

出國報告（出國類別：出席研討會）

偏微分方程及其應用國際會議

服務機關：國立中興大學

姓名職稱：郭紅珠教授

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摘要

本人郭紅珠受邀參加由中國科學技術大學所舉辦之「偏微分方程及其應用國際會議」。本次會議主題為 elliptic partial differential equations。這是本人主要的研究領域。此會議共有 20 位學者受邀發表，每場演講為 50 分鐘，講者皆是該領域研究表現傑出學者。台灣應邀出席的研究者僅 2 人，除了本人外，另一位是台灣大學的林長壽院士，林院士亦應邀擔任此會議的第一位演講者。

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目的

本人郭紅珠受邀參加由位在合肥的中國科學技術大學主辦的「偏微分方程及其應用國際會議」。此會議共有 20 場學術發表。台灣應邀出席的研究者僅 2 人，除了本人外，另一位是台灣大學的林長壽院士，林院士亦應邀擔任此會議的第一位演講者。本次會議主題為 elliptic partial differential equations。這是本人主要的研究領域。另，與本人有長期研究合作的 Prof. Neil Trudinger 澳洲院士也受邀參加此會議。本人參加此會議的主要目的之一即是在此與他會面，討論所合作的論文及執行研究計畫。

過程

大部分演講是有關 Conformal geometry, Curvature flow problems, Nonlinear PDES related to pattern formation, and Geometric properties of solutions to Nonlinear PDEs among others. Scientific Committee 含 1. Alice Chang 2. XiuXiong Chen 3. Pengfei Guan 4. Yanyan Li 5. Organizing Committee 6. Jiayu Li 7. Xinan Ma 8. Juncheng Wei 9. Meijun Zhu

本次會議演講議程如下:

1. 8/1 Mon.	8/2 Tue.	8/3 Wed.	8/4 Thur.	8/5-8/6
8:30-9:20	Changshou Lin	Neil Trudinger	Yanyan Li	Gabriella Tarantello
9:20-10:10	Lei Zhang	Jiakun Liu	Junfang Li	Guan Bo
10:10-10:30	Tea Break			
10:30-11:20	Songying Li	Luc Nguyen	Susanna Terracini	Qing Han
11:30-14:00	Lunch			
14:30-15:20	Takashi Suzuki	Jose Antonio Galvez	Arrigo Cellina	Juncheng Wei
15:20-16:10	Tonia Ricciardi	Shusen Yan	Frank Pacard	
16:10-16:30	Tea Break			
16:30-17:20	Xingwang Xu	Congming Li		
17:30-19:00	dinner	Supper		

此會議的演講題目及摘要如下:

·Changshou Lin

Title: Bubbling solutions to the Relativistic SU(3) Chern-Simons-Higgs System

Abstract: In this talk I will talk about a joint work with Shusen Yan on the existence of a nonlinear system which arises from the Relativistic version of SU(3) gauge field theory to explain some superconductivity phenomenon. This is the start of a series of papers to understand the limiting patterns of this system as the coupling constant tends to 0.

·Lei Zhang

Title: A Topological Degree Counting for Some Liouville Systems of Mean Field Equations

Abstract: Let M be a Riemann surface and $\chi(M)$ be its Euler characteristic. The following generalized Liouville system is related to many different research areas in mathematics and physics:

$$\Delta_g u_i + \sum_{j=1}^n a_{ij} \rho_j \left(\frac{h_j e^{u_j}}{\int h_j e^{u_j}} - 1 \right) = 0 \quad \text{in } M, i, \dots, n \quad (0.1)$$

where $A = (a_{ij})_{n \times n}$ is a nonnegative matrix, h_1, \dots, h_n are positive C^1 functions and

ρ_1, \dots, ρ_n are positive constants. We prove that, under some assumptions on the matrix A , the Leray-Schauder degree of (0.1) is equal to

$$\frac{(-\chi(M) + 1) \cdots (-\chi(M) + N)}{N!}$$

if $\rho = (\rho_1, \dots, \rho_n)$ satisfies

$$8\pi N \sum_{i=1}^n \rho_i < \sum_{1 \leq i, j \leq n} a_{ij} \rho_i \rho_j < 8\pi(N+1) \sum_{i=1}^n \rho_i$$

the derivation of the degree counting formula is involved with the classification theorems of some Liouville systems defined in R^2 , delicate blowup analysis and the strong interaction between bubbling solutions. I will present the main idea of each part briefly. This is a joint work with Chang-shou Lin.

·Songying Li

Title: Rigidity type questions and theorems associated to the exact bottom of spectrum of Laplace-Beltrami operators and solution of degenerate Monge-Ampère operators.

Abstract: In this talk, I will present some recent works of mine along with my collaborators in the area of estimating the sharp bound on the bottom of the spectrum of Laplace-Beltrami operators, and its connection to degenerate Monge-Ampère operators and pseudo-Hermitian CR manifold.

Takashi Suzuki

Title: A competitive system of chemotaxis

Abstract: We study a competitive system of chemotaxis in two-space dimensions. If the existence time of the solution is finite, there is a formation of collapse, possibly degenerate, to each component, the total mass quantization, and a formation of subcollapses. Several related results are also shown.

Tonia Ricciardi

Title: Blow up analysis and a Trudinger-Moser inequality for a mean field equation arising in 2D turbulence

Abstract: In the context of Onsager's theory of equilibrium turbulence. Such an equation contains a probability measure describing the distribution of vortex intensities. By blow-up analysis and duality arguments we study a related Trudinger-Moser type inequality with probability measure. In particular, we identify the optimal constant for boundedness from below. This is joint work with Takashi Suzuki.

Xingwang Xu

Title: Existence results for the Einstein-scalar field Lichnerowicz equations

Abstract: TBA

Neil Trudinger

Title: On the local theory of prescribed Jacobian equations

Abstract: We present a local theory of nonlinear elliptic partial differential equations arising from the prescription of the Jacobian determinant of mappings depending on functions and their gradients. The relevant mappings are defined through generating functions which extend the notion of cost function in optimal transportation. Specifically we consider the associated convexity theory, the existence of generalized solutions and their regularity and the relationship to recent work in near field geometric optics with Jia-kun Liu.

Jiakun Liu

Title: Schauder estimates in optimal transportation

Abstract: In this talk we study the continuity of second derivatives of solution to a Monge-Ampere equation arising from optimal transportation. When the inhomogeneous term is merely Holder or Dini continuous, we obtain Holder or general continuity estimates for second derivatives of solution, which implies the corresponding regularity of optimal mapping in optimal transportation. This is a joint work with Neil Trudinger and Xu-Jia Wang.

Luc Nguyen

Title: Singular harmonic map and near horizon geometry of degenerate horizons in stationary vacuum spacetimes

Abstract: In a regular axially symmetric and stationary vacuum spacetime, Hajicek showed that the near horizon geometry of a degenerate horizon is identical to that of a Kerr spacetime, and thus is completely determined by, e.g., the horizon angular momentum. We generalize this to Weinstein solutions which could contain conical singularities along the axis of symmetry: The near horizon geometry of a degenerate horizon is determined by the angular momentum and the

conical singularities. The analysis boils down to a study of point singularity of certain “singular” harmonic map with values in the real hyperbolic plane.

Jose Antonio Galvez

Title: Elliptic Monge-Ampère equations in the presence of isolated singularities.

Abstract: This talk is devoted to the behavior of solutions to some geometric elliptic PDEs of Monge-Ampère type in two variables, in the presence of non-removable isolated singularities. We will describe local classification theorems around such an isolated singularity, as well as global classification theorems for the case of finitely many isolated singularities.

Shusen Yan

Title: Bubbling Solutions for the Chern-Simons Model on a Torus

Abstract: In this talk, I will introduce some of the recent results on the existence of bubbling solutions for the Chern—Simons field theories. This is joint work with C.S.Lin.

Congming Li

Title: Qualitative analysis of some differential and integral systems

Abstract: This talk will focus on the study of some systems of differential and integral equations related to the Hardy-Littlewood-Sobolev (HLS) inequality. These partial differential/integral systems have been the focal point in the field of analysis of nonlinear elliptical PDEs including the Yamabe problem and the Nirenberg problem. We study the symmetry, monotonicity, regularity, the asymptotic and the classification of the solutions.

Yanyan Li : 身體不適 取消參加此會議

Junfang Li

Title: Hardy inequalities on mean convex domains

Abstract: I will report a recent joint work with Roger Lewis and Yanyan Li. In this work, we prove that Hardy inequalities with a sharp constant hold on weakly mean convex domains. Moreover, we show that the weakly mean convexity condition cannot be weakened. We also prove various improved Hardy inequalities on mean convex domains along the line of Brezis-Marcus \cite{BM}. The key fact we observed is the distance function of a C^2 domain with boundary is superharmonic in the distribution sense if and only if the boundary is {\em weakly mean convex}.

Susanna Terracini

Title: Boundary singular solutions associated with connecting thin tubes

Abstract: Consider two domains connected by a thin tube so that the mass of a given eigenfunction (linear or nonlinear) concentrates in the first of the two domains. The restriction to the second domain develops a singularity at the junction of the tube, as the section of the channel shrinks to zero. The asymptotics for this type of solutions can be precisely described.

Arrigo Cellina

Title:

Abstract:

Frank Pacard

Title: Sign changing solutions without any symmetry for the some nonlinear Schroedinger equation

Abstract: I will explain the construction of sign changing solutions for some semilinear elliptic equation which arises in the study of stationary waves for nonlinear Schoedinger equations. This construction which is inspired by similar construction for compact constant mean curvature surfaces in Euclidean space gives rise to the existence of solutions whose energy is finite and which do not have any symmetry.

This is a joint work with W. Ao, M. Musso and J. Wei.

Gabriella Tarantello

Title: On a class of planar Liouville type problems.

Abstract: Motivated by the study of self-dual gauge field vortices and strings, we consider a class of planar Liouville type equations, that include (via stereographic projection) a singular mean field equations on the two-sphere. We discuss optimal existence, uniqueness and symmetry results according to a relevant parameter, that in the applications corresponds to the total “mass” (or total “curvature”). We also present several of the remaining open problems.

Guan Bo

Title: Dirichlet problem for eqautions of prescribed curvature

Abstract: TBA

Juncheng Wei

Title: Some new results on supercritical problems

Abstract: I will report some new results on two problems involving nonlinear elliptic equations with supercritical exponents. In the first problem, we show that Morse index solution to $\Delta u + \lambda u^p = 0$ in B_1 , $u = 0$ on $\partial B_1(0)$ is either 0 or 1, when $p \geq p_{\text{JL}}(N)$. For second problem, we find infinitely many nonradial singular solutions to $\Delta u + u^p = 0$ in $\mathbb{R}^N \setminus \{0\}$ for $p \in (\frac{N+1}{N-3}, p_{\text{JL}}(N-1))$. (Here p_{JL} is the Joseph-Lungren exponent.) (joint work with Guo, Dancer-Guo)

Meijun Zhu

Title: Hardy inequality revisited – New proof, improvement and generalization

Abstract: In the talk, we shall present a new proof for Hardy inequality with $k=2$. Then new proof yields improved Hardy inequalities due to Brezis and Vazquez with optimal constants. A generalized version of Hardy inequality will also be described.

心得及建議

參加此會議讓我的學術研究收穫良多，了解其他研究機構學者最新研究成果，面對面交換心得與想法。藉由參加會議的機會，得以再與澳洲院士 Neil Trudinger 有充分的時間進一步討論未來合作的研究及共同發表論文，會中也認識其他未曾見面同行學者。大會亦安排於會議結束後，與其他學者一起參訪李鴻章在合肥的故居，觀賞黃山美景及李安拍攝的 *Crunching tiger and tiding dragon* 現場其中一景。所有受邀者都必須自籌機票，但會後的黃山文化之旅則由研討會主辦單位免費安排。非常感謝中興大學能補助本人機票費，使本人能參加這個有意義的研討會。