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EUROSYSTEM

Rating System

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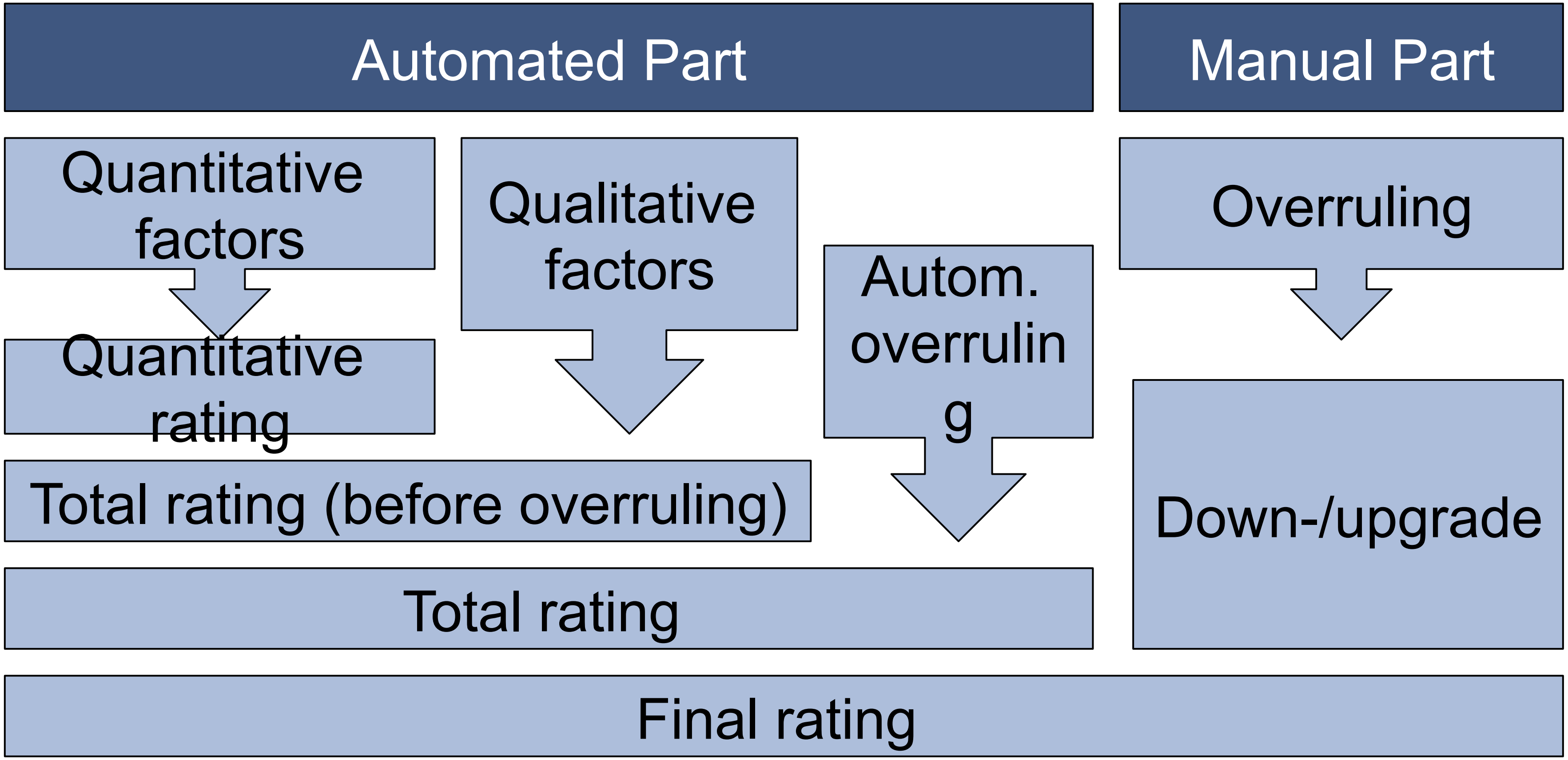
AGENDA

Rating system



How to develop a rating system

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Definition of potential explanatory factors	Collection of data	Single factor analysis	Multi factor analysis	(additionally)	Calibration
	▌Benchmarks ▌Quantitative and qualitative factors	▌Pre-selection ▌Transformatio n (Logit)	▌Regression (multiple models) ▌Choice of model	Stressful events not captured by the rating model ▌Selection of factors ▌Threshold values and impact on rating	▌Average default rates ▌Mapping to Master-scale

Definition of risk factors

- **Discriminatory power**
  - (Adjusted) Powerstat
  - Statistical error (1st / 2nd degree)
- **Correlation**
- **Explanatory power of single factor (R^2)**
- **Quality of data**
  - Missing values
  - Expert opinion
  - Availability / collection of data in future times
- **Economic plausibility**
- **Continuity of relationships**

Data

- **Good-bad-analysis**
  - Data source (internal or / and external)
  - Default information (internal or / and external)
  - Assignment to good and bad sample
- **Shadow rating**
  - External Ratings
  - Explanatory factors
- **Quality of data**
  - Definitino of eligible data sources
  - Processing
  - Editing
- **Backfilling data**
  - Systematic approach
  - Expert opinion

Overruling (manual)

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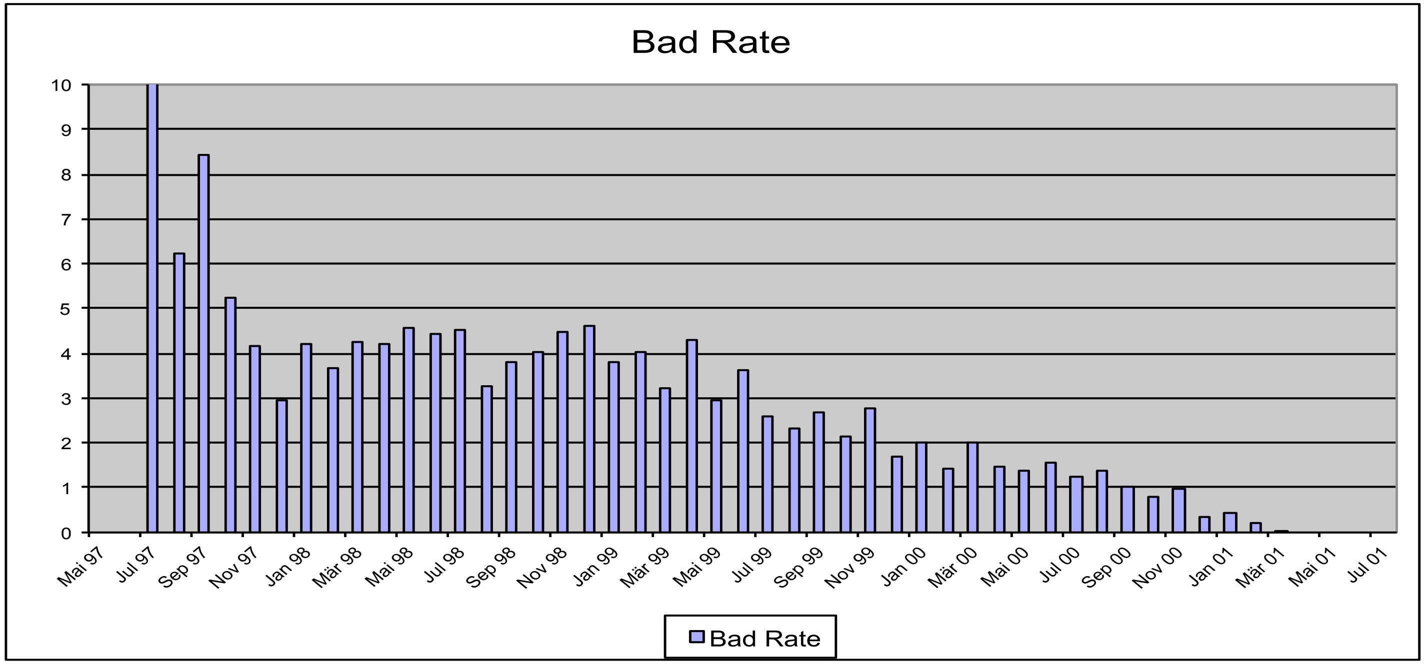
- **Political issues, e.g.**
  - War, conflicts, terrorism
  - Volatile political environment
- **Economic issues, e.g.**
  - Susceptibility to economic shocks
  - Interpretability of data / risk factors limited
- **Other issues, e.g.**
  - Data quality
  - Currentness of data
  - Payment behaviour
  - Catastrophes

PREDICTION OF DEFAULT AND TRANSITION RATES

Observed default rates

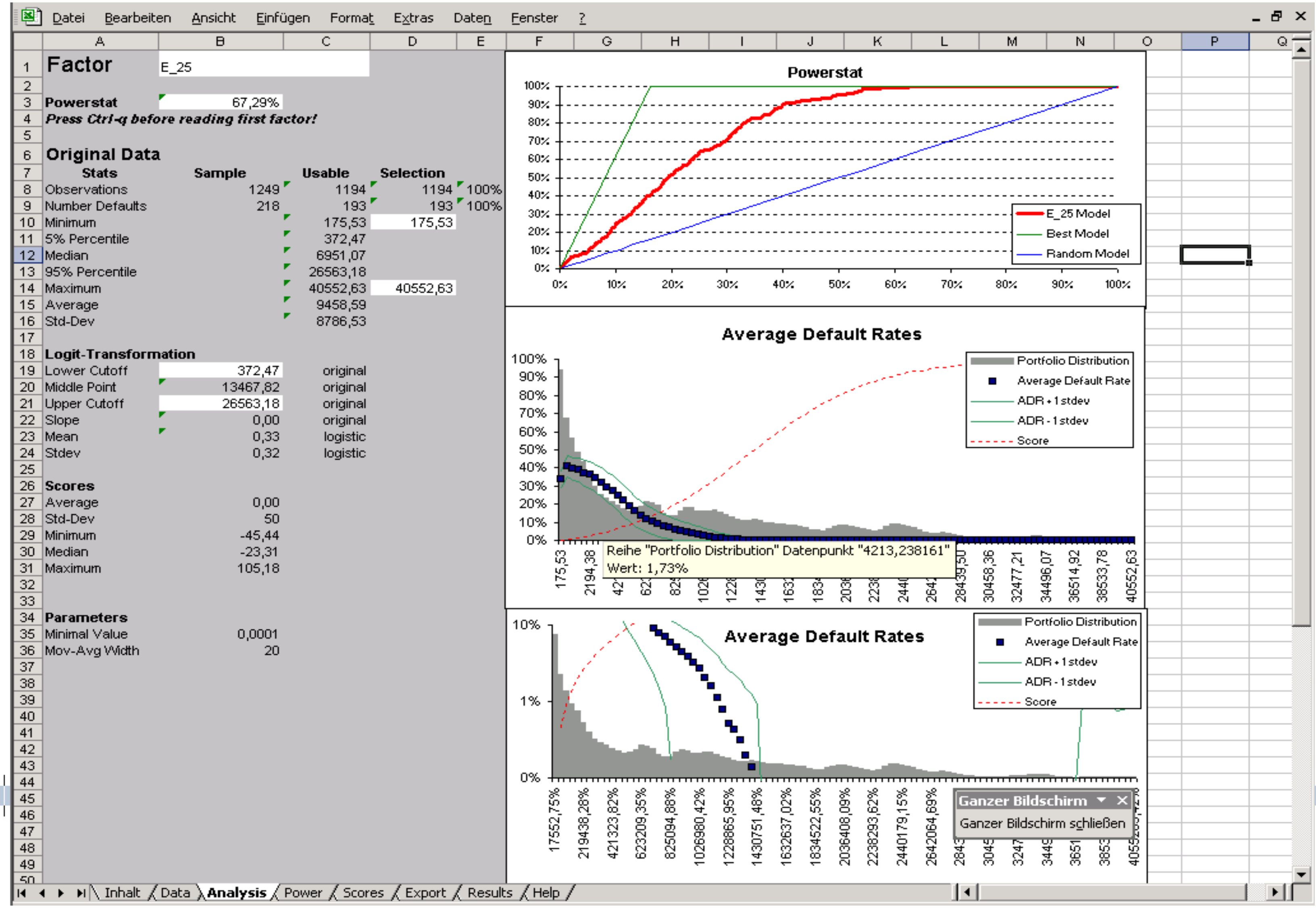
- **The Data sample to use depends on the following:**
  - „Riping Time“
  - Structure of the sample
    (as a rule, the number of non-performing loans determines the size of the sample period. We would like to have enough non-performing loans in our sample, in order to get a stable relationship.)
  - Point of time of the good-bad-definition

Riping time



- **In this case, the bank chose the sample period to start April 1999 and end March 2000, i.e. non-performing loans are from April 1999 to June 2001 with a riping period of 15 Months. Riping period is chosen based on expert-knowledge.**

Candidate variables



Predicting default rates with linear regression

- **Having collected the candidate variables we could linearly combine the variables to produce the default rate forecast:**

$$\mathrm{PD}_{t,t+1} = b_0 + b_1 f_1 + \ldots$$

- **The corresponding estimation equation is as follows**

$$\mathrm{PD}_{t,t+1} = b_0 + b_1 f_1 + \cdots + \epsilon_t;\ \epsilon_t \sim \mathrm{i.\ i.\ d.}, E(\epsilon) = 0$$

- **A straightforward way to estimate this equation is linear regression.**
  - Drawback: Values not restricted to {0,1}, so a direct interpretation as a measure of probability is not possible (use Poisson regression as an alternative...)

Discussion of different models

- **There are a number of indicators available to describe the goodness of fit of a linear estimation**
  - SE: Standard error of coefficient b
  - R^2: Coefficient of determination (fraction of the variance of the dependent variable that is explained by the explanatory variables). $R^2 = 1 – RSS/(MSS+RSS)$ ϵ_t
  - RMSE: Standard deviation of the residuals
  - F-statistics: tests the significance of the entire regression
  - DF: degrees of freedom
  - MSS: Model sum of squares (variation of PD that is explained by the model)
  - RSS: Residual sum of squares (variation of PD that is not explained by the model)
  - t-stat: Test, whether a coefficient b equals zero

Example

- **Macroeconomic conditions**
  - Liquidity and profits of corporates are affected by overall economic conditions

$$\text{PRF}_t = \frac{1 + \text{Forecasted change in corporate profits}_{t,t+1}}{1 + \text{Forecasted change in GDP deflator}_{t,t+1}}$$

- **Corporate bond spreads**
  - Yields of corporate bonds should be set such that the expected return from holding a bond is at least as large as the return from holding a risk-free government bond

$$\text{SPR}_t = \text{Yield of corporate bonds}_t - \text{Yield of US treasuries}_t$$

Example

- **Aging effect**
  - Issures who first entered the bond market three to four years ago are observed to be relatively likely to default. A possible explanation is that the debt issue provides firms with enough cash to survive for several ears even if the business plan envisaged at the time of the bond issue did not work out. So, liquidity problems with new issures will only appear with a certain delay.

$$\mathrm{AGE}_t = \frac{\text{No. newly rated issures}_{t-4,t-3}}{\text{No. rated issuers}_t}$$

- **Average risk**
  - When analysing average default rates of a sample comprising several rating categories, we should take into account that the composition of the group can change over time

$$\mathrm{BBB}_t = \frac{\text{No. BBB-rated issures}_t}{\text{No. investment grade issures}_t}$$

Example

Example from Löffler / Posch, pp. 75ff.

Model 1	SPR	BBB	AGE	PRF	CON
Coeff	0,051	0,004	0,019	-0,016	-0,229
SE(coeff)	0,033	0,003	0,009	0,004	0,098
R²/RMSE	0,602	0,078	#NV	#NV	#NV
F/DF	6,050	16	#NV	#NV	#NV
t-stat	1,536	1,325	1,993	-3,609	-2,336
p-value	0,144	0,204	0,064	0,002	0,033

Model 2	AGE	PRF	CON
	0,024	-0,013	-0,058
	0,009	0,005	0,076
	0,439	0,088	#NV
	7,046	18	#NV
t-stat	2,655	-2,863	-0,761
p-value	0,016	0,010	0,457

Coefficients meet the expectations:

- High spreads, a large fraction of risky BBB issuers and a large fraction of recently rated issures should be associated with higher default rtes, and thereforde positive b's.
- Higher profit expectations should be coupled with lower default rates.
- Note, constants cannot be interpreted directly since they are not average default rates

Example

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- **t-statistics / p-values**
  - Profit forecasts (PRF) and aging effect (AGE) are the most significant variables.
  - p-values of PRF and AGE are below 7%, so we can reject the hypothesis that the coefficients are zero with a significance level of 7% and better.
  - Bond spreads (SPR) and the fraction of BBB-rated issures (BBB) also have some explanatory power, but with a lower significance → let‘s try without them (model 2)

Example

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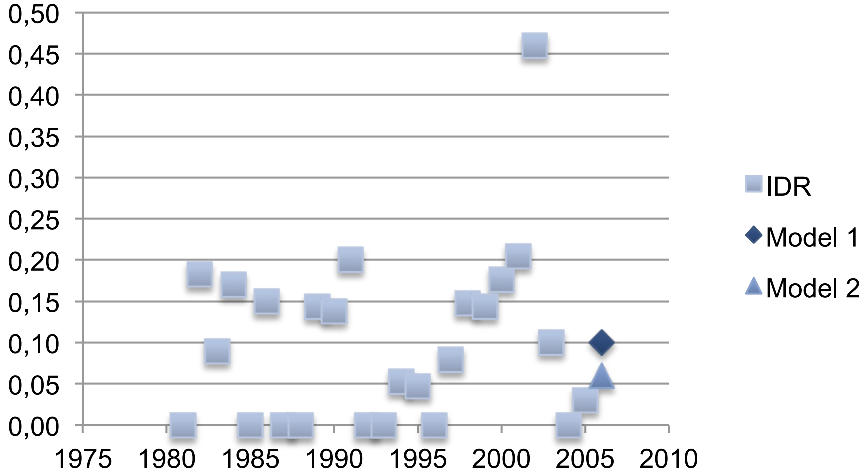
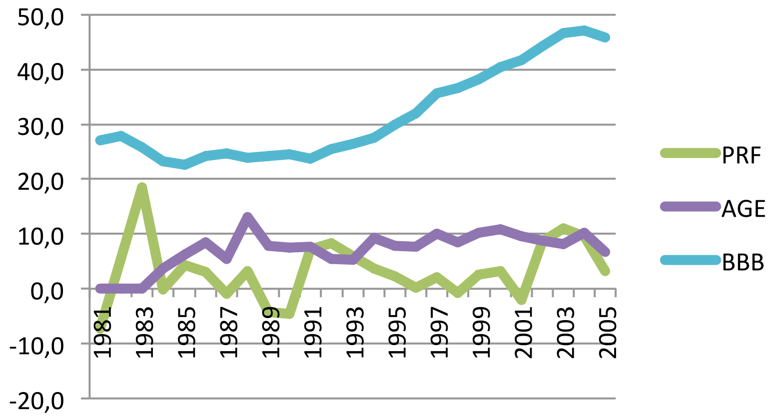
- **Dropping AGE and PRF from model 1 reduces R^2 from 60% to 44%**
  - Is this loss of explanatory power important (significant)?
  - F-test yields an F-value of 3.274 with a p-value of 6.4%. So if we start with model 2, inclusion of the two variables SPR and BBB do not add explanatory power to the model with a probability of 6.4%.
    - In practice, this exercise is performed by stepwise regression routines with a cut-off level assigned to the added explanatory power
  - A stringent standard of statistical significance, e.g. variables should be significant at the 5% level of better could lead us to favour model 2

Example





- **Explanation of the differences between the two models**
  - Profit forecast PRF ist close to the average for the last 25 years while the aging effect AGE ist less present in the past.
  - Therefore the default rate prediction based on model 2 should be below the average default rate, wich is 0.1%
  - The fraction of BBB-rated issuers, having increased over the 1990s, is at a high level. Once we include the variable BBB like in model 1, the default rate forecast increases

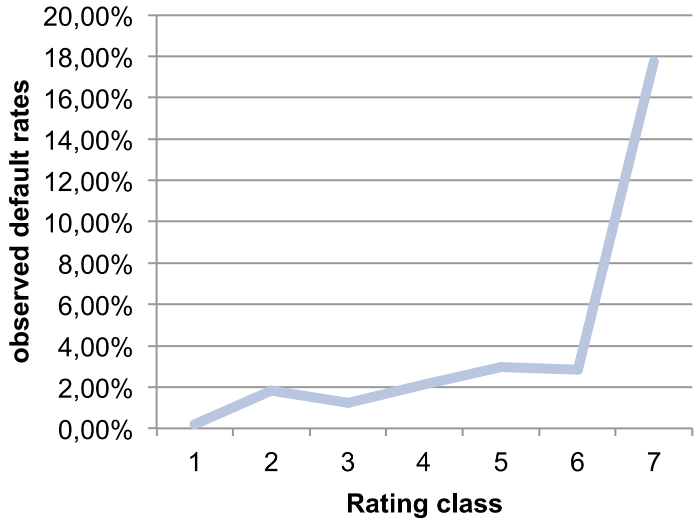
CALIBRATION

Observed default rates

- **Observed default rates typically do not increase monotonically in rating classes, so they cannot be used directly for model calibration:**

- **Sample: March 2000 – Mai 2003**
- **Default rates calculated on monthly basis**

- **How to calibrate:**
  - 1st step: Propopse a monotonically increasing function for the PDs
  - 2nd step: Test whether the deviance between the proposed and the observed default rates appears to be appropriate



1st step

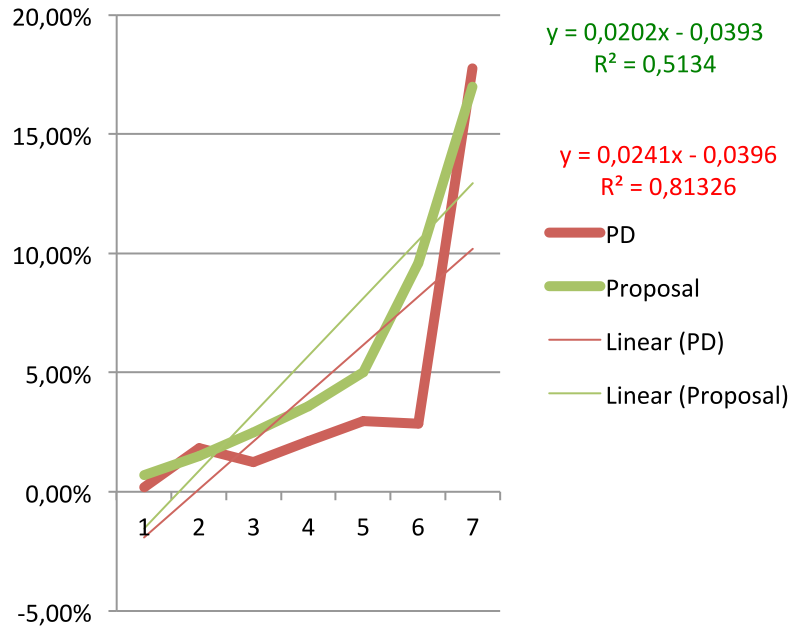
- **Exponential function for proposed PD per rating class i**

$$\mathrm{PD}_i = \frac{1}{1 + \exp\left(-\left(\alpha + i\beta\right)\right)}$$

- **Boundary conditions for estimating parameters**

$$\bar{\mathrm{PD}} = \bar{\mathrm{PD}}_{\mathrm{emp}} \forall i$$

$$\mathrm{PD}(7) = \bar{\mathrm{PD}}_{\mathrm{emp}}(7)$$



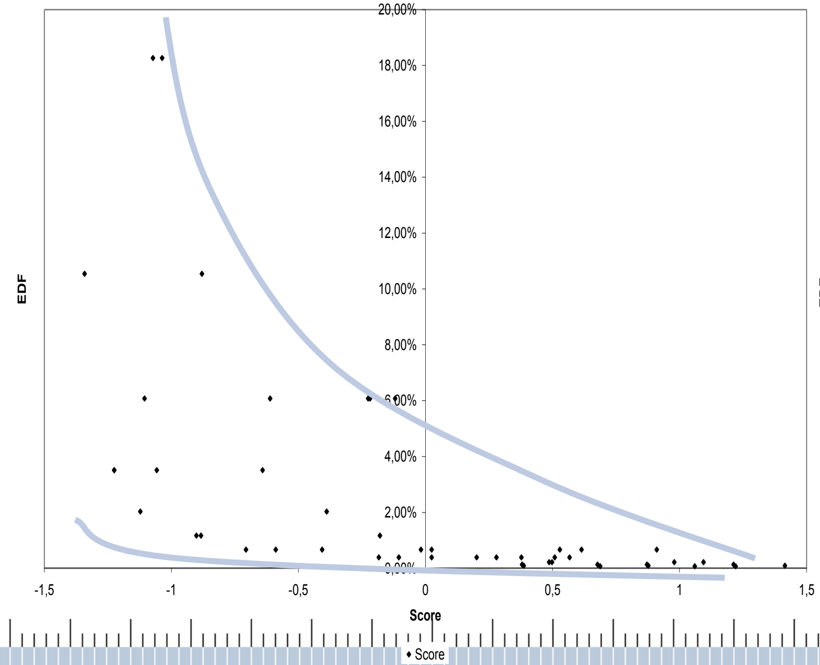
2nd step: Binomial test

- **Assumes, that defaults per rating class are statistically independent from each other and that all members of a rating class have the same PD**
  - In real life, there are correlations between defaults
  - As a consequence and as a rule of thumb, binomial test is a conservative indicator for the quality of a calibration of the pd of a rating class.
  - The test states, that the true value of the PD is not larger than the proposed value with probability α
  - If the hypothesis is not supported, the proposed PD must be higher.

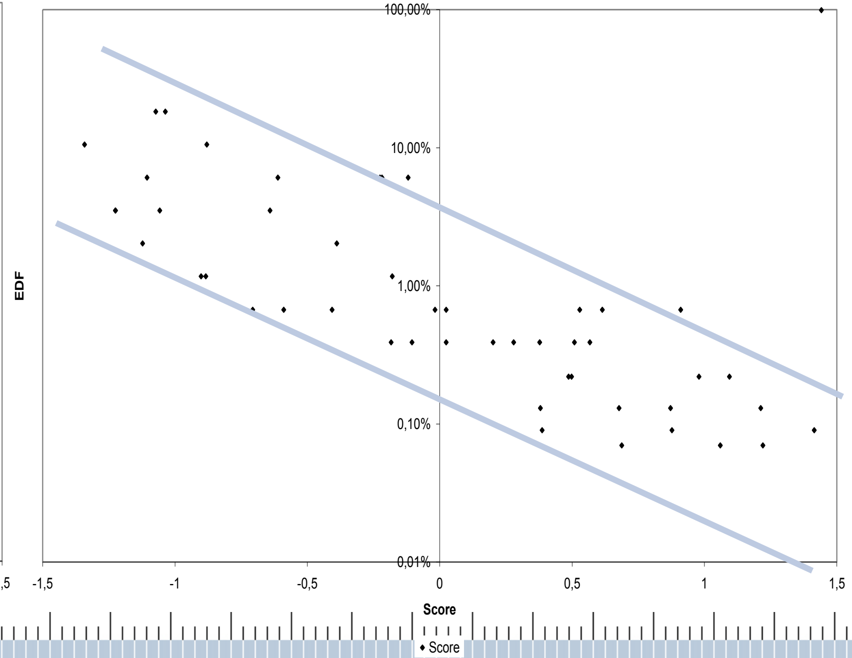
Use of logarithm

- Goal: Allotment of PDs to scores / risk factor values
- Means: Linear regression (log PD)

Original data



log representation



Buckets

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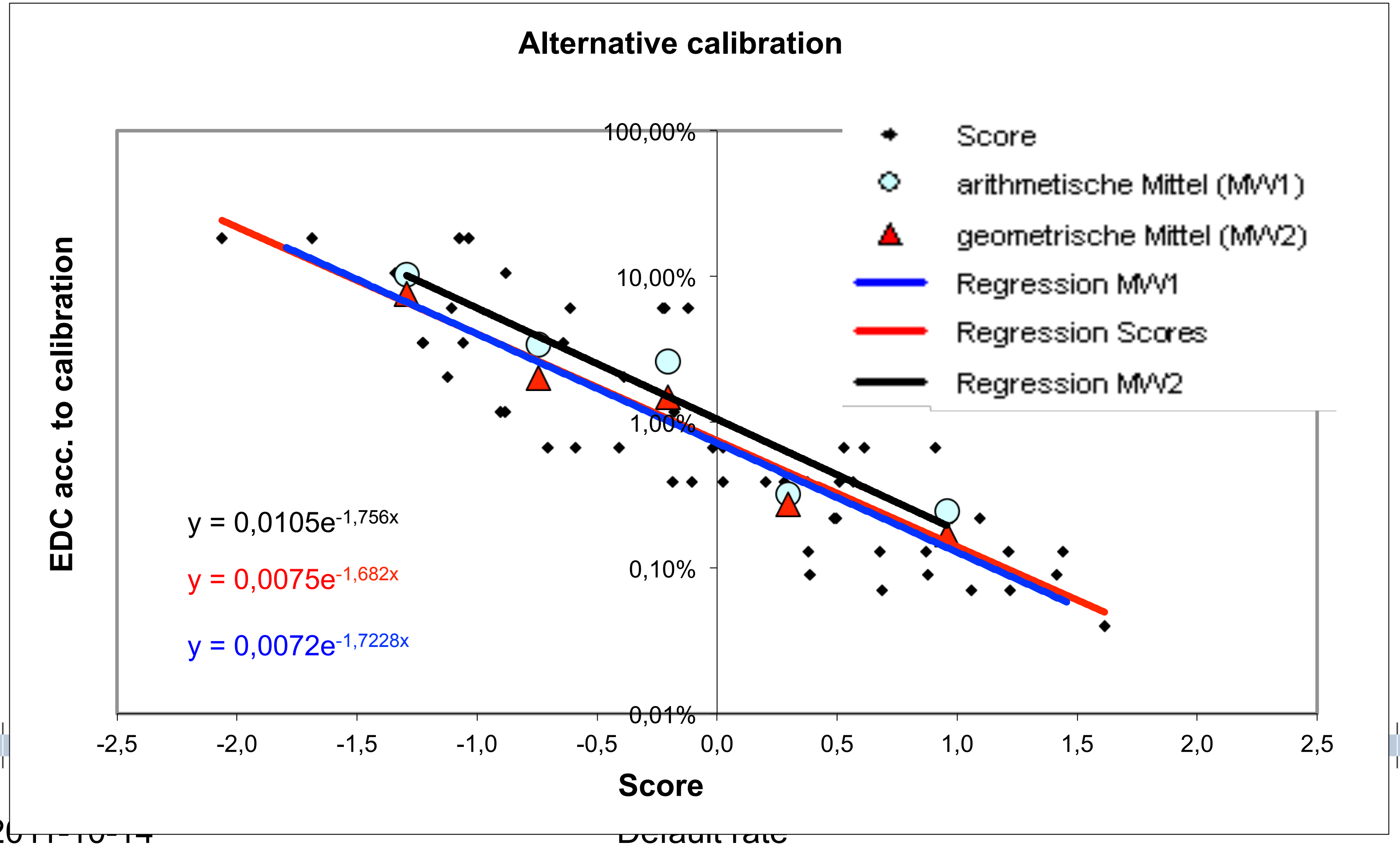
- **Reduction of complexity: Assignment of single observations to score classes (buckets)**
  - Susceptibility to outliers?
  - Ease in (ocular) analysis?
- **Definition of buckets**
  - Representativeness?
  - Equal distance?

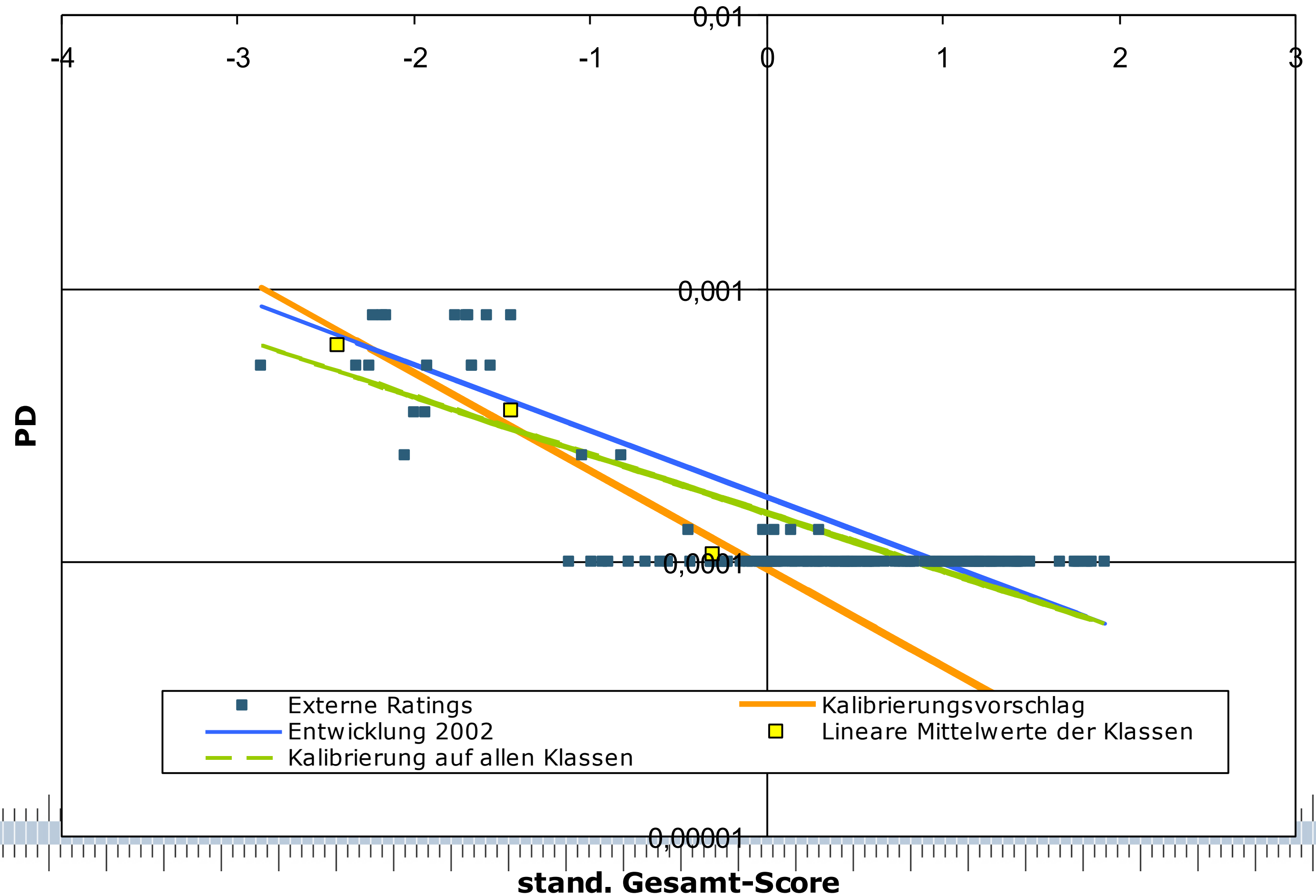
Example

All-over distribution of scores



Example

Non-even distribution of scores



Central tendency

- **Definition of CT:**
  - external CT: (Arthm.) mean of EDFs in portfolio acc. to base calibration
  - internal CT: (Arithm.) mean of internal PDs
- **CT is a measure of absolute differences in PDs**
  - CT is dominated by classes with high PDs
- **Least squares estimator minimizes relative differences in PDs**
- **Discussion:**
  - Calculation of internal PD: score-PDs vs. mid-PDs?
  - Calibration to CT can effectively lower PD resp. to base calibration?
  - Inconsitency: Base calibration on log PDs but CT ref. to PDs?

LOW DEFAULT PORTFOLIOS

Low default portfolios

- **Especially good rating grades might enjoy many years without any defaults.**
- **Even if we observe defaults, the observed default rates might exhibit a high degree of valatility due to the relatively low number of borrowers in that grade.**
- **Even portfolios with low or zero defaults are not uncommon, e.g. sovereign or bank portfolios, and especially high-volume low-number portfolios e.g. specialised lending**
- **Solution methods:**
  - Qualitative mapping mechanisms to bank-wide master scales
  - External ratings

No defaults, assumption of independence

- Obligors are distributed to rating grades A, B, and C with frequencies n_A, n_B, and n_C.
- The grade with the highest credit-worthiness is denoted by A.
- No defaults occured in A, B or C during the last observation period.
- The PDs (still to be estimated) of grades A to C reflect the decreasing credit-worthiness of the grades:

$$p_A \leq p_B \leq p_C$$

- As a consequence, the most prudent estimate of p_A is obtained under the assumption that the probabilities p_A and p_C are equal. Then

$$p_A = p_B = p_C$$

No defaults, assumption of independence

- Assuming the above equality we are looking for a confidence region for p_A at confidence level γ
- This confidence region ist the set of all admissible valies of pA with the property that the probability of not observing any default during the observation period ist not less than 1 $-\gamma$
- If the above equality holds, then the three rating grades A, B, and C do not differ in their respective risiness.
- Hence we deal with a homogeneous sample of size $n_A + n_B + n_C$ without any default during the observation period.
- Assuming unconditional independence of the default events, the probability of observing no defaults turns out to be

$$(1 - p_A)^{n_A + n_B + n_C}$$

No defaults, assumption of independence

- **Consequently we have to solve the inequality**

$$1-\gamma \leq (1-p_A)^{n_A+n_B+n_C}$$

- **for p_A in order to obtain the confidence region at level** γ
- **as the set of all the values of pA such that**

$$p_A \leq 1-(1-\gamma)^{\frac{1}{n_A+n_B+n_C}}$$

- **As an example, choose**

$$n_A = 100, n_B = 400, n_C = 300$$

- **For some confidence levels with the corresponding maximum values (upper confidence bounds) of p_A we have**

gamma	95%	99%	99,0%
p_A^{est}	0.37	0.57	0.86

No defaults, assumption of independence

- **By inequality** $p_A \leq p_B \leq p_C$
- **the PD of B cannot be grater than the PD of C.**
- **Consequently the most prudent estimate of pA is obtained by assuming** $p_B = p_C$
- **We cannot have** $p_A = p_B = p_C$
- **any more, because p_A is the lower bound of p_B.**
- **Therefore the confidence region for pB is obtained from**

$$p_B \leq 1 - (1 - \gamma)^{\frac{1}{n_B + n_C}}$$

gamma	95%	99%	99.9%
p_B^{est}	0.43%	0.66%	0.98%

No defaults, assumption of independence

- Analogously, we achieve for p_C

gamma	95%	99%	99.9%
p_C^{est}	0.99%	1.52%	2.28%

- Comparison of the tables shows that the applicable samples size is a main driver of the upper confidence bound.
- The smaller the sample size, the greater will be the upper confidence bound. (Intuitively the credit-worthiness ought to be better, if the number of obligors in a portfolio without any default observation is better).
- It is suggested to use the upper confidence bounds as described in the tables for PD estimates.