000	142%	\$ 1,702.2229
3	1,200.0000	134%	\$ 1,605.8707
4	1,200.0000	126%	\$ 1,514.9724
5	1,200.0000	119%	\$ 1,429.2192
6	1,200.0000	112%	\$ 1,348.3200
7	1,200.0000	106%	\$ 1,272.0000
			\$ 10,676.9615

40.88

**Finding interest rates, i
Ordinary Annuity**

N	5	Future Value	\$ 10,000.0000
i	25.783945%	PMT =	\$1,200
N	Payment	FVIF	FV
1	1,200.0000	250%	\$ 3,003.8766
2	1,200.0000	199%	\$ 2,388.1240
3	1,200.0000	158%	\$ 1,898.5921
4	1,200.0000	126%	\$ 1,509.4073
5	1,200.0000	100%	\$ 1,200.0000
			\$ 10,000.0000

41.88

Perpetuities (永續年金)

- Perpetuity: a stream of equal payments at fixed intervals expected to continue forever
- PV of a perpetuity = PMT / i
- Consol
 - A perpetual bond issued by the British government to consolidate past debts
 - In general, any perpetual bond is called consol.

42.88

The Consol's Value

Face Value = \$1,000; Coupon rate = 2.5%

- $i=2.5\%$ in 1888
 $\text{Consol value}_{1888} = \$1,000 * 2.5\% / 2.5\% = \$1,000$
- $i=5.2\%$ in 2004
 $\text{Consol value}_{2004} = \$1,000 * 2.5\% / 5.2\% = \480.77

44.88

The Consol's Value

Face Value = \$1,000; Coupon rate = 2.5%

- $i=2.5\%$ in 1888
 $\text{Consol value}_{1888} = \$1,000 * 2.5\% / 2.5\% = \$1,000$
- $i=5.2\%$ in 2004
 $\text{Consol value}_{2004} = \$1,000 * 2.5\% / 5.2\% = \480.77
- $i=2\%$ now

44.88

The Consol's Value

Face Value = \$1,000; Coupon rate = 2.5%

- $i=2.5\%$ in 1888
 $\text{Consol value}_{1888} = \$1,000 * 2.5\% / 2.5\% = \$1,000$
- $i=5.2\%$ in 2004
 $\text{Consol value}_{2004} = \$1,000 * 2.5\% / 5.2\% = \480.77
- $i=2\%$ now
 $\text{Consol value} = \$1,000 * 2.5\% / 2\% = \$1,250.00$

44.88

N	i	Rate value	\$ 1,000
N	i	Payments	PV
1	100.00	80.81%	\$ 80.81
2	100.00	83.64%	\$ 83.64
3	100.00	78.13%	\$ 78.13
4	100.00	88.30%	\$ 88.30
5	100.00	82.08%	\$ 82.08
6	100.00	88.48%	\$ 88.48
7	100.00	81.33%	\$ 81.33
8	100.00	98.88%	\$ 98.88
9	100.00	92.41%	\$ 92.41
10	100.00	38.88%	\$ 38.88
11	100.00	38.88%	\$ 38.88
12	100.00	31.88%	\$ 31.88
13	100.00	38.87%	\$ 38.87
14	100.00	38.33%	\$ 38.33
15	100.00	33.94%	\$ 33.94
16	100.00	31.78%	\$ 31.78
17	100.00	18.78%	\$ 18.78
18	100.00	17.88%	\$ 17.88
19	100.00	18.38%	\$ 18.38
20	100.00	14.88%	\$ 14.88
21	100.00	13.81%	\$ 13.81
22	100.00	13.38%	\$ 13.38
23	100.00	11.17%	\$ 11.17
24	100.00	10.18%	\$ 10.18
25	100.00	8.33%	\$ 8.33
			\$ 927.70

44.88

Uneven cash flows

- Non-constant cash flows

1. Annuity plus additional final payment
2. Irregular cash flows

47.88

Annuity plus additional final payment

N	5		
i	12%	Payment =	\$ 100.00
N	Payment	PVIF	PV
1	100.00	89.29%	\$ 89.29
2	100.00	79.72%	\$ 79.72
3	100.00	71.18%	\$ 71.18
4	100.00	63.55%	\$ 63.55
5	1,100.00	56.74%	\$ 624.17
			\$ 927.90

48.88

Irregular Cash Flows

N	5		
i	12%	Payment =	\$ 100.00
N	Payment	PVIF	PV
1	100.00	89.29%	\$ 89.29
2	300.00	79.72%	\$ 239.16
3	300.00	71.18%	\$ 213.53
4	300.00	63.55%	\$ 190.66
5	500.00	56.74%	\$ 283.71
			\$1,016.35

40/89

Future value of an uneven cash flow stream

N	5		
i	12%		
N	Payment	FVIF	FV
1	100	157%	\$ 157.35
2	300	140%	\$ 421.48
3	300	125%	\$ 376.32
4	300	112%	\$ 336.00
5	500	100%	\$ 500.00
			\$ 1,791.15

50/89

Solving for i with uneven cash flows

- A trial-and-error process
 - Using financial calculator
 - Using Excel's IRR function

51/89

Solving for i with uneven CFs

i	12.5465%		
N	Payment	PVIF	PV
1	100.00	88.85%	\$ 88.85
2	300.00	78.95%	\$ 236.84
3	300.00	70.15%	\$ 210.44
4	300.00	62.33%	\$ 186.98
5	500.00	55.38%	\$ 276.89
		Cost	\$1,000.00

52/89

Finding interest rates, i Ordinary Annuity

N	5	Future Value	\$ 10,000.0000
i	25.783945%	PMT =	\$1,200
N	Payment	FVIF	FV
1	1,200.0000	250%	\$ 3,003.8766
2	1,200.0000	199%	\$ 2,388.1240
3	1,200.0000	158%	\$ 1,898.5921
4	1,200.0000	126%	\$ 1,509.4073
5	1,200.0000	100%	\$ 1,200.0000
		FV	\$ 10,000.0000

53/89

Semiannual and other compounding periods: Comparing interest rates

- Nominal (quoted or stated) interest rate (i_{NOM}) 名目利率
 - stated in contracts
 - the annual percentage rate (APR)
- Periodic rate (i_{PER}) 期間利率
 - amount of interest charged each period
 - $i_{\text{PER}} = i_{\text{NOM}} / m$ (m : 每年複利計息次數)
- Effective or equivalent annual rate (EAR) 有效年利率 $= (1 + i_{\text{NOM}} / m)^m - 1$

54/89

Semiannual & other compounding periods

- Annual compounding
 $FV = PMT (1 + i_{NOM})^n$
- Semiannual compounding
 $FV = PMT (1 + i_{NOM} / 2)^{2n}$
- Quarterly compounding
 $FV = PMT (1 + i_{NOM} / 4)^{4n}$
- Monthly compounding
 $FV = PMT (1 + i_{NOM} / 12)^{12n}$
- Daily compounding
 $FV = PMT (1 + i_{NOM} / 365)^{365n}$

Comparing interest rates

- An investment opportunity offers a nominal interest rate of 10.25% with annual compounding.
- Another one offers a nominal interest rate of 10% with semiannual compounding.
- No difference between them?**

Comparing interest rates

- An investment opportunity offers a nominal interest rate of 10.25% with annual compounding.
- Another one offers a nominal interest rate of 10% with semiannual compounding.
- No difference between them.

$(1 + 10.25\%)^1 = 110.25\%$
 $(1 + \frac{10\%}{2})^{1 \times 2} = 110.25\%$

Fractional Time Periods

Suppose that you deposited \$1 million in a bank that pays a nominal rate of 12%. How much would you get from the bank after 274 days, i.e. 9 months?

- Based on a 365-day year 按日计息
- Based on a 12-month year 按月计息

Fractional Time Periods

Suppose that you deposited \$1 million in a bank that pays a nominal rate of 12%. How much would you get from the bank after 274 days, i.e. 9 months?

- Based on a 365-day year 按日计息
 - Periodical (daily) rate = $12\% / 365$
 - Number of days = $(9/12) \times 365 = 273.75 \approx 274$
 - Ending amount = $\$100 (1 + 12\% / 365)^{274} = \109.4248

Fractional Time Periods

Suppose that you deposited \$1 million in a bank that pays a nominal rate of 12%. How much would you get from the bank after 274 days, i.e. 9 months?

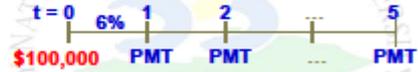
- Based on a 365-day year 按日计息
 - Periodical (daily) rate = $12\% / 365$
 - Number of days = $(9/12) \times 365 = 273.75 \approx 274$
 - Ending amount = $\$100 (1 + 12\% / 365)^{274} = \109.4248
- Based on a 12-month year 按月计息
 - Periodical (monthly) rate = $12\% / 12 = 1\%$
 - Ending amount = $\$100 (1 + 12\% / 12)^9 = \109.3685

Loan Amortization 借(貸)款攤還

- Amortization tables are widely used for home mortgages, auto loans, business loans, retirement plans, etc.
- Fully amortized loan**
An amortization schedule for a \$100,000 6% annual rate loan with 5 equal annual payments.

61/89

Full Amortized Loan



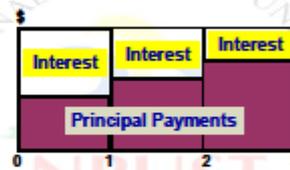
62/89

Constructing a loan amortization table

N	5		PV	\$ 100,000	
i	6%		PMT =	\$ 23,739.64	
Amortization Table for the loan					
n	Loan amort.	Payment	Interest	Principal	Balance
1	\$ 100,000.00	\$ 23,739.64	\$ 6,000.00	\$ 17,739.64	\$ 82,260.36
2	\$ 82,260.36	\$ 23,739.64	\$ 4,935.62	\$ 18,804.02	\$ 63,456.34
3	\$ 63,456.34	\$ 23,739.64	\$ 3,807.38	\$ 19,932.26	\$ 43,524.08
4	\$ 43,524.08	\$ 23,739.64	\$ 2,611.44	\$ 21,128.20	\$ 22,395.89
5	\$ 22,395.89	\$ 23,739.64	\$ 1,343.75	\$ 22,395.89	\$ 0.00

63/89

Where does the money go?



- Constant payments
- Declining interest payments
- Declining balance

64/89

附件四：Bond Valuation (共 15 頁)



What is a bond?

- A long-term debt instrument
 - Original maturity longer than 1 year
 - A contract between the borrower (債權人) and the lender (債權人)
- The borrower agrees to make payments of principal and interest, on specific dates, to the holders of the bond, i.e. the lenders.

Who issues bonds?

- Governments
 - Including federal or central government and local or state governments
 - Treasury bonds and municipal bonds (munis)
- Corporations
 - Corporate bonds
- Foreign governments and corporations
 - Foreign bonds

Key Characteristics of Bonds

- Par value
 - State of face value of the bond
 - Paid at maturity
- Coupon interest rate
 - The stated annual interest rate on a bond
 - Multiply by par to get dollar payment of interest
- Maturity date
- Call provisions
- Sinking fund provisions

Other Features of Bonds

- Convertible bond
 - May be exchanged for common stock of the firm
- Warrant
 - An option to buy a certain number of shares of common stock at a specified price
- Puttable bond
 - Allowing holders to sell the bond back to the issuer prior to maturity
- Interest bond
 - Paying interest only when interest is needed by the firm
- Indexed, or participating, premium, bond
 - Interest rate paid to bondholders is rate + inflation

The Value of Financial Assets

$$\text{Value}_{\text{Asset}} = \frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \dots + \frac{CF_n}{(1+k)^n}$$

The Value of A Bond

$$\text{Value}_{\text{Bond}} = \frac{INT}{(1+r_d)^1} + \frac{INT}{(1+r_d)^2} + \dots + \frac{INT + M}{(1+r_d)^n}$$

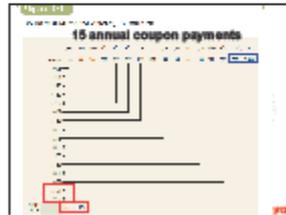
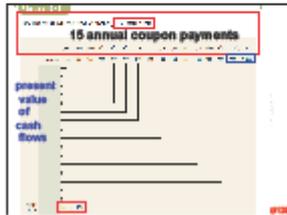
15 annual coupon payments

Timeline diagram showing 15 annual coupon payments (INT) and a final payment of INT + M at time 15.

15 annual coupon payments

Cash Flows generated from the bond investment

Timeline diagram showing 15 annual coupon payments (INT) and a final payment of INT + M at time 15.



Coupon = 10.00%		Face value = \$ 1,000	
Yield rate = 10.00%	mm = 1		
	CPI	PVCF	PV
1	100.00	0.909091	90.91
2	100.00	0.826450	82.64
3	100.00	0.751310	75.13
4	100.00	0.683013	68.30
5	100.00	0.620921	62.09
6	100.00	0.564473	56.45
7	100.00	0.513160	51.32
8	100.00	0.466807	46.68
9	100.00	0.425090	42.51
10	100.00	0.387507	38.75
11	100.00	0.353664	35.37
12	100.00	0.323181	32.32
13	100.00	0.295696	29.57
14	100.00	0.270831	27.08
15	1,100.00	0.248300	273.33
Bond price			1,000.00

Coupon = 10.00%		Face value = \$ 1,000	
Yield rate = 10.00%	mm = 1		
	CPI	PVCF	PV
1	100.00	0.909091	90.91
2	100.00	0.826450	82.64
3	100.00	0.751310	75.13
4	100.00	0.683013	68.30
5	100.00	0.620921	62.09
6	100.00	0.564473	56.45
7	100.00	0.513160	51.32
8	100.00	0.466807	46.68
9	100.00	0.425090	42.51
10	100.00	0.387507	38.75
11	100.00	0.353664	35.37
12	100.00	0.323181	32.32
13	100.00	0.295696	29.57
14	100.00	0.270831	27.08
15	1,100.00	0.248300	273.33
Bond price			1,000.00

Change in Interest Rate & Bond Price

- Par bond
 - A bond is sold at par.
 - The going rate of interest = coupon rate
- Discount bond
 - A bond is sold below its par value.
 - The going rate of interest > coupon rate
- Premium bond
 - A bond is sold above its par value.
 - The going rate of interest < coupon rate

Coupon = 10.00%		Face value = \$ 1,000	
Yield rate = 10.00%	mm = 1		
	CPI	PVCF	PV
1	100.00	0.909091	90.91
2	100.00	0.826450	82.64
3	100.00	0.751310	75.13
4	100.00	0.683013	68.30
5	100.00	0.620921	62.09
6	100.00	0.564473	56.45
7	100.00	0.513160	51.32
8	100.00	0.466807	46.68
9	100.00	0.425090	42.51
10	100.00	0.387507	38.75
11	100.00	0.353664	35.37
12	100.00	0.323181	32.32
13	100.00	0.295696	29.57
14	100.00	0.270831	27.08
15	1,100.00	0.248300	273.33
Bond price			1,000.00

Change in Interest Rate & Bond Price

- Par bond
 - A bond is sold at par.
 - The going rate of interest = coupon rate
- Discount bond
 - A bond is sold below its par value.
 - The going rate of interest > coupon rate
- Premium bond
 - A bond is sold above its par value.
 - The going rate of interest < coupon rate

Coupon = 10.00%		Face value = \$ 1,000	
Yield rate = 10.00%	mm = 1		
	CPI	PVCF	PV
1	100.00	0.909091	90.91
2	100.00	0.826450	82.64
3	100.00	0.751310	75.13
4	100.00	0.683013	68.30
5	100.00	0.620921	62.09
6	100.00	0.564473	56.45
7	100.00	0.513160	51.32
8	100.00	0.466807	46.68
9	100.00	0.425090	42.51
10	100.00	0.387507	38.75
11	100.00	0.353664	35.37
12	100.00	0.323181	32.32
13	100.00	0.295696	29.57
14	100.00	0.270831	27.08
15	1,100.00	0.248300	273.33
Bond price			1,000.00

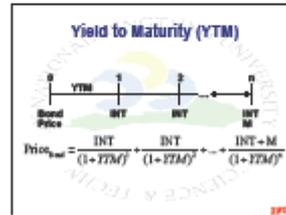
Change in Interest Rate & Bond Price

- Par bond
 - A bond is sold at par.
 - The going rate of interest = coupon rate
- Discount bond
 - A bond is sold below its par value.
 - The going rate of interest > coupon rate
- Premium bond
 - A bond is sold above its par value.
 - The going rate of interest < coupon rate

Coupon = 10.00%		Face value = \$1,000	
Yield or $r_m = 6.00%$		$m = 1$	
t	CR	PVCF	PVt
1	\$ 100.00	0.939956	93.99
2	\$ 100.00	0.883256	88.33
3	\$ 100.00	0.829832	82.98
4	\$ 100.00	0.779280	77.93
5	\$ 100.00	0.731193	73.12
6	\$ 100.00	0.685170	68.52
7	\$ 100.00	0.641820	64.18
8	\$ 100.00	0.600850	60.09
9	\$ 100.00	0.562060	56.21
10	\$ 100.00	0.525240	52.52
11	\$ 100.00	0.490200	49.02
12	\$ 100.00	0.456750	45.68
13	\$ 100.00	0.424710	42.47
14	\$ 100.00	0.393900	39.39
15	\$ 1,100.00	0.364250	364.25
		Bond price	1,234.55

Bond Yields 債券收益率

- Yield to maturity (YTM) 到期收益率**
= The rate of return earned on a bond if it is held to maturity
- Yield to call (YTC) 贖回收益率**
= Price of callable bond
- Current yield 現行收益率**
= The annual interest payment on a bond divided by the bond's current price



Coupon = 10.00%		Face value = \$1,000	
Yield or $r_m = 6.00%$		$m = 1$	
t	CR	PVCF	PVt
1	\$ 100.00	0.939956	93.99
2	\$ 100.00	0.883256	88.33
3	\$ 100.00	0.829832	82.98
4	\$ 100.00	0.779280	77.93
5	\$ 100.00	0.731193	73.12
6	\$ 100.00	0.685170	68.52
7	\$ 100.00	0.641820	64.18
8	\$ 100.00	0.600850	60.09
9	\$ 100.00	0.562060	56.21
10	\$ 100.00	0.525240	52.52
11	\$ 100.00	0.490200	49.02
12	\$ 100.00	0.456750	45.68
13	\$ 100.00	0.424710	42.47
14	\$ 100.00	0.393900	39.39
15	\$ 1,100.00	0.364250	364.25
		Bond price	1,234.55

Example: Yield to Maturity (YTM)

Coupon = 10.00% Face value = \$1,000
Yield = 6.00% $m = 1$

t	CR	PVCF	PVt
1	\$ 100.00		
2	\$ 100.00		
3	\$ 100.00		
4	\$ 100.00		
5	\$ 1,100.00		
		Bond price	

Coupon = 10.00%		Face value = \$1,000	
Yield = 6.00%		$m = 1$	
t	CR	PVCF	PVt
1	\$ 100.00	0.939956	93.99
2	\$ 100.00	0.883256	88.33
3	\$ 100.00	0.829832	82.98
4	\$ 100.00	0.779280	77.93
5	\$ 1,100.00	0.731193	731.19
		Bond price	1,234.55

Example: Yield to Maturity (YTM)

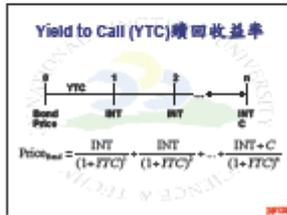
Coupon = 10.00%		Face value = \$1,000	
Yield = 6.00%		$m = 1$	
t	CR	PVCF	PVt
1	\$ 100.00	0.940596	94.06
2	\$ 100.00	0.890666	89.07
3	\$ 100.00	0.840619	84.07
4	\$ 100.00	0.790264	79.07
5	\$ 1,100.00	0.740228	740.23
		Bond price	1,234.55

Example: Yield to Maturity (YTM)

Coupon = 10.00%		Face value = \$1,000	
Yield = 10.00%		$m = 1$	
t	CR	PVCF	PVt
1	\$ 100.00	0.909091	90.91
2	\$ 100.00	0.826446	82.64
3	\$ 100.00	0.751315	75.13
4	\$ 100.00	0.682913	68.29
5	\$ 1,100.00	0.620921	620.92
		Bond price	1,000.00

Example: Yield to Maturity (YTM)

Coupon = 10.00%		Face value = \$1,000	
Yield = 12.00%		$m = 1$	
t	CR	PVCF	PVt
1	\$ 100.00	0.892857	89.29
2	\$ 100.00	0.797194	79.72
3	\$ 100.00	0.711779	71.18
4	\$ 100.00	0.636418	63.64
5	\$ 1,100.00	0.567427	567.43
		Bond price	827.90



Coupon = 10.00%		Face value = \$ 1,000	
Yield or $y_c = 4.2549%$		$m = 1$	
t	CFt	PV(Ft)	call price
1	\$ 100.00	0.959556	95.96
2	\$ 100.00	0.920747	92.07
3	\$ 100.00	0.883590	88.36
4	\$ 100.00	0.847775	84.78
5	\$ 100.00	0.813280	81.33
6	\$ 100.00	0.780027	78.00
7	\$ 100.00	0.748006	74.80
8	\$ 100.00	0.717223	71.87
9	\$ 1,100.00	0.687635	827.59
Bond price			1,494.83

Example: Yield to Call (YTC)

- YTM (k_d) = 12%

Coupon = 10.00%		Face value = \$ 1,000	
Yield = 12.00%		$m = 1$	
t	CFt	PV(Ft)	call price
1	\$ 100.00	0.892857	89.29
2	\$ 100.00	0.797184	79.72
3	\$ 100.00	0.711739	71.18
4	\$ 100.00	0.635519	63.55
5	\$ 1,000.00	0.567427	567.43
Bond price			927.36

Example: Yield to Call (YTC)

- YTM (k_d) = 12%

Coupon = 10.00%		Face value = \$ 1,000	
YTC = 13.6719%		$m = 1$	
t	CFt	PV(Ft)	call price
1	\$ 100.00	0.879726	87.97
2	\$ 100.00	0.773917	77.39
3	\$ 1,100.00	0.688835	762.54
4	\$ -	0.598948	0.00
5	\$ -	0.528919	0.00
Bond price			927.36

Example: Yield to Call (YTC)

- YTM (k_d) = 12%

Coupon = 10.00%		Face value = \$ 1,000	
YTC = 13.6719%		$m = 1$	
t	CFt	PV(Ft)	call price
1	\$ 100.00	0.879726	87.97
2	\$ 100.00	0.773917	77.39
3	\$ 1,100.00	0.688835	762.54
4	\$ -	0.598948	0.00
5	\$ -	0.528919	0.00
Bond price			927.36

What's your refunding strategy if interest rates are going to change?

- A decrease in bond yield
12% \rightarrow 10%
- An increase in bond yield
12% \rightarrow 16%

Example: Yield to Call (YTC)

- YTM (k_d) = 16%

Coupon = 10.00%		Face value = \$ 1,000	
Yield = 16.00%		$m = 1$	
t	CFt	PV(Ft)	call price
1	\$ 100.00	0.854804	85.48
2	\$ 100.00	0.736896	73.69
3	\$ 100.00	0.635255	63.53
4	\$ 100.00	0.547547	54.75
5	\$ 1,000.00	0.463362	463.36
Bond price			1,000.00

Example: Yield to Call (YTC)

- YTM (k_d) = 16%

Coupon = 10.00%		Face value = \$ 1,000	
YTC = 16.0007%		$m = 1$	
t	CFt	PV(Ft)	call price
1	\$ 100.00	0.854803	85.42
2	\$ 100.00	0.736893	73.75
3	\$ 1,100.00	0.635259	827.84
4	\$ -	0.547545	0.00
5	\$ -	0.463362	0.00
Bond price			1,000.00

Example: Yield to Call (YTC)

- YTM (k_d) = 16%

Coupon = 10.00%		Face value = \$ 1,000	
YTC = 16.0007%		$m = 1$	
t	CFt	PV(Ft)	call price
1	\$ 100.00	0.854803	85.42
2	\$ 100.00	0.736893	73.75
3	\$ 1,100.00	0.635259	827.84
4	\$ -	0.547545	0.00
5	\$ -	0.463362	0.00
Bond price			1,000.00

Current Yield

Coupon = 7.00%
Yield = 9.00%
Face value = \$1,000
m = 1

Year	CPI	PVCF	PVt
1	70.00	0.659091	46.13
2	70.00	0.604660	47.35
3	70.00	0.559315	48.58
4	70.00	0.513970	49.81
5	70.00	0.468625	51.04
6	70.00	0.423280	52.27
7	70.00	0.377935	53.50
8	70.00	0.332590	54.73
9	70.00	0.287245	55.96
10	70.00	0.241900	57.19
11	70.00	0.196555	58.42
12	1,070.00	0.151210	59.65
		Red price	795.99

Current Yield = $70 / 795.99 = 8.8\%$

Expected Capital Gain Yield

Coupon = 7.00%
Yield = 9.00%
Face value = \$1,000
m = 1

Expected capital gain yield for one year, given no change in interest rate and assuming market equilibrium

$= 1\%$

Year	CPI	PVCF	PVt
1	70.00	0.659091	46.13
2	70.00	0.604660	47.35
3	70.00	0.559315	48.58
4	70.00	0.513970	49.81
5	70.00	0.468625	51.04
6	70.00	0.423280	52.27
7	70.00	0.377935	53.50
8	70.00	0.332590	54.73
9	70.00	0.287245	55.96
10	70.00	0.241900	57.19
11	70.00	0.196555	58.42
12	1,070.00	0.151210	59.65
		Red price	795.99

Yield to Maturity

Yield to Maturity	Expected Current Yield	Expected Capital Gain Yield	Expected Total Return
1%	8.8%	1%	9.8%
2%	8.8%	1%	9.8%
3%	8.8%	1%	9.8%
4%	8.8%	1%	9.8%
5%	8.8%	1%	9.8%
6%	8.8%	1%	9.8%
7%	8.8%	1%	9.8%
8%	8.8%	1%	9.8%
9%	8.8%	1%	9.8%
10%	8.8%	1%	9.8%
11%	8.8%	1%	9.8%
12%	8.8%	1%	9.8%

Coupon = 7.00%
Yield = 9.00%
Face value = \$1,000
m = 1

Year	CPI	PVCF	PVt
1	70.00	0.659091	46.13
2	70.00	0.604660	47.35
3	70.00	0.559315	48.58
4	70.00	0.513970	49.81
5	70.00	0.468625	51.04
6	70.00	0.423280	52.27
7	70.00	0.377935	53.50
8	70.00	0.332590	54.73
9	70.00	0.287245	55.96
10	70.00	0.241900	57.19
11	70.00	0.196555	58.42
12	1,070.00	0.151210	59.65
		Red price	795.99

Expected Capital Gain Yield

Coupon = 7.00%
Yield = 9.00%
Face value = \$1,000
m = 1

Expected capital gain yield for one year, given no change in interest rate and assuming market equilibrium

$= (795.99 - 788.9) / 788.9 = 0.9\%$

Year	CPI	PVCF	PVt
1	70.00	0.659091	46.13
2	70.00	0.604660	47.35
3	70.00	0.559315	48.58
4	70.00	0.513970	49.81
5	70.00	0.468625	51.04
6	70.00	0.423280	52.27
7	70.00	0.377935	53.50
8	70.00	0.332590	54.73
9	70.00	0.287245	55.96
10	70.00	0.241900	57.19
11	70.00	0.196555	58.42
12	1,070.00	0.151210	59.65
		Red price	795.99

Yield to Maturity

Yield to Maturity	Expected Current Yield	Expected Capital Gain Yield	Expected Total Return
1%	8.8%	0.9%	9.7%
2%	8.8%	0.9%	9.7%
3%	8.8%	0.9%	9.7%
4%	8.8%	0.9%	9.7%
5%	8.8%	0.9%	9.7%
6%	8.8%	0.9%	9.7%
7%	8.8%	0.9%	9.7%
8%	8.8%	0.9%	9.7%
9%	8.8%	0.9%	9.7%
10%	8.8%	0.9%	9.7%
11%	8.8%	0.9%	9.7%
12%	8.8%	0.9%	9.7%

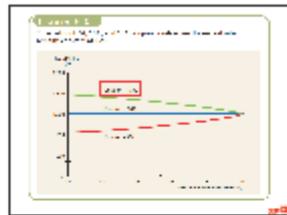
Expected Total Return

Coupon = 7.00%
Yield = 9.00%
Face value = \$1,000
m = 1

Expected total return for one year, given no change in interest rate and assuming market equilibrium

= Current Yield + Exp. Capital Gain Yield = $8.8\% + 0.9\% = 9.7\%$

Year	CPI	PVCF	PVt
1	70.00	0.659091	46.13
2	70.00	0.604660	47.35
3	70.00	0.559315	48.58
4	70.00	0.513970	49.81
5	70.00	0.468625	51.04
6	70.00	0.423280	52.27
7	70.00	0.377935	53.50
8	70.00	0.332590	54.73
9	70.00	0.287245	55.96
10	70.00	0.241900	57.19
11	70.00	0.196555	58.42
12	1,070.00	0.151210	59.65
		Red price	795.99

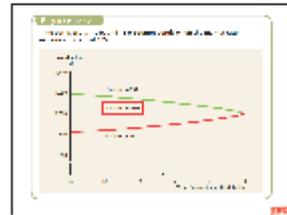


Coupon = 7.00%
Yield = 9.00%
Face value = \$1,000
m = 1

Year	CPI	PVCF	PVt
1	70.00	0.659091	46.13
2	70.00	0.604660	47.35
3	70.00	0.559315	48.58
4	70.00	0.513970	49.81
5	70.00	0.468625	51.04
6	70.00	0.423280	52.27
7	70.00	0.377935	53.50
8	70.00	0.332590	54.73
9	70.00	0.287245	55.96
10	70.00	0.241900	57.19
11	70.00	0.196555	58.42
12	1,070.00	0.151210	59.65
		Red price	795.99

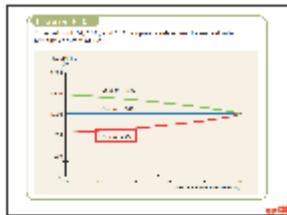
Coupon = 10.00%			
Yield = 10.00%			
Face value = \$ 1,000			
m = 1			
t	CPI	PMVP	PVI
1	100.00	0.999991	99.99
2	100.00	0.99988	99.98
3	100.00	0.99976	99.97
4	100.00	0.99963	99.96
5	100.00	0.99950	99.95
6	100.00	0.99937	99.93
7	100.00	0.99924	99.92
8	100.00	0.99911	99.91
9	100.00	0.99898	99.90
10	100.00	0.99885	99.89
11	100.00	0.99872	99.88
12	100.00	0.99859	99.87
13	1,100.00	0.99846	99.86
			1,000.00

Coupon = 10.00%			
Yield = 10.00%			
Face value = \$ 1,000			
m = 1			
t	CPI	PMVP	PVI
1	100.00	0.999991	99.99
2	100.00	0.99988	99.98
3	100.00	0.99976	99.97
4	1,100.00	0.99963	99.96
			1,000.00



Coupon = 10.00%			
Yield = 10.00%			
Face value = \$ 1,000			
m = 1			
t	CPI	PMVP	PVI
1	100.00	0.999991	99.99
2	100.00	0.99988	99.98
3	100.00	0.99976	99.97
4	100.00	0.99963	99.96
5	100.00	0.99950	99.95
6	100.00	0.99937	99.93
7	100.00	0.99924	99.92
8	100.00	0.99911	99.91
9	100.00	0.99898	99.90
10	100.00	0.99885	99.89
11	100.00	0.99872	99.88
12	100.00	0.99859	99.87
13	100.00	0.99846	99.86
14	100.00	0.99833	99.85
15	1,100.00	0.99820	99.84
			1,000.00

Coupon = 10.00%			
Yield = 10.00%			
Face value = \$ 1,000			
m = 1			
t	CPI	PMVP	PVI
1	100.00	0.999991	99.99
2	100.00	0.99988	99.98
3	100.00	0.99976	99.97
4	100.00	0.99963	99.96
5	100.00	0.99950	99.95
6	100.00	0.99937	99.93
7	100.00	0.99924	99.92
8	100.00	0.99911	99.91
9	100.00	0.99898	99.90
10	100.00	0.99885	99.89
11	100.00	0.99872	99.88
12	100.00	0.99859	99.87
13	100.00	0.99846	99.86
14	1,100.00	0.99833	99.85
			1,000.00



Coupon = 7.50%			
Yield = 10.00%			
Face value = \$ 1,000			
m = 1			
t	CPI	PMVP	PVI
1	70.00	0.999991	69.99
2	70.00	0.99988	69.98
3	70.00	0.99976	69.97
4	70.00	0.99963	69.96
5	70.00	0.99950	69.95
6	70.00	0.99937	69.93
7	70.00	0.99924	69.92
8	70.00	0.99911	69.91
9	70.00	0.99898	69.90
10	70.00	0.99885	69.89
11	70.00	0.99872	69.88
12	70.00	0.99859	69.87
13	70.00	0.99846	69.86
14	70.00	0.99833	69.85
15	1,070.00	0.99820	69.84
			971.90

Coupon = 7.50%			
Yield = 10.00%			
Face value = \$ 1,000			
m = 1			
t	CPI	PMVP	PVI
1	70.00	0.999991	69.99
2	70.00	0.99988	69.98
3	70.00	0.99976	69.97
4	70.00	0.99963	69.96
5	70.00	0.99950	69.95
6	70.00	0.99937	69.93
7	70.00	0.99924	69.92
8	70.00	0.99911	69.91
9	70.00	0.99898	69.90
10	70.00	0.99885	69.89
11	70.00	0.99872	69.88
12	70.00	0.99859	69.87
13	1,070.00	0.99846	69.86
			971.90

Coupon = 7.00%		Face value = \$1,000	
Yield = 10.00%		n = 4	
t	CPI	PMFt	PVt
1	\$ 70.00	0.909091	63.64
2	\$ 70.00	0.826446	57.85
3	\$ 70.00	0.751315	52.59
4	\$ 1,070.00	0.683913	738.82
Bond price			934.90

Bonds with Semiannual Coupons

- The value of a semiannual bond

$$V_{\text{Bond}} = \sum_{t=1}^{2n} \frac{\text{INT}/2}{(1+r_f/2)^t} + \frac{M}{(1+r_f/2)^{2n}}$$

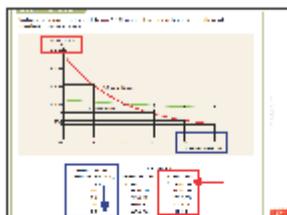
Coupon = 10.00%		Face value = \$1,000	
Yield = 10.00%		n = 4	
t	CPI	PMFt	PVt
1	\$ 100.00	0.909091	90.91
2	\$ 100.00	0.826446	82.64
3	\$ 100.00	0.751315	75.13
4	\$ 100.00	0.683913	68.39
5	\$ 100.00	0.624568	62.46
6	\$ 100.00	0.572031	57.20
7	\$ 100.00	0.526104	52.61
8	\$ 100.00	0.485644	48.56
9	\$ 100.00	0.449414	44.94
10	\$ 100.00	0.417266	41.73
11	\$ 100.00	0.388044	38.80
12	\$ 100.00	0.361693	36.17
13	\$ 100.00	0.338067	33.81
14	\$ 100.00	0.316933	31.70
15	\$ 100.00	0.297177	29.72
16	\$ 100.00	0.278696	27.87
17	\$ 100.00	0.261301	26.13
18	\$ 100.00	0.245821	24.58
19	\$ 100.00	0.232199	23.22
20	\$ 1,100.00	0.220136	242.14
Bond price			1,000.00

Coupon = 7.00%		Face value = \$1,000	
Yield = 10.00%		n = 1	
t	CPI	PMFt	PVt
1	\$ 70.00	0.909091	63.64
2	\$ 70.00	0.826446	57.85
3	\$ 70.00	0.751315	52.59
4	\$ 1,070.00	0.683913	738.82
Bond price			934.90

Coupon = 7.00%		Face value = \$1,000	
Yield = 10.00%		n = 2	
t	CPI	PMFt	PVt
1	\$ 35.00	0.952381	33.33
2	\$ 35.00	0.869565	31.75
3	\$ 35.00	0.793832	28.23
4	\$ 35.00	0.724638	25.79
5	\$ 35.00	0.661576	23.42
6	\$ 35.00	0.604170	21.15
7	\$ 35.00	0.551972	19.07
8	\$ 1,035.00	0.503683	520.53
Bond price			902.95

Assessing a Bond's Riskiness

- Interest rate risk → Figure 3
- The current market value of the bond portfolio



Coupon = 10.00%		Face value = \$1,000	
Yield = 10.00%		n = 5	
t	CPI	PMFt	PVt
1	\$ 100.00	0.909091	90.91
2	\$ 100.00	0.826446	82.64
3	\$ 100.00	0.751315	75.13
4	\$ 100.00	0.683913	68.39
5	\$ 100.00	0.624568	62.46
6	\$ 100.00	0.572031	57.20
7	\$ 100.00	0.526104	52.61
8	\$ 100.00	0.485644	48.56
9	\$ 100.00	0.449414	44.94
10	\$ 100.00	0.417266	41.73
11	\$ 100.00	0.388044	38.80
12	\$ 100.00	0.361693	36.17
13	\$ 100.00	0.338067	33.81
14	\$ 100.00	0.316933	31.70
15	\$ 1,100.00	0.297177	297.18
Bond price			1,000.00

Coupon = 10.00%		Face value = \$1,000	
Yield = 10.00%		n = 1	
t	CPI	PMFt	PVt
1	\$ 100.00	0.909091	90.91
2	\$ 100.00	0.826446	82.64
3	\$ 100.00	0.751315	75.13
4	\$ 100.00	0.683913	68.39
5	\$ 100.00	0.624568	62.46
6	\$ 100.00	0.572031	57.20
7	\$ 100.00	0.526104	52.61
8	\$ 100.00	0.485644	48.56
9	\$ 100.00	0.449414	44.94
10	\$ 100.00	0.417266	41.73
11	\$ 100.00	0.388044	38.80
12	\$ 100.00	0.361693	36.17
13	\$ 100.00	0.338067	33.81
14	\$ 100.00	0.316933	31.70
15	\$ 1,100.00	0.297177	297.18
Bond price			1,000.00

Coupon = 10.00%		Face value = \$1,000	
Yield = 10.00%		n = 1	
t	CPI	PMPV	PV
1	0	0.000000	95.00
2	0	0.781548	74.01
3	0	0.607916	64.79
4	0	0.477438	57.09
5	0	0.367677	50.75
6	0	0.283208	45.53
7	0	0.218867	41.29
8	0	0.169903	37.89
9	0	0.132493	35.09
10	0	0.102708	32.72
11	0	0.078443	30.79
12	0	0.100000	39.09
13	0	0.169309	56.39
14	0	0.148339	44.43
15	1,100.00	0.110000	133.09
		Bond price	707.28

Coupon = 10.00%		Face value = \$1,000	
Yield = 10.00%		n = 1	
t	CPI	PMPV	PV
1	0	0.000000	95.33
2	0	0.600000	45.45
3	0	0.477964	37.87
4	0	0.367333	34.33
5	0	0.281876	31.55
6	0	0.209900	29.49
7	0	0.150000	27.68
8	0	0.109900	26.39
9	0	0.100000	26.39
10	0	0.100000	26.39
11	0	0.100000	26.39
12	0	0.110000	27.63
13	0	0.169309	45.39
14	0	0.148339	37.79
15	1,100.00	0.110000	74.49
		Bond price	287.28

Coupon = 10.00%		Face value = \$1,000	
Yield = 10.00%		n = 1	
t	CPI	PMPV	PV
1	0	0.000000	95.00
2	0	0.600000	45.00
3	0	0.410000	34.39
4	0	0.280000	29.09
5	0	0.207000	24.77
6	0	0.150000	21.21
7	0	0.109000	18.07
8	0	0.107700	16.79
9	0	0.104200	15.43
10	0	0.107000	16.74
11	0	0.080000	8.08
12	0	0.087000	8.87
13	0	0.040000	3.00
14	0	0.040000	4.49
15	1,100.00	0.090000	28.79
		Bond price	287.21

Assessing a Bond's Riskiness

- Interest rate risk → Figure 3
 - The current market value of the bond portfolio
 - The higher the interest rate, the lower the bond price.
- Reinvestment rate risk
 - The income produced by the bond portfolio
- Comparing interest rate and reinvestment rate risk based on investment horizon 投資期限
 - Short-term bonds → Higher reinvestment rate risk
 - Long-term bonds → Higher interest rate risk

Default Risk

We discuss bond's default risk in terms of:

- Various types of corporate bonds
- Bond ratings
- Bankruptcy 倒閉 and reorganization 重整

Various Types of Corporate Bonds

- Mortgage bonds 抵押債券
 - Subprime 次級債券
- Debentures 無擔保債券
- Subordinated debentures 次順位債券
- Investment-grade bonds 投資等級債券
- Junk bonds 垃圾債券

Bond Ratings 債券評等

Based on both qualitative and quantitative factors, bond ratings are designed to reflect the probability of a bond issuer going into default.

Rating Agency	Investment-Grade Bonds 投資等級債券	Junk Bonds 垃圾債券
Moody's	Aaa Aa Aa1 Aa2	Ba B Caa C
S&P	AAA AA+ AA A BBB	BB B CCC D

Bond Rating Criteria

- Quantitative factors
 - Financial ratios
- Qualitative factors
 - Bond contract terms
 - Restrictive covenants
- Miscellaneous qualitative factors
 - Environmental factors, Product liability, Pension liability, Labor unrest and Potential ambient problem, and International operations

Table 10-10: Bond Rating 1990-2009 Annual % of Total Issuance

	A	A-	A+	B+	B	B-	C	D
Total Issuance	10	10	10	10	10	10	10	10
Investment-Grade	10	10	10	10	10	10	10	10
Subprime	10	10	10	10	10	10	10	10
Mortgage	10	10	10	10	10	10	10	10
Other	10	10	10	10	10	10	10	10

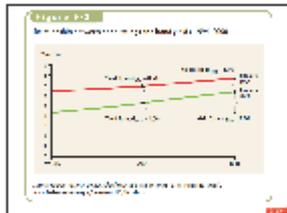
Importance of Bond Ratings

- Ratings are important both to issuers and to investors.
- Institutional investors' concern
 - Investment-grade sensitive
- Figure 4 shows the importance of required rate of return on bonds.
 - Spread or default risk premium
 - Spread may vary with macroeconomic conditions.



Importance of Bond Ratings

- Ratings are important both to issuers and to investors.
- Institutional investors' concern
 - Investment-grade sensitive
- Figure 4 shows the importance of required rate of return on bonds.
 - Spread or default risk premium
- Figure 5 reflects:
 - The decline in both real interest rates and expected inflation over time
 - The change in investors' risk aversion



Bankruptcy 倒産 and Reorganization 重整

- Insolvency
 - A business does not have enough cash to meet its interest and principal payments.
- The choice of insolvency case
 - Chapter 11 Reorganization → restructuring the debt
 - Chapter 7 Liquidation 清算
- Typically, a company wants Chapter 11, while creditors may prefer Chapter 7.

Bond Markets

- Corporate bonds are traded primarily in the OTC market.
 - Information on bond markets in the OTC market is not published.
- Some bonds are listed and traded on the bond division of the NYSE.
 - Table 4 shows the "corporate bonds" section from The Wall Street Journal.
 - Coupon rates are set at levels that reflect the going rate of interest on the day the bonds are issued.

Table 4: A summary of the corporate bond market, 10/1/07

Market	Total	Outstanding	Issued	Issued
	(\$Bn)	(\$Bn)	(\$Bn)	(\$Bn)
Invested	10,000	10,000	10,000	10,000
Open	100	100	100	100
1-3	100	100	100	100
3-6	100	100	100	100
6-12	100	100	100	100
1-3	100	100	100	100
3-6	100	100	100	100
6-12	100	100	100	100
Other	100	100	100	100

Table 4: A summary of the corporate bond market, 10/1/07

Market	Total	Outstanding	Issued	Issued
	(\$Bn)	(\$Bn)	(\$Bn)	(\$Bn)
Invested	10,000	10,000	10,000	10,000
Open	100	100	100	100
1-3	100	100	100	100
3-6	100	100	100	100
6-12	100	100	100	100
1-3	100	100	100	100
3-6	100	100	100	100
6-12	100	100	100	100
Other	100	100	100	100

Table 4: A summary of the corporate bond market, 10/1/07

Market	Total	Outstanding	Issued	Issued
	(\$Bn)	(\$Bn)	(\$Bn)	(\$Bn)
Invested	10,000	10,000	10,000	10,000
Open	100	100	100	100
1-3	100	100	100	100
3-6	100	100	100	100
6-12	100	100	100	100
1-3	100	100	100	100
3-6	100	100	100	100
6-12	100	100	100	100
Other	100	100	100	100

Bond Price Volatility

Main factors determine a bond's price volatility to changing interest rates:

- Time to maturity
- Coupon
- Prevailing level of market interest rate

NPST UNIVERSITY OF SCIENCE & TECHNOLOGY

Bond Price Volatility

- **The maturity effect**
 - The longer the time to maturity, the greater a bond's price sensitivity.
 - Price volatility increases at a decreasing rate with maturity.
- **The coupon effect**
 - The **greater** the coupon rate, the **lower** a bond's price sensitivity.
- **The yield level effect**
 - For the same change in bond yield, there is **greater price sensitivity of lower yield bonds.**

Coupon	6.00%	Face value	\$ 1,000		
Yield	6.00%	# of payment	2		
1	CPY	PPY	PPY	SP-CPY	
1	\$ 30.00	0.0706759	29.10	29.10	68.20
2	\$ 30.00	0.1302559	26.26	66.00	104.47
3	\$ 30.00	0.1819147	27.45	62.26	129.46
4	\$ 1,000.00	0.3004879	915.16	288.87	1,000.00
	Price	\$ 1,000.00	282.65	1,000.00	
	Duration in half years	3.23291155	Convexity	4.444268	
	Macaulay's Duration	1.61645578			
	Modified Duration	1.60894919			
	Yield change (%)	1.000000%			
	Modified Duration estimate	-1.608949%			\$ 160.89

Coupon	6.00%	Face value	\$ 1,000		
Yield	6.00%	# of payment	2		
1	CPY	PPY	PPY	SP-CPY	
1	\$ 30.00	0.0706759	29.10	29.10	68.20
2	\$ 30.00	0.1302559	26.26	66.00	104.47
3	\$ 30.00	0.1819147	27.45	62.26	129.46
4	\$ 30.00	0.3004879	28.64	68.42	154.45
5	\$ 30.00	0.4200716	29.83	74.58	179.44
6	\$ 1,000.00	0.5406553	92.01	80.74	204.43
	Price	\$ 1,000.00	282.65	1,000.00	
	Duration in half years	4.23291155	Convexity	5.717252	
	Macaulay's Duration	2.11645578			
	Modified Duration	2.09894919			
	Yield change (%)	1.000000%			
	Modified Duration estimate	-2.098949%			\$ 209.89

Bond Price Volatility

- **The maturity effect**
 - The longer the time to maturity, the greater a bond's price sensitivity.
 - Price volatility increases at a decreasing rate with maturity.
- **The coupon effect**
 - The **greater** the coupon rate, the **lower** a bond's price sensitivity.
- **The yield level effect**
 - For the same change in bond yield, there is **greater price sensitivity of lower yield bonds.**

Coupon	6.00%	Face value	\$ 1,000		
Yield	6.00%	# of payment	2		
1	CPY	PPY	PPY	SP-CPY	
1	\$ 30.00	0.0706759	29.10	29.10	68.20
2	\$ 30.00	0.1302559	26.26	66.00	104.47
3	\$ 30.00	0.1819147	27.45	62.26	129.46
4	\$ 1,000.00	0.3004879	91.26	299.27	1,000.00
	Price	\$ 1,000.00	282.65	1,000.00	
	Duration in half years	4.23291155	Convexity	4.444268	
	Macaulay's Duration	1.61645578			
	Modified Duration	1.60894919			
	Yield change (%)	1.000000%			
	Modified Duration estimate	-1.608949%			\$ 160.89

Coupon	6.00%	Face value	\$ 1,000		
Yield	6.00%	# of payment	2		
1	CPY	PPY	PPY	SP-CPY	
1	\$ 40.00	0.0706759	28.26	28.26	72.50
2	\$ 40.00	0.1302559	27.29	76.41	108.73
3	\$ 40.00	0.1819147	26.01	100.00	145.00
4	\$ 1,000.00	0.3004879	92.00	288.14	1,000.00
	Price	\$ 1,022.17	282.65	1,022.17	
	Duration in half years	3.23291155	Convexity	4.309662	
	Macaulay's Duration	1.61645578			
	Modified Duration	1.60492536			
	Yield change (%)	1.000000%			
	Modified Duration estimate	-1.604925%			\$ 160.49

Bond Price Volatility

- **The maturity effect**
 - The longer the time to maturity, the greater a bond's price sensitivity.
 - Price volatility increases at a decreasing rate with maturity.
- **The coupon effect**
 - The **greater** the coupon rate, the **lower** a bond's price sensitivity.
- **The yield level effect**
 - For the same change in bond yield, there is **greater price sensitivity of lower yield bonds.**

Coupon	6.00%	Face value	\$ 1,000		
Yield	6.00%	# of payment	2		
1	CPY	PPY	PPY	SP-CPY	
1	\$ 30.00	0.0706759	29.10	29.10	68.20
2	\$ 30.00	0.1302559	26.26	66.00	104.47
3	\$ 30.00	0.1819147	27.45	62.26	129.46
4	\$ 1,000.00	0.3004879	915.16	288.87	1,000.00
	Price	\$ 1,000.00	282.65	1,000.00	
	Duration in half years	3.23291155	Convexity	4.444268	
	Macaulay's Duration	1.61645578			
	Modified Duration	1.60894919			
	Yield change (%)	1.000000%			
	Modified Duration estimate	-1.608949%			\$ 160.89

Coupon	6.00%	Face value	\$ 1,000
Yield	6.00%	# of payment	2
CF1	30.00	Price	999.01
1	\$ 30.00	Pay	999.01
2	\$ 30.00	Pay	1030.00
3	\$ 30.00	Pay	1030.00
4	\$ 1,030.00	Pay	1030.00
Duration in half years	2.0000000	Convexity	4.0000000
Macaulay's Duration	1.9194004		
Modified Duration	1.0000000		
Yield change (Y)	1.0000000%		
Modified Duration estimate	1.0000000%		41.7297

Bond Price Volatility

The Duration Measure:

- Price volatility of a bond varies inversely with its coupon and directly with its time to maturity.
- A composite measure considering both coupon and maturity
- Developed by Frederick R. Macaulay (1938)

Macaulay Duration

$$D_{Mac} = \frac{\sum_{t=1}^n t \times CF_t(t)}{\sum_{t=1}^n CF_t} = \frac{\sum_{t=1}^n t \times PV_t}{Price}$$

where:
 D_{Mac} = Macaulay Duration
 t = time period in which the coupon or principal payment occurs
 CF_t = interest or principal payment that occurs in period t
 P_0 = yield to maturity on the bond

Duration公式之推導

$$P = \frac{C}{(1+R)} + \frac{C}{(1+R)^2} + \dots + \frac{C+F}{(1+R)^n}$$

$$\frac{dP}{dR} = -\frac{C}{(1+R)^2} - \frac{2C}{(1+R)^3} - \dots - \frac{N(C+F)}{(1+R)^{n+1}}$$

$$\frac{dP}{dR} = -\frac{1}{(1+R)} \left[\frac{C}{(1+R)} + \frac{2C}{(1+R)^2} + \dots + \frac{N(C+F)}{(1+R)^n} \right]$$

Duration公式之推導

$$\frac{dP}{dR} = -\frac{1}{(1+R)} \left[\frac{C}{(1+R)} + \frac{2C}{(1+R)^2} + \dots + \frac{N(C+F)}{(1+R)^n} \right]$$

$$D_{Mac} = \frac{C}{(1+R)^2} + \frac{2C}{(1+R)^3} + \dots + \frac{N(C+F)}{(1+R)^{n+1}}$$

$$\Rightarrow \frac{dP}{dR} = -\frac{1}{(1+R)} [D_{Mac} \times P]$$

Bond Price and Interest Rates

$$\frac{dP}{dR} = -\frac{1}{(1+R)} [D \times P]$$

$$\Rightarrow \frac{\Delta P}{\Delta R} = -\frac{1}{(1+R)} [D \times P] = -\frac{D}{(1+R)} \times P$$

$$\Delta P = -\frac{D}{(1+R)} \times \Delta R \times P$$

Modified Duration

修正後存續期間

$$D_{mod} = D_{Mac} \times \left(\frac{1}{1+R/m} \right)$$

where:
 D_{mod} = the modified duration of the bond
 D_{Mac} = the Macaulay duration of the bond
 R = interest rate
 m = the number of coupon payments each year

Bond Price and Interest Rates

債券價格與利率變動之關係

$$\frac{\Delta P}{P} = -D_{mod} \times \frac{\Delta R}{1+R/m} = -\frac{D_{mod}}{1+R/m} \times \Delta R$$

$$= -D_{mod} \times \Delta R$$

$$\Rightarrow \Delta P = -D_{mod} \times \Delta R \times P$$

Coupon	6.00%	Face value	\$ 1,000
Yield	6.00% <td># of payment</td> <td>2</td>	# of payment	2
CF1	30.00	Price	999.01
1	\$ 30.00	Pay	999.01
2	\$ 30.00	Pay	1030.00
3	\$ 30.00	Pay	1030.00
4	\$ 1,030.00	Pay	1030.00
Duration in half years	2.0000000	Convexity	4.0000000
Macaulay's Duration	1.9194004		
Modified Duration	1.0000000		
Yield change (Y)	1.0000000%		
Modified Duration estimate	1.0000000%		41.7297

Coupon	6.00%	Face value	\$ 1,000	
Yield	6.00%	# of payment	2	
1	CF1	PV0	FV1	SP-SP1
1	\$ 30.00	0.970728	30.13	29.23
2	\$ 30.00	0.942599	30.28	28.58
3	\$ 30.00	0.915417	30.45	28.05
4	\$ 1,030.00	0.889193	915.14	942.54
	Price	\$ 1,000.00	970.87	985.50
Duration in half years	3.2201118	Convexity	4.44458	
Macaulay's Duration	1.6100559			
Modified Duration	1.5145598			
Yield change (%)	1.000000%			
Modified Duration estimate	-0.171316%			\$ 17.016

Characteristics of Macaulay Duration

- Duration of a bond with coupons is always less than (\leq) its term to maturity.
- A zero-coupon bond's duration equals its maturity.
- There is an **inverse** relation between duration and coupon rate.

Coupon	0.00%	Face value	\$ 1,000	
Yield	6.00%	# of payment	2	
1	CF1	PV0	FV1	SP-SP1
1	\$ -	0.970728	0.00	0.00
2	\$ -	0.942599	0.00	0.00
3	\$ -	0.915417	0.00	0.00
4	\$ 1,000.00	0.889193	889.19	915.14
	Price	\$ 1,000.00	889.19	915.14
Duration in half years	4.0000000	Convexity	4.712888	
Macaulay's Duration	2.0000000			
Modified Duration	1.9114157			
Yield change (%)	1.000000%			
Modified Duration estimate	-0.241046%			\$ 17.2822

Coupon	6.00%	Face value	\$ 1,000	
Yield	6.00%	# of payment	2	
1	CF1	PV0	FV1	SP-SP1
1	\$ 30.00	0.970728	30.13	29.23
2	\$ 30.00	0.942599	30.28	28.58
3	\$ 30.00	0.915417	30.45	28.05
4	\$ 1,030.00	0.889193	915.14	942.54
	Price	\$ 1,000.00	970.87	985.50
Duration in half years	3.2201118	Convexity	4.44458	
Macaulay's Duration	1.6100559			
Modified Duration	1.5145598			
Yield change (%)	1.000000%			
Modified Duration estimate	-0.171316%			\$ 17.016

Coupon	6.00%	Face value	\$ 1,000	
Yield	6.00%	# of payment	2	
1	CF1	PV0	FV1	SP-SP1
1	\$ 40.00	0.970728	38.83	37.87
2	\$ 40.00	0.942599	37.79	36.41
3	\$ 40.00	0.915417	36.81	34.97
4	\$ 1,040.00	0.889193	924.62	960.54
	Price	\$ 1,021.77	970.87	985.50
Duration in half years	3.7295118	Convexity	4.58162	
Macaulay's Duration	1.8647559			
Modified Duration	1.7674228			
Yield change (%)	1.000000%			
Modified Duration estimate	-0.181765%			\$ 18.288

Characteristics of Macaulay Duration

- A **positive** relation between term to maturity and duration, but duration increases at a decreasing rate with maturity.
- **Inverse** relation between YTM and duration.
- **Sinking bonds and call provisions** can have a dramatic effect on a bond's duration.
- To accelerate the total cash flows and to reduce the duration.

Coupon	6.00%	Face value	\$ 1,000	
Yield	6.00%	# of payment	2	
1	CF1	PV0	FV1	SP-SP1
1	\$ 30.00	0.970728	30.13	29.23
2	\$ 30.00	0.942599	30.28	28.58
3	\$ 30.00	0.915417	30.45	28.05
4	\$ 1,030.00	0.889193	915.14	942.54
	Price	\$ 1,000.00	970.87	985.50
Duration in half years	3.2201118	Convexity	4.44458	
Macaulay's Duration	1.6100559			
Modified Duration	1.5145598			
Yield change (%)	1.000000%			
Modified Duration estimate	-0.171316%			\$ 17.016
Convexity Adjustment = 0.5(C)²/(Y)	0.000000%			
Estimated change in bond price (%)	-0.169327%			
Theoretical price change (%)	-0.169327%			\$ 16.93

Coupon	6.00%	Face value	\$ 1,000	
Yield	6.00%	# of payment	2	
1	CF1	PV0	FV1	SP-SP1
1	\$ 30.00	0.970728	30.13	29.23
2	\$ 30.00	0.942599	30.28	28.58
3	\$ 30.00	0.915417	30.45	28.05
4	\$ 1,030.00	0.889193	915.14	942.54
	Price	\$ 1,000.00	970.87	985.50
Duration in half years	3.2201118	Convexity	4.44458	
Macaulay's Duration	1.6100559			
Modified Duration	1.5145598			
Yield change (%)	1.000000%			
Modified Duration estimate	-0.171316%			\$ 17.016

Coupon	6.00%	Face value	\$ 1,000	
Yield	6.00%	# of payment	2	
1	CF1	PV0	FV1	SP-SP1
1	\$ 30.00	0.970728	30.13	29.23
2	\$ 30.00	0.942599	30.28	28.58
3	\$ 30.00	0.915417	30.45	28.05
4	\$ 30.00	0.889193	30.45	28.05
5	\$ 30.00	0.863433	30.45	27.12
6	\$ 1,030.00	0.839134	861.61	878.85
	Price	\$ 1,000.00	970.87	985.50
Duration in half years	4.8215232	Convexity (%)	6.87273	
Macaulay's Duration	2.4107616			
Modified Duration	2.2959173			
Yield change (%)	1.0000000%			
Modified Duration estimate	-0.2496821%			\$ 24.9682

Characteristics of Macaulay Duration

- A positive relation between term to maturity and duration, but duration increases at a decreasing rate with maturity.
- Inverse relation between YTM and duration
- Sinking bonds and call provisions can have a dramatic effect on a bond's duration.
- To accelerate the total cash flows and to reduce the duration

Yield	0.00%	Face value	\$ 1,000		
Yield	0.00%	# of payments	2		
1	CR	PPR	PR	SP/PPR	
1	\$ 30.00	0.0000000	20.10	20.10	90.20
2	\$ 30.00	0.0000000	20.20	40.30	100.50
3	\$ 30.00	0.0000000	21.45	62.30	120.45
4	\$ 1,000.00	0.0000000	915.51	2000.00	1000.00
	Price	\$ 1,000.00	2000.00	1000.00	
	Duration in half years	3.2207110	Convexity	4.44000	
	Macaulay's Duration	1.6103555			
	Modified Duration	1.6095495			
	Yield change (%)	1.000000%			
	Modified Duration effects	-1.609549% / \$ 1609.55			

Yield	0.00%	Face value	\$ 1,000		
Yield	0.00%	# of payments	2		
1	CR	PPR	PR	SP/PPR	
1	\$ 30.00	0.0000000	20.05	20.05	80.10
2	\$ 30.00	0.0000000	21.74	41.79	100.42
3	\$ 30.00	0.0000000	20.07	40.07	100.04
4	\$ 1,000.00	0.0000000	905.05	2000.00	1000.00
	Price	\$ 1,000.00	2000.00	1000.00	
	Duration in half years	3.2207110	Convexity	4.44000	
	Macaulay's Duration	1.6103555			
	Modified Duration	1.6095495			
	Yield change (%)	1.000000%			
	Modified Duration effects	-1.609549% / \$ 1609.55			

Duration and Bond Price Volatility

Bond price movements will vary with modified duration for small changes in yields:

$$\frac{\Delta P}{P} \% \approx -D_{mod} \times \Delta Y_m$$

Where:
 ΔP = change in price for the bond
 P = beginning price for the bond
 D_{mod} = the modified duration of the bond
 ΔY_m = yield change in basis points divided by 100

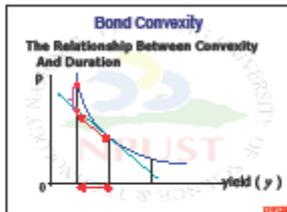
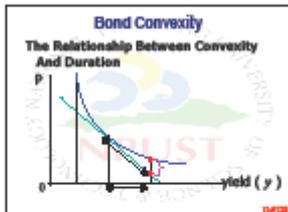
Yield	0.00%	Face value	\$ 1,000		
Yield	0.00%	# of payments	2		
1	CR	PPR	PR	SP/PPR	
1	\$ 30.00	0.0000000	20.10	20.10	90.20
2	\$ 30.00	0.0000000	20.20	40.30	100.50
3	\$ 30.00	0.0000000	21.45	62.30	120.45
4	\$ 1,000.00	0.0000000	915.51	2000.00	1000.00
	Price	\$ 1,000.00	2000.00	1000.00	
	Duration in half years	3.2207110	Convexity	4.44000	
	Macaulay's Duration	1.6103555			
	Modified Duration	1.6095495			
	Yield change (%)	1.000000%			
	Modified Duration effects	-1.609549% / \$ 1609.55			

Bond Convexity

- The percentage price change formula using duration is a linear approximation of bond price change for small changes in market yields

$$\frac{\Delta P}{P} \% \approx -D_{mod} \times \Delta Y_m$$

- Price changes are not linear, but a convex (convex) function.



Bond Convexity

- The measure of the curvature
- Convexity is the percentage change in D_{mod} for a given change in yield.
- It is the second derivative of bond price with respect to yield (d^2P/dY^2)

$$Convexity = \frac{d^2P/dY^2}{P}$$

$$= \frac{1}{(1+y)^2} \left[\sum_{t=1}^n \frac{t^2 + t}{(1+y)^t} \right]$$

Coupon	6.00%	Face value	\$ 1,000
Yield	6.00%	# of payments	2
1	CPI	PVPI	FPI
1	\$ 30.00	0.9798738	28.19
2	\$ 30.00	0.9409139	28.29
3	\$ 30.00	0.9046487	27.48
4	\$ 1,030.00	0.8698970	895.14
	Price	\$ 1,000.00	2693.81
Duration in half years	3.82881425	Convexity	
Macaulay's Duration	1.914407125	4.641268	
Modified Duration	1.89954825		
Yield change (Y)	0.250000%		
Modified Duration estimate	0.469887%		
Convexity Adjustment = 0.5(Y)(Y)(Y)	0.001268%		
Estimated change in bond price (%)	-0.468619%		
Actual price change (%)	-0.468619%		

Coupon	6.00%	Face value	\$ 1,000
Yield	6.00%	# of payments	2
1	CPI	PVPI	FPI
1	\$ 30.00	0.9798738	28.19
2	\$ 30.00	0.9409139	28.29
3	\$ 30.00	0.9046487	27.48
4	\$ 1,030.00	0.8698970	895.14
	Price	\$ 1,000.00	2693.81
Duration in half years	3.82881425	Convexity	
Macaulay's Duration	1.914407125	4.641268	
Modified Duration	1.89954825		
Yield change (Y)	0.250000%		
Modified Duration estimate	0.469887%		
Convexity Adjustment = 0.5(Y)(Y)(Y)	0.001268%		
Estimated change in bond price (%)	-0.468619%		
Actual price change (%)	-0.468619%		

Bond Convexity

Determinants of Convexity

- Inverse relationship between coupon and convexity
- Direct relationship between maturity and convexity
- Inverse relationship between yield and convexity