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THESIS

OPTIMAL OPERATION OF SURVEILLANCE TOWERS
WITH LIMITED MANPOWER

by

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September 2010

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Tower-based surveillance systems have been employed by the U.S. military to enhance intelligence, surveillance, and reconnaissance capabilities in Iraq and Afghanistan. We consider a scenario wherein two surveillance towers are installed in separate locations; however, the surveillance team does not have enough operators to operate both towers to their capacity. Two strategies can be used to operate these two towers: stationary and dynamic. We formulate a two-person nonzero-sum game to analyze these strategies, in which the surveillance team wants to maintain regional stability while insurgents carry out attacks to disrupt it.

Our analysis suggests that the dynamic allocation strategy can improve the performance of surveillance towers over stationary allocation under most circumstances. The improvement tends to be more significant when the surveillance team has more surveillance resource. The dynamic allocation tends to be less effective when (1) a detected attack has a smaller negative impact on the insurgent operations, or when (2) a detected attack brings a larger immediate benefit to the surveillance team.

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OPTIMAL OPERATION OF SURVEILLANCE TOWERS WITH LIMITED MANPOWER

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ABSTRACT

Tower-based surveillance systems have been employed by the U.S. military to enhance intelligence, surveillance, and reconnaissance capabilities in Iraq and Afghanistan. We consider a scenario wherein two surveillance towers are installed in separate locations; however, the surveillance team does not have enough operators to operate both towers to their capacity. Two strategies can be used to operate these two towers: stationary allocation and dynamic allocation. We formulate a two-person nonzero-sum game to analyze these strategies, in which the surveillance team wants to maintain regional stability while insurgents carry out attacks to disrupt it.

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EXECUTIVE SUMMARY

Dominant intelligence, surveillance, and reconnaissance capability is one of the key enablers in irregular warfare. This thesis is motivated by the deployment of Ground-Based Operational Surveillance System—a 24-hour all-weather tower-based surveillance system—to enhance situation awareness in Iraq and Afghanistan in the late 2000s. When the number of operators is not enough to staff all surveillance towers, we examined whether it is helpful to dynamically move the operators between them, in the hope that an understaffed surveillance tower can still deter insurgency activities.

We considered the following scenario: two surveillance towers are installed in separate locations. However, the surveillance team does not have enough operators to operate both towers to their capacity. We compared two strategies: stationary allocation and dynamic allocation. With stationary allocation, the team splits up so that each tower is partially operational; with dynamic allocation, the team moves back and forth between the two towers at random intervals. We formulated a two-person nonzero-sum game, in which the surveillance team wants to maintain regional stability while the insurgents carry out attacks to disrupt it.

Our analysis suggests that the dynamic allocation strategy can improve the performance of the surveillance towers over stationary allocation under most circumstances. The improvement tends to be more significant when the surveillance team has more surveillance resource. The dynamic allocation tends to be less effective when (1) a detected attack has a smaller negative impact on the insurgent operations, or when (2) a detected attack brings a larger immediate benefit to the surveillance team. These findings can provide suggestions for decision makers in allocating resources to enhance ISR capabilities on the battlefield.

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I.   INTRODUCTION

A.   BACKGROUND

1.   Intelligence, Surveillance, and Reconnaissance

In modern warfare, military forces seek not only to advance weapon systems, but also to enhance their capability in intelligence, surveillance, and reconnaissance (ISR). According to Joint Publication 1-02, ISR is an activity that synchronizes and integrates the planning and operation of sensors and assets, and processing, exploitation, and dissemination systems in direct support of current and future operations [1]. ISR is an integrated intelligence and operations function. For U.S. military operations in Iraq and Afghanistan, ISR capabilities are more critical than ever before, due to the nature of insurgent activities. Since insurgents can blend in easily with non-combatant citizens, an ability to dominate ISR on the battlefield is critical. A great deal of effort has been exerted to enhance ISR capabilities. For example, unmanned aerial vehicles and Ground-Based Operational Surveillance Systems (G-BOSS) have been deployed for some time with many documented success stories. Generally speaking, these systems provide surveillance and reconnaissance on the battlefield by collecting video and audio intelligence to enhance the commander's situational awareness on the battlefield. This thesis focuses on G-BOSS.

2.   Ground-Based Operational Surveillance System

G-BOSS is a tower-based surveillance system derived from the sensor suite utilized on the Rapid Aerostat Initial Deployment, as shown in Figure 1. This system consists of four major assemblies:

- Cameras, including one primary infrared camera (FLIR T-3000), and one electro-optical infrared camera (FLIR Star SAFIRE IIIFP).
- One mobile tower (approximately 107 feet tall).
- One Man-Portable Surveillance and Target Acquisition Radar (MSTAR).
- One Ground Control Station (GCS).

Currently, the United States Marine Corps (USMC) deploys G-BOSS to Iraq and Afghanistan to enhance ISR capabilities. The USMC awarded the initial $60 million G-BOSS contract to Raytheon on April 9, 2008. The goal is to use these surveillance systems to detect and disrupt insurgent activities. According to a news release from the Quantico Sentry [2], G-BOSS is being deployed in four phases.

1. Phase One is deployment of G-BOSS to coalition outposts. During this phase, the system is operated manually at the base of each tower, with radio communications to the Combat Operation Center (COC) as shown in Figure 2.

2. During Phase Two, G-BOSS is operated but the data/information is automatically fed to the COC.

3. During Phase Three, G-BOSS is controlled from within the COC, with automatic slewing or rotating capabilities. During this phase, video storage capabilities are integrated.

4. By Phase Four, the surveillance crew in charge of monitoring G-BOSS can track not only what is happening in their own region, but also that of the entire province through an integrated network.

As this system is phased in, more monitors will be installed in COCs for consistent surveillance. It will thus be necessary to increase the number of operators to staff all systems in order to have better surveillance results. Increasing the number of operators, however, is usually a carefully considered constraint in combat situations. If it is not possible to increase the number of operators, the workload of current operators will then increase accordingly. According to Parasuraman and Mouloua [3], "the most significant factor that may influence the accuracy of monitoring under automation is task loads imposed on the operator." What can be done to make the best use of G-BOSS when facing a manpower constraint?

This thesis explores the idea of assigning operators in a dynamic manner. Instead of assigning a single operator to one G-BOSS, the operator is moved among systems from time to time. Because the tower-based surveillance system is prominent in the areas where it is installed, the tower may still produce a deterrent effect for insurgents, even if it is not actively being monitored. With that idea in mind, one group of operators can be assigned to shift their attention back and forth between towers; the tower without operators will serve as a decoy. The objective in this thesis is to use mathematical models

to determine whether dynamic allocation of manpower can improve the performance of multiple systems, and whether a decoy tower can provide a deterrent effect with insurgents.

Figure 1.   Top of surveillance tower
(From: FLIR http://www.gs.flir.com/datasheets/land.cfm)

Figure 2.   G-BOSS control room (From: Quantico Sentry)

B. OBJECTIVE

The goal of this thesis is to determine whether it is helpful to dynamically move manpower between surveillance towers when manpower is limited. The following scenario is considered: two surveillance towers are installed in two desired locations. However, there is only one surveillance team to operate one tower at its full capacity. It is possible either to move the surveillance team back and forth between the two towers, or to split the team so that each tower can be partially operational. The thesis develops mathematical models to study these two strategies. The findings in this thesis can provide suggestions for decision makers while employing surveillance systems on the battlefield.

C. RELATED WORKS

A significant amount of work has been done to improve the performance and effectiveness of surveillance systems in ISR and perimeter protection. The work can be divided into three categories.

First, from the perspective of technology, the advances of cameras have had a significant influence on system performance, particularly the combination of a high-resolution charge-coupled device and electro-optics with an infrared sensor system [4]. With a longer surveillance range, higher image resolution, and information integration, a camera could remotely monitor an adversary's activity day and night. These new surveillance technologies not only mitigate false detection rates, but also help reduce crew requirements.

Second, there is a stream of work that uses mathematical modeling and optimization to improve surveillance results. Szechtman *et al.* [5] used mathematical models to analyze optimal strategies for a moving surveillance sensor to detect infiltrators on a border. Midgette [6] proposed an agent-based simulation model to elevate the operational effectiveness of G-BOSS as guidance for system fielding. Also, William [7] carried out a surveillance and interdiction model with a game-theoretic approach to fight against vehicle-borne improvised explosive devices.

Third, there are studies that compare the frequency of criminal activity before and after installation of surveillance monitors. Gill and Spriggs [8] summarized their research on the impact of using closed-circuit television (CCTV) in different cities throughout Great Britain thusly:

> The use of CCTV needs to be supported by a strategy outlining the objectives of the system and how these will be fulfilled. This needs to take account of local crime problems and prevention measures already in place.

Welsh and Farrington [9] concluded their research about using CCTV in crime prevention as follows:

> Overall, it might be concluded that CCTV reduces crime to a small degree. In light of the successful results, future CCTV schemes should be carefully implemented in different settings and should employ high quality evaluation designs with long follow-up periods.

Conclusions from previous research indicates that having appropriate surveillance equipment is a key enabler toward better detection results in a surveillance plan, which corresponds to the objective of this thesis.

D.   THESIS ORGANIZATION

The rest of this thesis is organized as follows. Chapter II formulates a two-person nonzero-sum game to model the interaction between coalition forces and insurgents. Coalition forces assign manpower between two surveillance towers, while insurgents launch attacks in order to interrupt regional stability. In Chapter III, numerical analysis is carried out to demonstrate the model. Situations are identified in which it is helpful to dynamically allocate manpower between the two surveillance towers. Finally, Chapter IV presents findings and suggests future research directions.

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II.   METHODOLOGY

A.   MODEL

Consider a situation in which Blue has established military bases in two towns. Blue's goal is to maintain peace and eliminate insurgent activities in these two towns. (From now on insurgent activities will be referred to as *attacks* for brevity.) Blue has one surveillance tower set up in each base, but Blue cannot detect all attacks in both towns at all times, due to a lack of resources (manpower, equipment, etc.). Denote by s ($s \leq 2$) the total resources available to Blue, such that Blue can allocate detection probability p_i to tower i, as long as $p_1 + p_2 \leq s$ and $0 \leq p_i \leq 1$, for $i = 1, 2$. The problem facing Blue is how to allocate s between the two surveillance towers.

In each town, an insurgent group attempts to carry out attacks for its own gain. The insurgent group operating in town i is referred to as Red i, for $i = 1, 2$. Red 1 and Red 2 operate independently from each other. For each Red team, the status quo is not to attack, in which case neither Red nor Blue receives a reward or a penalty. If a Red team launches an attack, there are two possible outcomes: either the attack is detected by Blue's surveillance tower, or it is not. The Red team earns reward of $+1$ for each undetected attack, and incurs a penalty $r > 0$ (reward $-r$) for each detected attack. Because Blue's goal is to maintain peace and ideally to eliminate attacks altogether, there is a penalty for each attack regardless of whether or not the attack is detected. However, detecting an attack is better than not detecting it, so Blue incurs a penalty 1 (reward -1) for an undetected attack and a smaller penalty $b \in (0,1)$ (reward $-b$) for a detected attack. Table 1 summarizes the reward for Blue and each Red team, respectively.

We model the interaction between Blue and two Red teams as a nonzero-sum game, where Blue moves first, and then each Red team moves second, independently, after observing Blue's strategy. The objective of each player is to maximize his own long-run average reward.

	No attack	Attack undetected	Attack detected
Red	0	+1	$-r$
Blue	0	-1	$-b$

Table 1.   Reward table

If the detection probability is p in a town, Blue's expected reward for each attack is

$$(-1)(1-p)+(-b)p = -1+(1-b)p, \tag{1}$$

and Red's expected reward for each attack is

$$(+1)(1-p)+(-r)p = 1-(1+r)p. \tag{2}$$

By setting Equation (2) to 0, we can solve

$$\hat{p} = \frac{1}{1+r}. \tag{3}$$

If $p > \hat{p}$, Equation (2) is negative, so it is optimal for a Red team to shut down its operation altogether. In the special case when $p = \hat{p}$, Red's expected reward for each attack is 0, so Red feels indifferent about whether to attack or not. For mathematical completeness, however, assume that Red will continue to attack if $p = \hat{p}$, as it gives Blue a negative expected reward.

Suppose each Red team can carry out attacks at a maximum rate x. Consider three cases for s:

1. $s \in (2\hat{p}, 2]$. If Blue allocates $p_1 = p_2 = s/2 > \hat{p}$, then both Red teams will stop their operations. The long-run reward rate is 0 for all two players.
2. $s \in [0, \hat{p}]$. No matter how Blue allocates s, both Red teams will continue to attack at the maximum rate x. The total long-run reward rate for both Red teams is
    $$x(1-(1+r)p_1 +1-(1+r)p_2) = x(2-(1+r)s). \tag{4}$$
    Blue's long-run reward rate is
    $$x(-1+(1-b)p_1 -1+(1-b)p_2) = x(-2+(1-b)s). \tag{5}$$
3. $s \in (\hat{p}, 2\hat{p}]$. In this case, it is possible for Blue to allocate the detection probability such that it is optimal for one Red team to stop its operation.

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The rest of this section focuses on the case when $s \in (\hat{p}, 2\hat{p}]$. In particular, it examines two strategies for Blue: stationary allocation and dynamic allocation.

1. Stationary Allocation

With a stationary allocation, Blue assigns p_i to surveillance tower i, $i = 1, 2$, on a permanent basis. It is reasonable to assume that each Red team will discover this allocation sooner or later, whether by intelligence or by computing its own success rate. Without loss of generality, assume $p_1 \geq p_2$. It does not help to set $p_1 \leq \hat{p}$, with which the optimal strategy for each Red team is to attack at the maximum rate x. If Blue sets $p_1 = \hat{p} + \varepsilon$, for some $\varepsilon > 0$, then it is optimal for Red 1 to cease the operation, and for Red 2 to attack at rate x.

Using Equation (2), Red 2's long-run reward rate is

$$x(1 - (1 + r)(s - \hat{p} - \varepsilon)) = x(2 - (1 + r)(s + \varepsilon)), \tag{6}$$

which converges to

$$x(2 - (1 + r)s) \tag{7}$$

as $\varepsilon \downarrow 0$.

Using Equation (1), Blue's long-run reward rate is

$$x(-1 + (1 - b)(s - \hat{p} - \varepsilon)) = x\left(-1 + (1 - b)\left(s - \frac{1}{1 + r} - \varepsilon\right)\right), \tag{8}$$

which converges to

$$x\left(-1 + (1 - b)\left(s - \frac{1}{1 + r}\right)\right), \tag{9}$$

as $\varepsilon \downarrow 0$.

2. Dynamic Allocation

With a dynamic allocation, Blue first assigns p to one tower and $s - p$ to the other tower, and then swaps these allocations from time to time. Without loss of generality, assume $p > s - p$. The idea of dynamic allocation is to make $p > \hat{p}$ so that sometimes it is optimal for a Red team to pause attacks, but each Red team needs to guess

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when to resume attacks. The tower with detection probability $s - p$ can be viewed as a decoy, which may provide a deterrence effect if a Red team does not know that detection probability has dropped from p to $s - p$.

Blue has two decision variables p and y, such that Blue allocates detection probability p to one tower and $s - p$ to the other, and swaps these allocations at a Poisson rate y. Assume that the battle goes on indefinitely, and that over time each Red team learns about Blue's choices of p and y, but does not discover Blue's real-time allocation. Because the two Red teams do not interact with each other, and because the parameters are identical in the two towns, from now on the analysis will focus on the interaction between Blue and one Red team, henceforth Red for brevity.

One feasible strategy for Red is to attack at a Poisson rate x. Alternatively, Red can set aside some effort to learn about the real-time detection probability at a Poisson rate z. Red can do this by sending a spy, bribing Blue's people, or probing the system in some way. We will impose a constraint that requires $x + \alpha z \leq c$, where $\alpha > 0$ models the tradeoff between the *attack rate x* and the *learning rate z*, and c is the maximum attack rate if Red sets the learning rate to 0. With a learning rate $z > 0$, Red would learn about the detection probability at time moments that constitute a Poisson process with rate z. In other words, the time between two consecutive learning points follows an exponential distribution with rate z, independent of everything else.

Recall that $p > s - p$. We say a surveillance tower is in state 1 if its detection probability is p, and in state 0 if its detection probability is $s - p$. In other words, each tower remains in state 1 for a random time that is exponentially distributed with mean $1/y$, and then switches to state 0 and stays in state 0 for another random time, which is also exponentially distributed with mean $1/y$, and so on. In the long run, each tower will be in each state 50% of the time. We say Red is in state 1 if Red is carrying out attacks at a Poisson rate x, and in state 0 if Red pauses its attacks. Red decides when it wants to move from one state to the other.

Because $p > \hat{p}$, when Red learns the tower is in state 1, Red should pause its attacks. Let $P_{jk}(t)$ denote the probability that the tower will be in state k after t time units if it is currently in state j, $j, k = 0, 1$. Using the result in Ross [10], we have

$$P_{11}(t) = \frac{1}{2} + \frac{1}{2}e^{-2yt} = 1 - P_{10}(t).$$

Red can compute the probability of detection after t time units once it learns that Blue is in state 1, if Red does not having a learning point in the next t time units, as

$$P_{11}(t) \cdot p + P_{10}(t) \cdot (s - p) = \left(\frac{1}{2} + \frac{1}{2}e^{-2yt} \right) p + \left(\frac{1}{2} - \frac{1}{2}e^{-2yt} \right)(s - p).$$

Red should attack if this detection probability is less than $\hat{p}$. After some algebra, we can show that Red should wait for another

$$\hat{t} = \frac{\ln\left(\dfrac{2p - s}{2\hat{p} - s} \right)}{2y} \tag{10}$$

time units before resuming attacks, if Red does not have another learning point in this time period. Consequently, Red's optimal strategy takes the following form: whenever Red learns that Blue's tower is in state 1, Red pauses its attacks until the next learning point or until $\hat{t}$ time units have elapsed. If Red learns Blue's state is 0 within the next $\hat{t}$ time units, Red should resume attacks immediately; if Red does not have a learning point within the next $\hat{t}$ time units, then Red resumes attacks after $\hat{t}$ time units. With this strategy, we can define a renewal reward process, where a renewal is a time moment when Red learns that Blue's tower is in state 1. Figure 3 depicts this renewal reward process.

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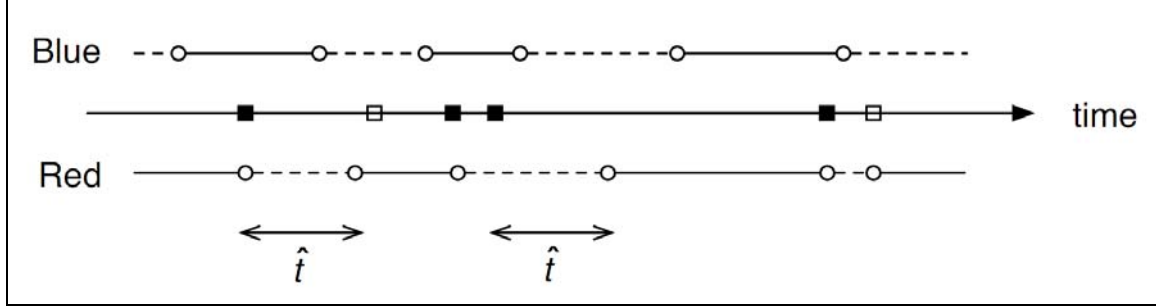
Figure 3.   Renewal reward process.

This diagram depicts the renewal reward process if Blue dynamically allocates its resource. For Blue, each circle represents a switch point, with solid lines being in state 1 (detection probability p) and dashed lines being in state 0 (detection probability s-p). For Red, solid lines indicate state 1 (attacking) and dashed lines indicate state 0 (not attacking). For the time line, each square represents a Red's learning point, with a solid square being a renewal (Blue in state 1).

Let T denote the cycle time in this renewal reward process. In addition, denote by T_k the time until the next renewal if Blue's current state is k, $k = 0,1$. To compute $E[T_1]$, consider the next event. If Blue's current state is 1, then the next event can either be Blue's switch to state 0, or Red's learning Blue's state. Because the time to each event is exponentially distributed, the time to *either* event, whichever occurs first, is also exponentially distributed with a rate equal to the sum of the two individual rates $y + z$. With probability $y / (y + z)$, the next event is Blue's switch to state 0, in which case the additional time until a renewal is distributed as T_0. With probability $z / (y + z)$, the next event is Red's learning Blue's state to be 1, which constitutes a renewal. Therefore, we can write

$$E[T_1] = \frac{1}{y + z} + \frac{y}{y + z} E[T_0].$$

With a similar argument, we can write

$$E[T_0] = \frac{1}{y} + E[T_1].$$

Solving the preceding yields

$$E[T_1] = \frac{2}{z}, \text{ and } E[T_0] = \frac{1}{y} + \frac{2}{z}.$$

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By definition, T and T_1 have identical distributions, so

$$E[T] = \frac{2}{z}.$$

Let X denote the number of detected attacks in a cycle, and Y the number of undetected attacks in a cycle. If Blue is in state k ($k = 0, 1$) and Red is in state 1 (attacking), then let X_k denote the number of detected attacks until the next renewal, and Y_k the number of undetected attacks until the next renewal.

To compute $E[X_1]$, consider whether Blue switches to state 0 first or Red learns Blue's state first. The time until either event occurs follows an exponential distribution with rate $y + z$, so the expected number of detections during this time period is $px/(y+z)$. Moreover, with probability $y/(y+z)$, Blue will switch to state 0 first, in which case the additional number of detected attacks in the cycle is distributed as X_0. With probability $z/(y+z)$, Red will learn that Blue is in state 1 first, which constitutes a renewal. Therefore, we can write

$$E[X_1] = \frac{x}{y+z}p + \frac{y}{y+z}E[X_0].$$

With a similar argument, we can write

$$E[X_0] = \frac{x}{y}(s-p) + E[X_1].$$

Solving from the preceding yields

$$E[X_1] = \frac{x}{z}s \text{, and } E[X_0] = \frac{x}{y}(s-p) + \frac{x}{z}s.$$

In a similar way, we can set up two linear equations involving $E[Y_1]$ and $E[Y_0]$ as follows:

$$E[Y_1] = \frac{x}{y+z}(1-p) + \frac{y}{y+z}E[Y_0],$$

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$$E[Y_0] = \frac{x}{y}(1 - (s - p)) + E[Y_1].$$

Solving from these two linear equations yields

$$E[Y_1] = \frac{x}{z}(2 - s), \text{ and } E[Y_0] = \frac{x}{y}(1 - (s - p)) + \frac{x}{z}(2 - s).$$

Now we proceed to compute $E[X]$ and $E[Y]$. Let Z denote the time of the first learning point after the renewal, which follows an exponential distribution with rate z. To compute $E[X]$, condition on the event $Z = t$. If $t < \hat{t}$, then at time t, either (1) the cycle ends if Blue is in state 1, or (2) Red resumes attacks (moves to state 1) if Blue is in state 0. If $t > \hat{t}$, then Red resumes attacks at time $\hat{t}$. Therefore,

$$E[X] = \int_0^{\hat{t}} P_{10}(t) E[X_0] z e^{-zt} dt + e^{-z\hat{t}} (P_{11}(\hat{t}) E[X_1] + P_{10}(\hat{t}) E[X_0])$$

$$= \int_0^{\hat{t}} \left(\frac{1}{2} - \frac{1}{2} e^{-2yt} \right) z e^{-zt} dt E[X_0] + e^{-z\hat{t}} \left(\frac{1}{2} + \frac{1}{2} e^{-2y\hat{t}} \right) E[X_1] + e^{-z\hat{t}} \left(\frac{1}{2} - \frac{1}{2} e^{-2y\hat{t}} \right) E[X_0]$$

$$= \frac{1}{2} \frac{x}{z} s(1 + e^{-z\hat{t}}) - \frac{1}{2} \frac{x}{z + 2y} (2p - s)(1 - e^{-(z+2y)\hat{t}}),$$

where $\hat{t}$ is given in Equation (10). Similarly,

$$E[Y] = \int_0^{\hat{t}} P_{10}(t) E[Y_0] z e^{-zt} dt + e^{-z\hat{t}} (P_{11}(\hat{t}) E[Y_1] + P_{10}(\hat{t}) E[Y_0])$$

$$= \frac{1}{2} \frac{x}{z}(2 - s)(1 + e^{-z\hat{t}}) + \frac{1}{2} \frac{x}{z + 2y}(2p - s)(1 - e^{-(z+2y)\hat{t}}).$$

Red's long-run reward rate is equal to (renewal reward theory)

$$R(p, y, x, z) = (+1) \frac{E[Y]}{E[T]} + (-r) \frac{E[X]}{E[T]}$$

$$= \frac{x}{4} \left((2 - (1 + r)s)(1 + e^{-z\hat{t}}) + (1 + r)(2p - s) \frac{z}{z + 2y}(1 - e^{-(z+2y)\hat{t}}) \right). \tag{11}$$

Red's decision variables are x and z, subject to $x + \alpha z \le c$. Blue's long-run reward rate is

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$$B(p, y, x, z) = (-1)\frac{E[Y]}{E[T]} + (-b)\frac{E[X]}{E[T]}$$

$$= \frac{x}{4}\left((-2 + (1-b)s)(1 + e^{-z\hat{t}}) - (1-b)(2p-s)\frac{z}{z+2y}(1 - e^{-(z+2y)\hat{t}})\right)$$

(12)

with decision variables p and y.

Remark 1. An important parameter to consider is the long-run proportion of time when Red is attacking. From the definition of the renewal process, at the beginning of each cycle Red will remain in state 0 until either the next learning point, or $\hat{t}$, whichever occurs first. In other words, the amount of time Red is in state 0 in each cycle is $\min(W, \hat{t})$, where W follows an exponential distribution with rate z. In each cycle, the expected time that Red is not attacking (state 0) is

$$E[\min(W, \hat{t})] = \int_0^{\hat{t}} w \cdot ze^{-zw}dw + \int_{\hat{t}}^{\infty} \hat{t} \cdot ze^{-zw}dw = \frac{1}{z}(1 - e^{-z\hat{t}}).$$

Consequently, the long-run proportion of time Red is not attacking (state 0) is

$$\frac{E[\min(W, \hat{t})]}{E[T]} = \frac{1}{2}(1 - e^{-z\hat{t}}).$$

(13)

The long-run proportion of time Red is attacking (state 1) is

$$\frac{1}{2}(1 + e^{-z\hat{t}}).$$

(14)

Remark 2. We assume that the two Red teams are operating independently, without any coordination. That is, when the Red team in one town learns the tower's detection probability, it does not give this information to the Red team in the other town. In the case when the two Red teams maintain real-time communication, essentially the learning rate at each town is doubled and the same analysis applies.

B. RED'S OPTIMAL STRATEGY

When Blue dynamically allocates its resources between the two surveillance towers, each player has two decision variables, as shown in Equations (11) and (12). In this model, Blue moves first and Red moves second, with each player trying to maximize his own long-run average reward. To compute this equilibrium, we first solve Red's optimization problem for given p and y. Although Red has two decision variables, at optimality the constraint $x + \alpha z \le c$ must be equality, because $R(p, y, x, z)$ strictly increases in x when z is held constant. Substituting $x = c - \alpha z$ into Equation (11), Red's objective function involves a single variable z as follows:

$$R(z) = \frac{c - \alpha z}{4}\left(\left(2 - (1+r)s\right)\left(1 + e^{-z\hat{t}}\right) + (1+r)(2p - s)\frac{z}{z + 2y}\left(1 - e^{-(z+2y)\hat{t}}\right)\right).$$

By letting

$$K_1 \equiv 2 - (1+r)s,$$
$$K_2 \equiv (1+r)(2p - s),$$

and using Equation (3) and (10) to get

$$\frac{K_1}{K_2} = \frac{\dfrac{2}{(1+r)} - s}{2p - s} = \frac{2\hat{p} - s}{2p - s} = e^{-2y\hat{t}} < 1,$$

we can simplify $R(z)$ to

$$
\begin{aligned}
R(z) &= \frac{c - \alpha z}{4}\left(K_1\left(1 + e^{-z\hat{t}}\right) + K_2\frac{z}{z + 2y}\left(1 - e^{-(z+2y)\hat{t}}\right)\right) \\
&= \frac{c - \alpha z}{4}K_2\left(e^{-2y\hat{t}}\left(1 + e^{-z\hat{t}}\right) + \frac{z}{z + 2y}\left(1 - e^{-(z+2y)\hat{t}}\right)\right) \\
&= K_2\frac{c - \alpha z}{4}\left(e^{-2y\hat{t}} + \frac{z}{z + 2y} + \frac{2y}{z + 2y}e^{-(z+2y)\hat{t}}\right).
\end{aligned}
$$
(15)

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Proposition 3. The function $R(z)$ is concave in z.

Proof: We will show $R''(z) < 0$ to complete the proof. To facilitate the computation, let

$$g(z) = \frac{c - \alpha z}{4},$$

$$h(z) = e^{-2y\hat{t}} + \frac{z}{z + 2y} + \frac{2y}{z + 2y}e^{-(z+2y)\hat{t}},$$

so $R''(z) = K_2(g''(z)h(z) + 2g'(z)h'(z) + g(z)h''(z))$.

For $g(z)$, compute

$$g'(z) = -\frac{\alpha}{4} < 0. \qquad g''(z) = 0.$$

Taking the first derivative of $h(z)$ yields

$$
\begin{aligned}
h'(z) &= \frac{2y}{(z+2y)^2}\left(1 - e^{-(z+2y)\hat{t}} - (z+2y)\hat{t}e^{-(z+2y)\hat{t}}\right) \\
&= \frac{2y}{(z+2y)^2}\left(1 - e^{-(z+2y)\hat{t}}\left(1 + (z+2y)\hat{t}\right)\right) \\
&> \frac{2y}{(z+2y)^2}\left(1 - e^{-(z+2y)\hat{t}}e^{(z+2y)\hat{t}}\right) = 0
\end{aligned}
$$

where the inequality follows by letting $\Delta = (z+2y)\hat{t} > 0$ in the inequality $1 + \Delta \le e^{\Delta}$. In addition,

$$
\begin{aligned}
h''(z) &= \frac{4y}{(z+2y)^3}\left(-1 + e^{-(z+2y)\hat{t}} + (z+2y)\hat{t}e^{-(z+2y)\hat{t}} + \frac{1}{2}\left((z+2y)\hat{t}\right)^2 e^{-(z+2y)\hat{t}}\right) \\
&= \frac{4y}{(z+2y)^3}\left(-1 + e^{-(z+2y)\hat{t}}\left(1 + (z+2y)\hat{t} + \frac{1}{2}\left((z+2y)\hat{t}\right)^2\right)\right) \\
&< \frac{4y}{(z+2y)^3}\left(-1 + e^{-(z+2y)\hat{t}}e^{(z+2y)\hat{t}}\right) = 0,
\end{aligned}
$$

where the inequality follows by letting $\Delta = (z+2y)\hat{t} > 0$ in the inequality $1 + \Delta + \frac{\Delta^2}{2} \le e^{\Delta}$. Consequently, $R''(z) < 0$, so $R(z)$ is concave in z. ∎

Red's objective is to choose $z \in [0, c/\alpha]$ to maximize $R(z)$. Because $R(z)$ is concave in z, to maximize $R(z)$, first compute

$$R'(0) = \frac{K_1}{4}\left(-2\alpha - c\hat{t}\right) + \frac{K_2}{4}\frac{c}{2y}\left(1 - e^{-2y\hat{t}}\right).$$

(16)

Consider two cases:

1. $R'(0) > 0$: In this case, it is optimal to set $z^* = 0$.

2. $R'(0) < 0$: Red wants to maximize $R(z)$ for $z \in [0, c/\alpha]$. Because $R(z)$ is concave and $R'(c/\alpha) < 0$, to maximize $R(z)$, it is equivalent to solve $R'(z) = 0$. A simple bisection algorithm is given below to compute z^* such that $R'(z^*) = 0$. The constant δ is the error bound on the solution.

(a) Let $a \leftarrow 0$ and $b \leftarrow c/\alpha$.

(b) Let $m \leftarrow (a + b)/2$, and compute $R'(m)$.

(c) If $R'(m) = 0$, then $z^* = m$ and exit. If $R'(m) > 0$, then let $a \leftarrow m$; if

$R'(m) < 0$, then let $b \leftarrow m$.

(d) If $b - a > \delta$, go to (b); otherwise $z^* = a$ and exit.

C.   BLUE'S OPTIMAL STRATEGY

Denote the optimal learning rate derived from the preceding algorithm by $z^*(p, y)$, and let $x^*(p, y) = c - \alpha z^*(p, y)$. Let

$$\hat{B}(p, y) = B(p, y, x^*(p, y), z^*(p, y)),$$

(17)

which Blue wishes to maximize by choosing p and y. To compute Blue's optimal strategy, we first plot $\hat{B}(p, y)$ and observe that the function is unimodal in each variable. We use the following algorithm to compute Blue's optimal strategy.

1. Let $i \leftarrow 0$, and $p_i \leftarrow \min(1, (s + \hat{p})/2)$. Use the golden section search to compute $y_i \leftarrow \arg\max_y \hat{B}(p_i, y)$

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2. Use the golden section search to compute $p_{i+1} \leftarrow \arg\max_p \hat{B}(p, y_i)$.

3. Use the golden section search to compute $y_{i+1} \leftarrow \arg\max_y \hat{B}(p_{i+1}, y)$.

4. If $\hat{B}(p_{i+1}, y_{i+1}) - \hat{B}(p_i, y_i) > \delta$, then let $i \leftarrow i+1$ and go to step 2. The parameter δ is the error bound.

5. Output $y^* = y_{i+1}$ and $p^* = p_{i+1}$ as Blue's optimal strategy.

We implement the preceding algorithm in Microsoft Excel using VBA. The end result is a decision aid that computes the optimal strategies for both players. For more details on the decision aid, see Appendix A.

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III.   NUMERICAL ANALYSIS

The model presented in this thesis has five parameters, namely r, b, s, c, and α. The parameters r and b model the trade-off between a detected attack and an undetected attack for Red and Blue, respectively. The parameter s represents the total resources available to Blue. The parameter c represents the maximum effort Red can divide between attacking and spying on Blue tower status. Without loss of generality, we can set $c = 1$, because using another value is equivalent to scaling the clock to a different time unit. Finally, the parameter α models the trade-off between Red's attack rate and its learning rate.

Intuitively, a small α implies that it is easy to learn about Blue's tower status, so Red can set aside more effort to attack. However, the effect of learning depends not only on Red's learning rate x but also Blue's switch rate y. Because Blue can set y freely, it turns out that the parameter α does not have any effect on the optimal solution. Mathematically, rewrite Equation (11) as $R(p, x, y, z, \alpha)$ and Equation (12) as $B(p, x, y, z, \alpha)$ to signify its dependence on α, and note that

$$R(p, y, x, z, \alpha) = R(p, \alpha y, x, \alpha z, 1)$$
$$B(p, y, x, z, \alpha) = B(p, \alpha y, x, \alpha z, 1)$$

In other words, if we treat αz (instead of z) as Red's decision variable and αy (instead of y) as Blue's decision variable, then we convert the original problem to an equivalent problem with $\alpha = 1$. Consequently, we can also set $\alpha = 1$ without loss of generality.

From Blue's standpoint, the optimal choice of y involves a delicate balance. If y is too small (say, once a year), then Red can easily take advantage of it by setting a moderate learning rate without much sacrifice to its attack rate. If y is too large (say, once an hour), then Red might as well give up learning altogether and attack at the maximum rate 1, which defeats Blue's purpose of using decoy surveillance towers. In

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other words, Blue's choice of y needs to be large enough to keep Red honest, and small enough so that Red has an incentive to set aside some effort to spy on Blue's operations.

By setting $c = \alpha = 1$, there are three parameters we need to consider. In Section A, we set $r = 4$, $b = 0.5$, and $s = 0.3$ as our main example in order to demonstrate how to compute the optimal strategy. In Section B, we vary the parameter s in order to show how dynamic allocation can improve Blue's performance beyond stationary allocation. Finally in Section C, we vary the parameters r and b in order to discuss some interesting observations.

A.   MAIN EXAMPLE

This section demonstrates how to compute the optimal strategies of Blue and Red while using dynamic allocation. We consider a plausible scenario by setting $r = 4$, $b = 0.5$. Recall that the dynamic case applies when $s \in (\hat{p}, 2\hat{p}]$, where $\hat{p} = 0.2$ and $2\hat{p} = 0.4$ according to Equation (3). We set $s = 0.3$ to demonstrate computation of the optimal strategy.

1.   Red's Optimal Strategy

Recall from Equation (15), Red has one decision variable, namely the learning rate z. Red decides on the value z after finding out Blue's detection probability p and switch rate y. For example, if Blue sets $p = 0.3$ and $y = 0.01980$, then Red can use Equation (15) to compute $R(z)$. Figure 4 depicts the function $R(z)$, which is concave in z as proved in Proposition 3. We then use bisection method to compute $z^* = 0.13237$, and from Red's constraint $x + \alpha z = c$, $x^* = 0.8696$ can be solved as well. In other words, with the optimal strategy, on average, Red will learn about Blue tower status once every 7.75 time units, and will attack once every 1.15 time units.
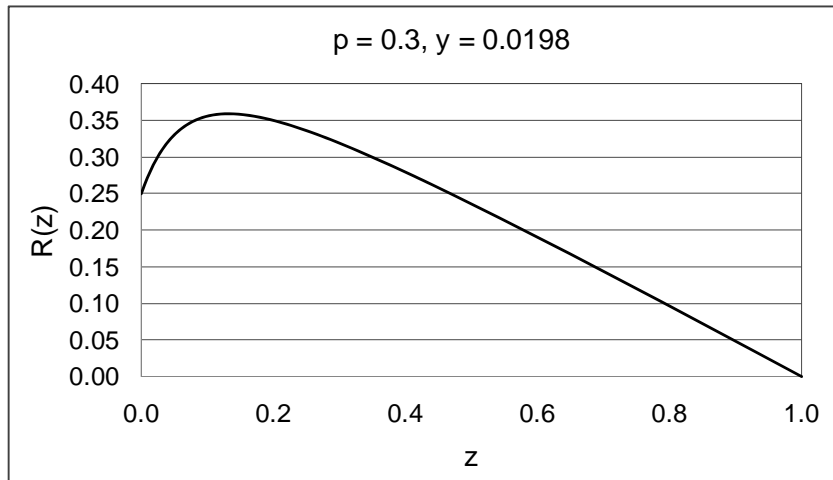
p = 0.3, y = 0.0198

Figure 4.   Red's optimal strategy: Given Blue's strategy is $p = 0.3$, $y = 0.0198$, Red should use $z* = 0.13237$.

2.   Blue's Optimal Strategy

Recall Blue's objective function $\hat{B}(p, y) = B(p, y, x*(p, y), z*(p, y))$ from Equation (17). For the main example when $r = 4$, $b = 0.5$, and $s = 0.3$, we can plot $\hat{B}(p, y)$, which is shown in Figure 5.

r = 4, b = 0.5, s = 0.3
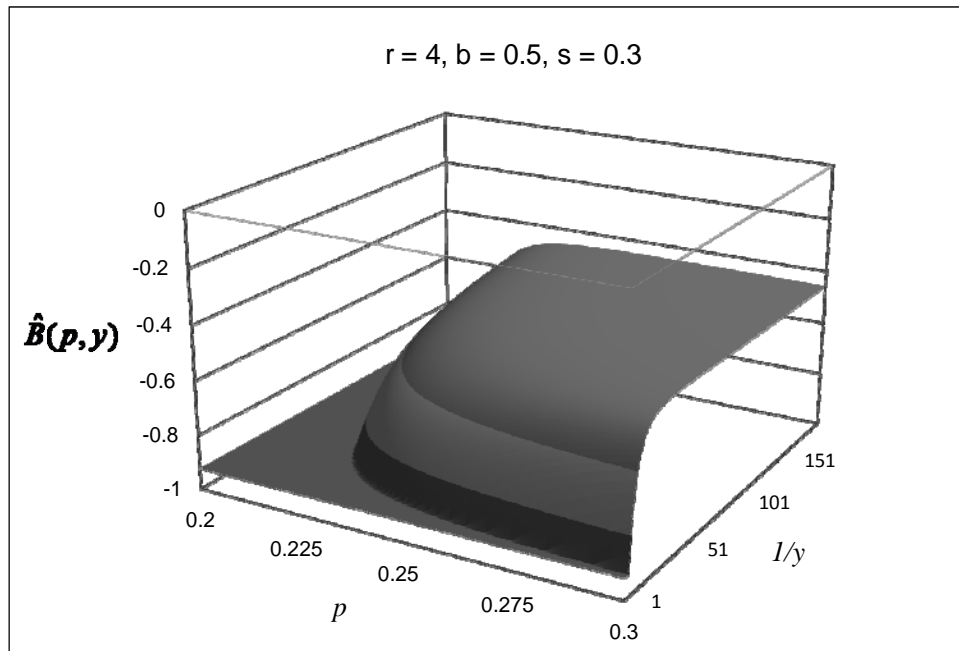
Figure 5.   Blue's objective function $\hat{B}(p, y)$

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Although it may be difficult to see, the function $\hat{B}(p,y)$ in Figure 5 is indeed unimodal in p and in y. Figure 6 shows the same function, when one of the variables is fixed. We use the golden section search method in each dimension iteratively to compute p^* and y^*, as discussed in Chapter II, Section B. In the main example, the algorithm produces $\hat{B}(p^*, y^*) = -0.43631$, when Blue set $p^* = 0.3$ and $y^* = 0.0198$. In other words, the model suggests that Blue's optimal strategy is to set one tower with detection probability $p = 0.3$ the other with detection probability $s - p = 0$ and, on average, switch between towers every 50.5 time units.
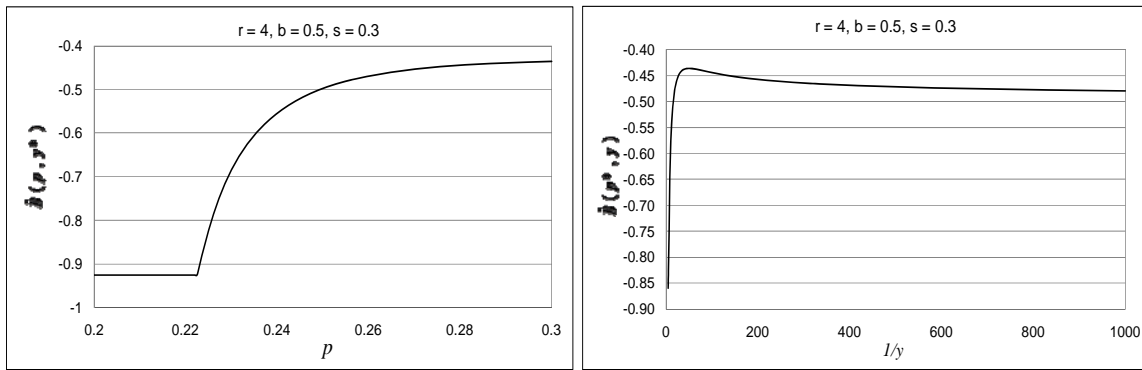
Figure 6. $\hat{B}(p, y^*)$ and $\hat{B}(p^*, y)$ are unimodal in p and y when $r = 4,\ b = 0.5,\ s = 0.3$.

Although we did not prove it mathematically, $\hat{B}(p,y)$ is unimodal in p and in y in all the numerical experiments we conducted. Another example when the optimal p does not lie on the boundary is shown in Figure 7. Consequently, our algorithm in Chapter II works well in computing the optimal solution.
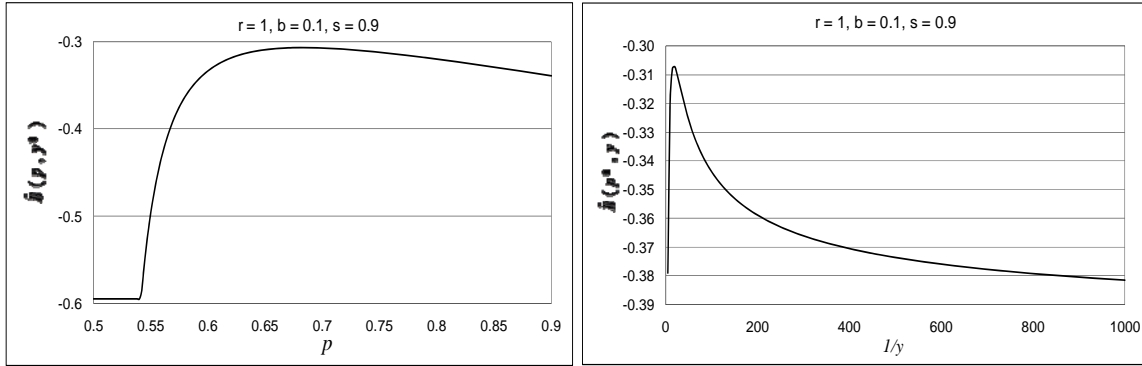
Figure 7. $\hat{B}(p, y*)$ and $\hat{B}(p*, y)$ are unimodal in p and y when $r = 1, \ b = 0.1, \ s = 0.9$.

B. EFFECTIVENESS OF DYNAMIC ALLOCATION

This section compares dynamic allocation with stationary allocation. Notice that when using dynamic allocation in two towns, the long-run reward rates derived from Equation (11) and (12) represent Red and Blue's reward in one town, respectively. For a fair comparison with stationary allocation, we multiply these two numbers by two. All the numbers reported in the remainder of this chapter refer to the total reward in the two towns.
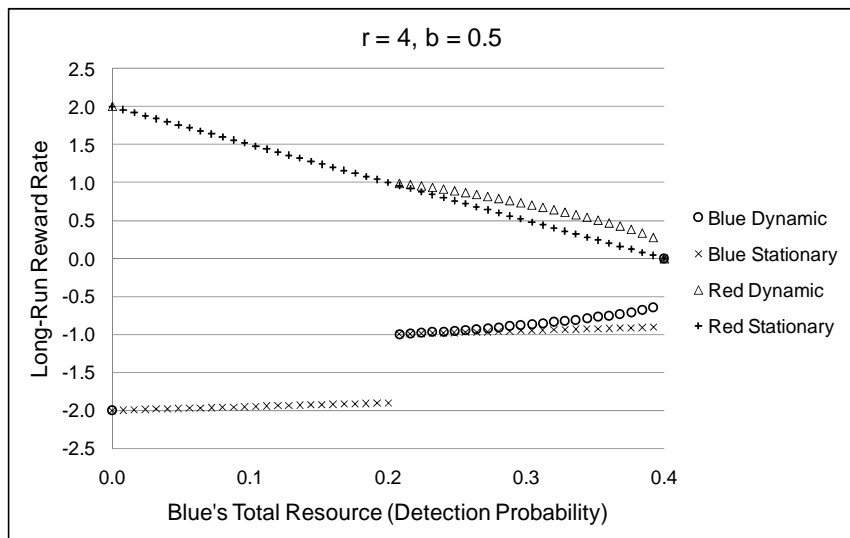
Figure 8. Comparison between dynamic allocation and stationary allocation with $r = 4, \ b = 0.5$

In the main example, we set $r = 4$, $b = 0.5$, which yields $\hat{p} = 0.2$ and $2\hat{p} = 0.4$, according to Equation (3). We consider three cases as in Chapter II, Section A.

First, in the case $s \in [0, \hat{p}]$, Blue's long-run reward rate is Equation (5), and Red's long-run reward rate is Equation (4), as derived in Chapter II, Section A. As shown in Figure 8, when $s \in [0, 0.2]$, Red's long-run reward rate decreased linearly, and Blue's long-run reward rate increases linearly, as Blue's resources increased.

Second, in the case $s \in (\hat{p}, 2\hat{p}]$, Blue has two options for resource allocation, either stationary allocation or dynamic allocation. If Blue chooses a stationary allocation strategy, we can use Equation (9) to plot Blue's long-run reward rate, and use Equation (7) to plot Red's long-run reward rate. The results of dynamic allocation come from the algorithm described in Chapter II, Section B and C. With dynamic allocation strategy, we can use Equation (12) to plot Blue's long-run reward rate, and use Equation (11) to plot Red's long-run reward rate.

Third, in the case of $s \in (2\hat{p}, 2]$, it is trivial for Blue to allocate $p_1 = p_2 = s/2 > \hat{p}$. It is optimal for both Red teams to stop their operations, so the payoffs of all players are zero. Therefore, this case is not shown in Figure 8.

As shown in Figure 8, dynamic allocation is better than stationary allocation for Blue. In our main example, compared with the stationary allocation, the Blue's long-run reward rate with dynamic allocation improves from 0.15% to 28.71% as s increased from 0.2 to 0.4.

With Blue's dynamic allocation strategy, however, Red's long-run reward rate also increases. As Red can learn the tower's status, Red will attack when attacks are less likely to be detected, and will pause when attacks are more likely to be detected. Although Red has a smaller attack rate x (because Red sets aside some effort on learning), its attacks become more effective. Consequently, Red's performance also improves when Blue uses dynamic allocation. In fact, Red's improvement as a percentage to that in stationary allocation is better than Blue's. This observation will be examined again in Section C of this chapter.

Blue's use of dynamic allocation also affects Red's operations. In Figure 9, we plot the long-run proportion of time Red is attacking as derived in Equation (14). The proportion of time Red is attacking is higher than 50% (stationary allocation), and increases when Blue's total resource s increases. The long-run attack rate, however, is computed by multiplying Equation (14) with the instantaneous attack rate x. As seen in Figure 10, Red's long-run attack rate is less than 50% (stationary allocation), and decreases when s increases.
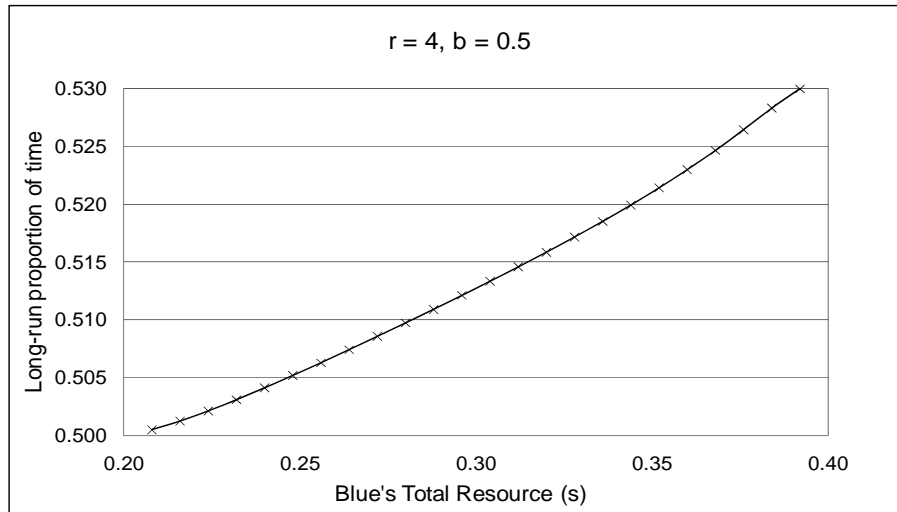
Figure 9.   Long-run proportion of time Red is attacking in dynamic allocation, compared with 0.5 in the case of stationary allocation.
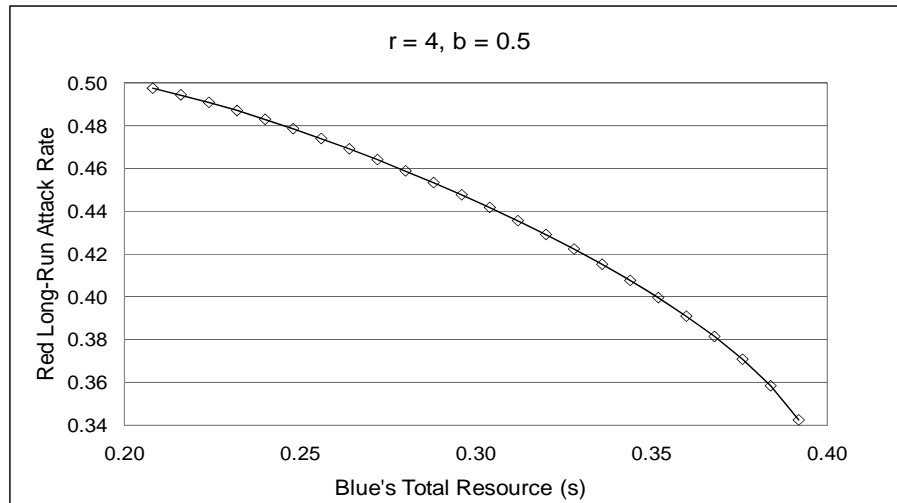
Figure 10.   Red long-run attack rate in dynamic allocation, compared with 0.5 in the case of stationary allocation.

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C.   DISCUSSIONS

In this section, we vary parameters b and r to see how they affect the optimal strategy. First, we fix $r = 4$, and compare three value of $b = 0.1,\ 0.5,\ 0.9$, as shown in Figure 11.
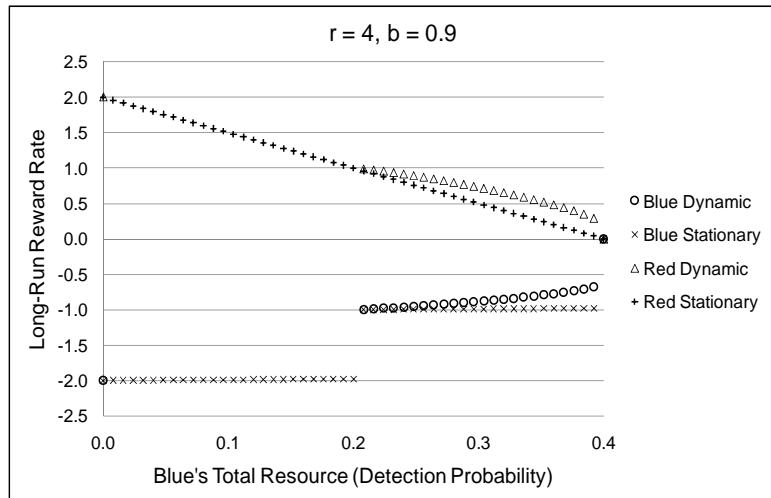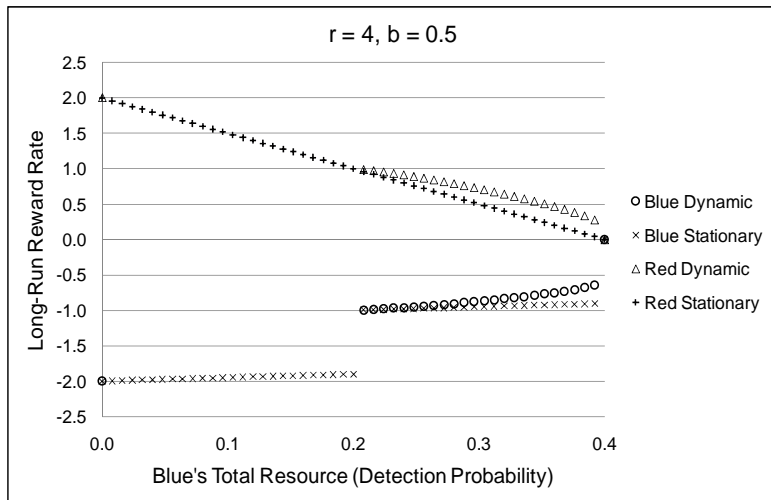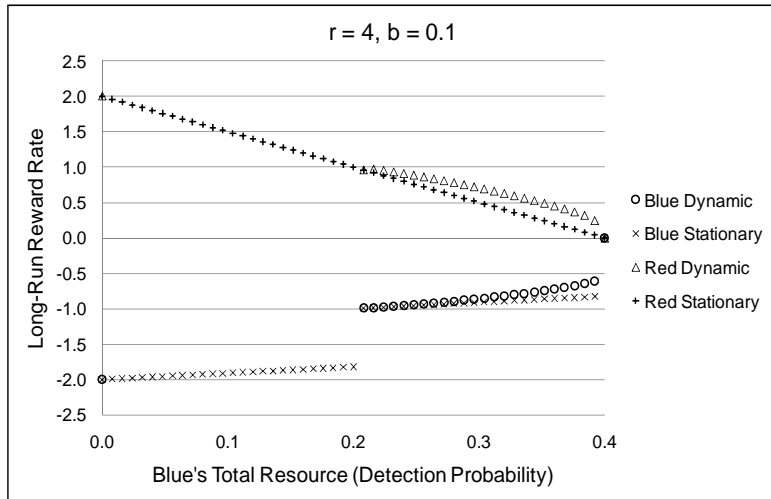
Figure 11.   Comparison between dynamic allocation and stationary allocation with
$r = 4,\ b = 0.1,\ 0.5,\ 0.9$

As shown in Figure 11, qualitatively the results look about the same when b changes. The dynamic allocation always provides some benefit over stationary allocation, and the improvement is more significant when s increases. Table 2 reports the reward rate of dynamic allocation as percentage over that of stationary allocation, for Blue and Red, respectively. It shows that, for both players, the improvement is more significant for a larger value of b. In other words, dynamic allocation is more effective for a larger s or for a larger b. Also seen in Table 2, Red's improvement is larger than Blue's.

$r = 4$			s			
			0.24	0.28	0.32	0.36
b	0.1	Blue	0.78%	3.61%	8.18%	15.31%
		Red	13.78%	30.26%	57.56%	126.69%
	0.5	Blue	1.96%	5.72%	10.98%	18.40%
		Red	14.21%	31.63%	60.71%	134.47%
	0.9	Blue	3.13%	7.76%	13.61%	21.23%
		Red	14.61%	32.91%	63.67%	141.83%

Table 2.   Improvement in dynamic allocation as b increases

Next, we repeat the experiment for $r = 1$ and $r = 9$, and plot the results in Figure 12 and Figure 13, respectively.

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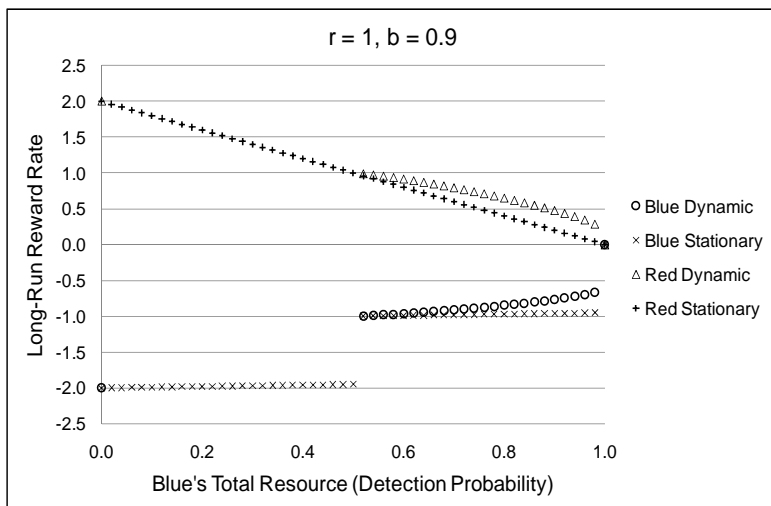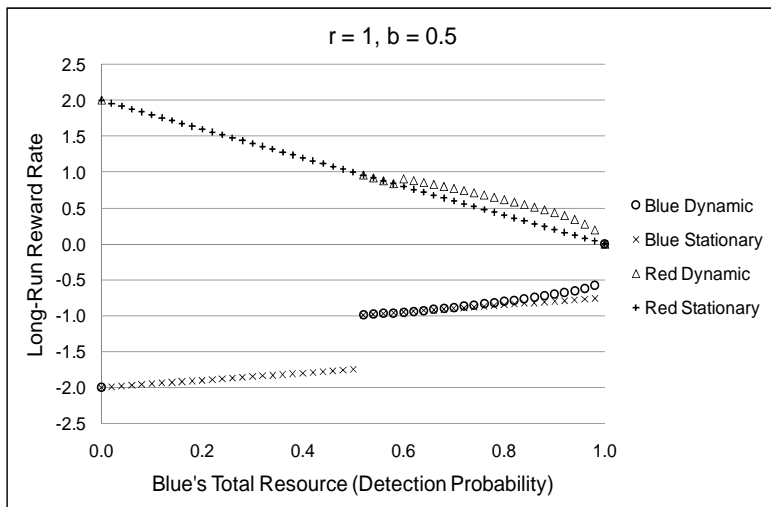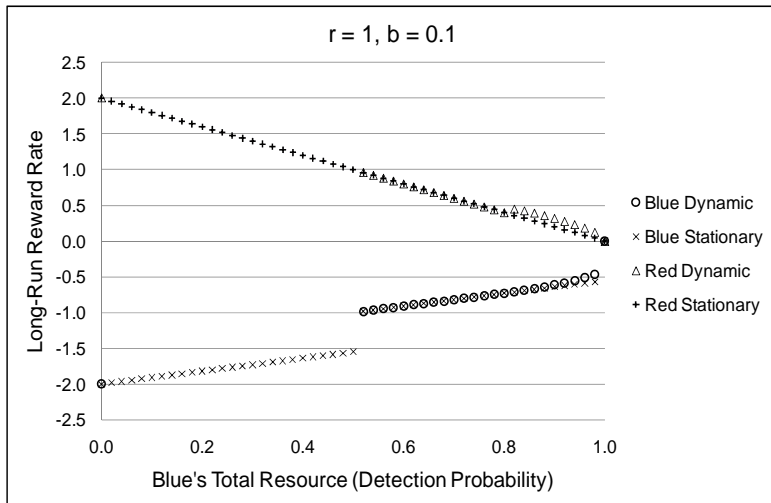
Figure 12. Comparison between dynamic allocation and stationary allocation with $r = 1,\ b = 0.1,\ 0.5,\ 0.9$
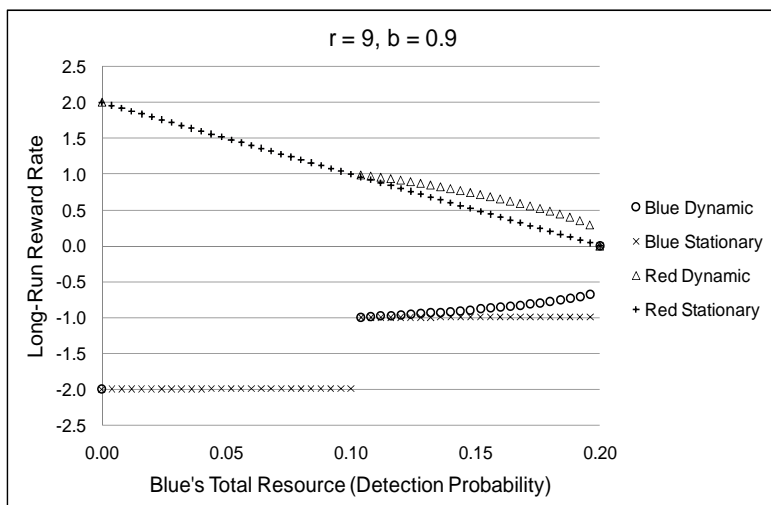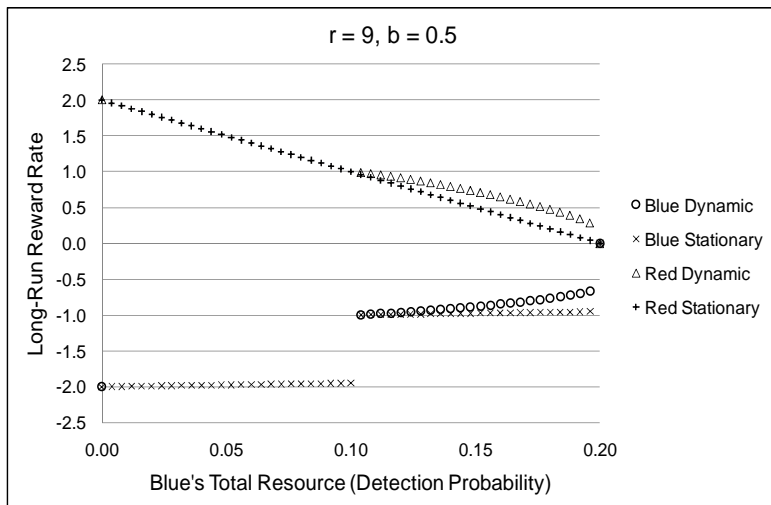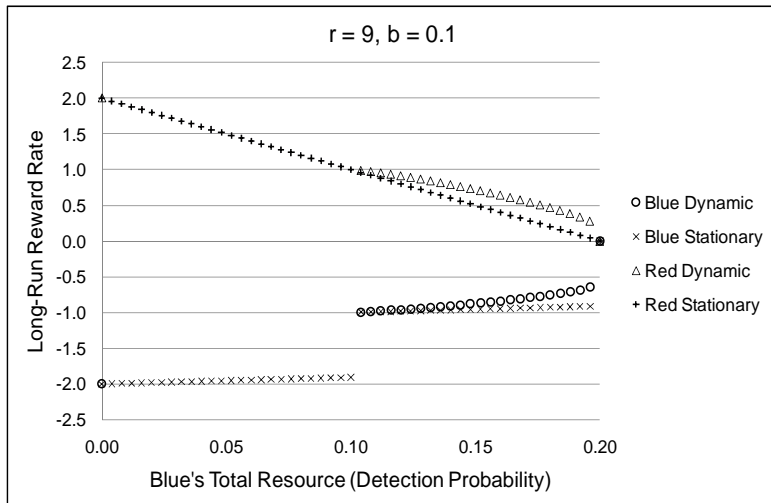
Figure 13. Comparison between dynamic allocation and stationary allocation with $r = 9, \ b = 0.1, \ 0.5, \ 0.9$

As shown in Figure 12, there are some cases where dynamic allocation does not provide additional benefits beyond those of stationary allocation. For instance, when $r = 1$, $b = 0.1$, the optimal solution to dynamic allocation coincides with stationary allocation, when s is between 0.5 and 0.8. We plot Red's optimal strategy in this case in Figure 14.
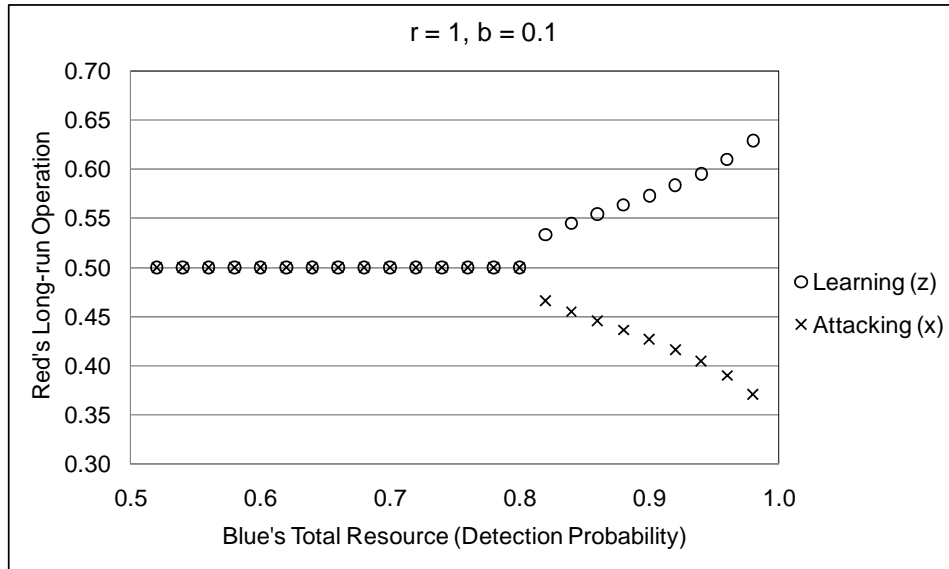
Figure 14.   Red's optimal strategy when the optimal solution to dynamic allocation coincides with stationary allocation

Qualitatively speaking, dynamic allocation tends to be less effective when r, b, and s are small. Below we offer some intuitive explanations. When r is small (close to 0), Red is not very concerned with a detected attack, so Red has less incentive to invest in the learning rate, which makes dynamic allocation less effective. When b is large (close to 1), Blue does not care much between detecting an attack or not. Instead, for Blue it is important to reduce Red's long-run attack rate, which can be accomplished by dynamic allocation. Finally, when s is small (close to $\hat{p}$), Red's expected payoff for each attack is only slightly less than 0, even if the detection probability is s. Therefore, Red has less incentive to find out Blue's tower status, which makes dynamic allocation less effective. In summary, dynamic allocation tends to be more effective when r, b, and s are larger.

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IV.   CONCLUSIONS

This thesis examined how to operate two surveillance towers most effectively with limited manpower (surveillance resources). In particular, a dynamic allocation strategy was studied, with which the surveillance team is moved between the two towers intermittently. Because it is difficult to tell from the outside whether a surveillance tower is fully functional, the understaffed tower can serve as a decoy to deter insurgent activities. The problem was formulated as a two-person nonzero-sum game between the insurgents and the government forces, with the latter moving first. After an algorithm is presented to compute the equilibrium in this game, this study's findings were demonstrated numerically.

Our analysis suggests that the dynamic allocation strategy can improve the performance of surveillance towers under most circumstances. The improvement tends to be more significant when government forces have more surveillance resources. Dynamic allocation tends to be less effective when (1) a detected attack has a smaller negative impact on insurgent operations, or when (2) a detected attack brings a larger immediate benefit to government forces. Our model applies not only to military operations but also to surveillance problems in general.

There are some limitations to our model. First, it assumes that attacks follow a Poisson process. Second, it assumes that there is no cost to switch the resource between surveillance towers. This assumption may be reasonable when the videos from two towers are fed to a single control room, but not if there are two separate control rooms. Third, the model assumes that the detection probability increases linearly in the allocated resource.

There are many possible future research directions. First, it may be worthwhile to study an asymmetric model with different parameters in the two towns. Second, extending analysis to more than two surveillance towers can give government forces more flexibility. Finally, this thesis studies Stackelberg equilibrium, in which government

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forces move first and insurgents move second. It is important to study whether there are other types of equilibriums, especially an equilibrium that results from simultaneous moves.

APPENDIX

A. DECISION AID

This Excel file implements the algorithms described in Chapter II, and consists of six worksheets. The green cells require the user to enter input values. The yellow cells are computation results from VBA codes. The red cells contain formulas, which should not be modified by the user. Below we explain each worksheet one at a time.

1. Parameter

This worksheet allows a user to enter model parameters, namely r, b, α, c and s. The reward table will be formulated and $\hat{p}$ computed. By clicking on the button, the program will check the value of s and show the corresponding result as Figure 15. In addition, the default precision for computation is 1.0E-8, which can be changed accoring to the user's desired calculation result.
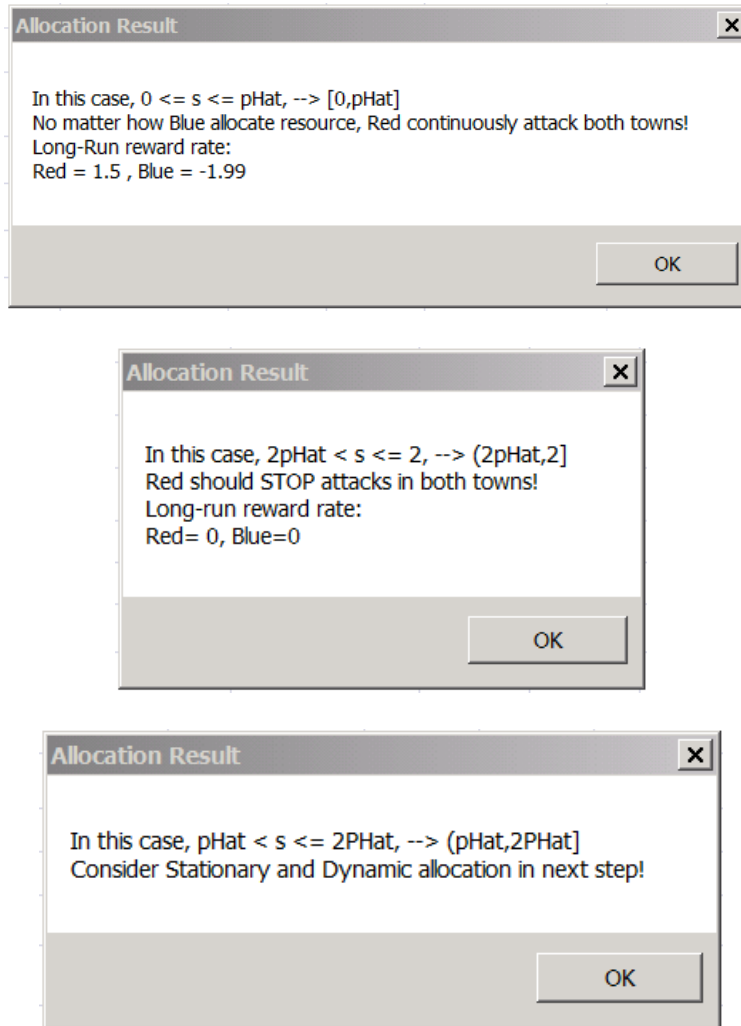
Figure 15.   Allocation recommendations for different s

2. Blue

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	pHat	Initial Cut "p"	s		Optimal for Blue in Dynamic Allocation			Iteration	p	y	x	z	B*	R*
2	0.2	0.25	0.3		p*	y*	B*(p,y)	1	0.250000	0.006426	0.921573	0.078427	-0.452082	0.313388
3					0.3	0.019801658	-0.872624202	2	0.266326	0.006426	0.918066	0.081934	-0.449589	0.345617
4					Optimal for Red in Dynamic Allocation			3	0.266326	0.010437	0.901380	0.098620	-0.446068	0.329380
5	Precision				x*	z*	R*(x,z)	4	0.288443	0.010437	0.896679	0.103321	-0.443155	0.370392
6	0.00000001				0.867628988	0.132371012	0.71904674	5	0.288443	0.016476	0.878207	0.121793	-0.439312	0.349545
7							Compute B+	6	0.300000	0.016476	0.875476	0.124524	-0.437105	0.369399
8								7	0.300000	0.019802	0.867629	0.132371	-0.436312	0.359523
9								8	0.300000	0.019802	0.867629	0.132371	-0.436312	0.359523
10		p*	y*	x*	z*	B*(p,y)	R*(x,z)							
11	Golden Search (Dynamic)	0.3	0.019801658	0.867628988	0.132371012	-0.436312101	0.35952337							
12			50.50082252			-0.872624202	0.71904674							
13														
14		p	y	x	z	B*	R*							
15	p=s (Dynamic)	0.3	0.019801658	0.867628988	0.132371012	-0.436312101	0.35952337							
16			50.50082252			-0.872624202	0.71904674							
17														
18		p	y	x	z	B*	R*							
19	p = pHat (Dynamic)	0.2	4.19E-09	1	0	-0.925	0.25							
20			238437702.7			-1.85	0.5							
21														
22	Stationary Allocation		y	x	z	B*	R*							
23			0	1	0	-0.95	0.5							
24														
25														
26														

Tabs: Parameter / Red / FindY / FindP / **Blue** / Experiment

This worksheet will be activated if $s \in (\hat{p}, 2\hat{p}]$. In this worksheet, the user needs to enter a initial cut "p" ($p \in [\hat{p}, s]$) as a starting point for program to perform the golden section search, as explained in Chapter II, Section C. The optimal strategies for Blue and Red will then be reported. Also, the iteration results of the golden section search will be listed for reference on the right-hand side. This worksheet implements the algorithms in Chapter II, Section C, "Blue's Optimal Strategy."

3. Red
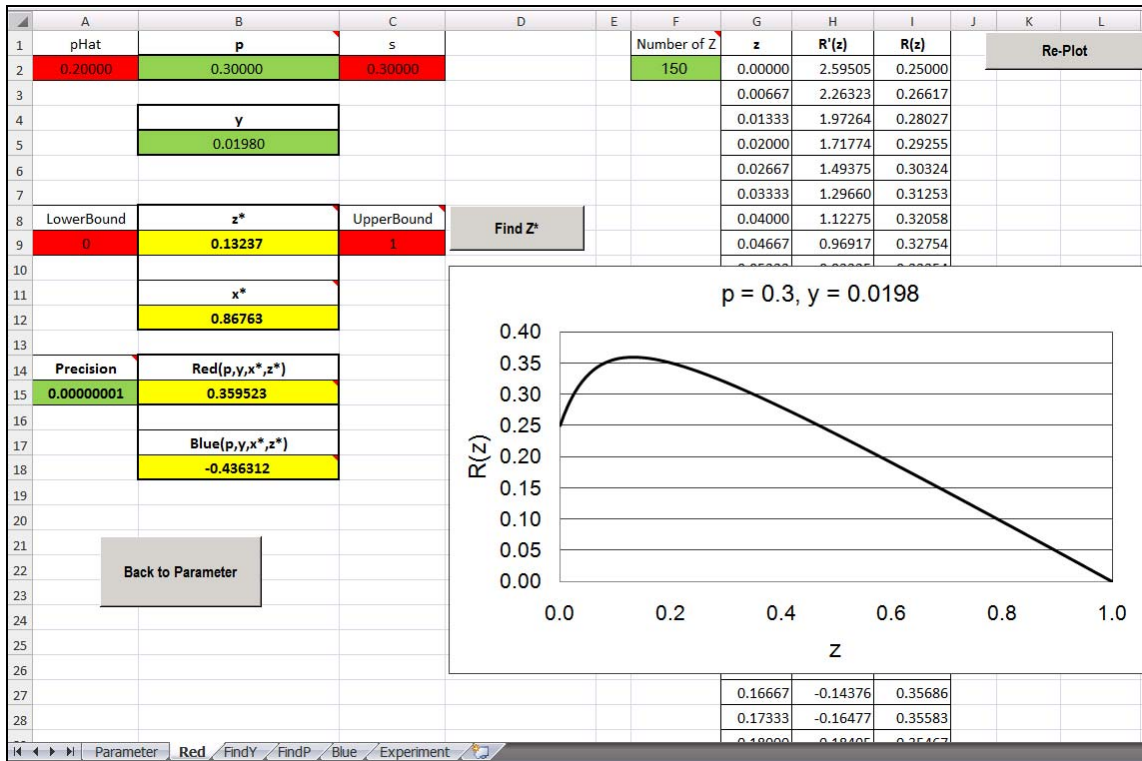
This worksheet plots Red's objective function $R(z)$ and computes Red's optimal strategy, z^* and x^*, as explained in Chapter II, Section B. Blue's strategy p and y are required inputs. This worksheet implements the algorithm in Chapter II, Section B, "Red's Optimal Strategy."

4.   FindY

A	n	y	z"	x"	1/y	B(p,y,x,z) fixed p
p						
0.3	1	0.2	0.081903	0.9681	5	-0.913948267
	2	0.1	0.138376	0.86162	10	-0.625013529
Y_UB	3	0.066667	0.157624	0.84238	15	-0.531187003
1000	4	0.05	0.159515	0.84048	20	-0.489344886
	5	0.04	0.156397	0.8436	25	-0.468002318
n	6	0.033333	0.151737	0.84826	30	-0.456344182
300	7	0.028571	0.146723	0.85328	35	-0.449780422
	8	0.025	0.141802	0.8582	40	-0.446094468
	9	0.022222	0.13714	0.86286	45	-0.444116875
Plot BlueP	10	0.02	0.132789	0.86721	50	-0.443189387
	11	0.018182	0.128754	0.87125	55	-0.442922803
	12	0.016667	0.125019	0.87498	60	-0.443078016
	13	0.015385	0.121562	0.87844	65	-0.443503628
	14	0.014286	0.118356	0.88164	70	-0.444101452
pHat	15	0.013333	0.115378	0.88462	75	-0.444806643
0.2	16	0.0125	0.112606	0.88739	80	-0.445575756
	17	0.011765	0.110019	0.88998	85	-0.4463794
s	18	0.011111	0.107599	0.8924	90	-0.447197602
0.8	19	0.010526	0.10533	0.89467	95	-0.448016754
	20	0.01	0.103198	0.8968	100	-0.448827653
	21	0.009524	0.10119	0.89881	105	-0.449624135
Precision	22	0.009091	0.099295	0.90071	110	-0.450402169
0.00000001	23	0.008696	0.097503	0.9025	115	-0.451159237
	24	0.008333	0.095806	0.90419	120	-0.451893861
	25	0.008	0.094196	0.9058	125	-0.452605326
	26	0.007692	0.092665	0.90733	130	-0.453293432
	27	0.007407	0.091208	0.90879	135	-0.453958352
	28	0.007143	0.089818	0.91018	140	-0.454600508
	29	0.006897	0.088492	0.91151	145	-0.455220492
	30	0.006667	0.087224	0.91278	150	-0.455819009
	31	0.006452	0.08601	0.91399	155	-0.456396826
	32	0.00625	0.084846	0.91515	160	-0.456954755
	33	0.006061	0.083729	0.91627	165	-0.45749361
	34	0.005882	0.082657	0.91734	170	-0.458014216
	35	0.005714	0.081626	0.91837	175	-0.458517374
	36	0.005556	0.080633	0.91937	180	-0.459003867
	37	0.005405	0.079677	0.92032	185	-0.459474457
	38	0.005263	0.078754	0.92125	190	-0.459929869

Given a "p", to find a "y" that maximize Blue's Long-run reward

p	y*	x"	z"	B*(p,y)	R*(x,z)
0.3	0.018083512	0.871477302	0.128522698	-0.442922069	0.3644525

time-unit per switch: 55.29899361

[Reset] [Compute y*]

Chart: r = 4, b = 0.5, s = 0.3 — $\hat{B}(p^*, y)$ versus $1/y$

Sheet tabs: Parameter / Red / **FindY** / FindP / Blue / Experiment

By holding p constant, this worksheet plots $\hat{B}(p, y)$ as a function of y, and computes y that maximizes it. It implements the golden section search method. The value of p is a required input. This worksheet implements the algorithm in Chapter II, Section C, "Blue's Optimal Strategy."

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5.   FindP

y	1/y	p	z*	x*	B(p,y,x,z*) fixed y	n
1.9802E-02	50.50082252	0.5	0	1	-0.595	1
		0.502	0	1	-0.595	2
p_LB	p_UB	0.504	0	1	-0.595	3
0.2	0.3	0.506	0	1	-0.595	4
		0.508	0	1	-0.595	5
n		0.51	0	1	-0.595	6
200		0.512	0	1	-0.595	7
		0.514	0	1	-0.595	8
		0.516	0	1	-0.595	9
		0.518	0	1	-0.595	10
Plot BlueY		0.52	0	1	-0.595	11
		0.522	0	1	-0.595	12
		0.524	0	1	-0.595	13
Precision		0.526	0	1	-0.595	14
0.00000001		0.528	0	1	-0.595	15
		0.53	0	1	-0.595	16
		0.532	0	1	-0.595	17
		0.534	0	1	-0.595	18
		0.536	0	1	-0.595	19
		0.538	0	1	-0.595	20
		0.54	0	1	-0.595	21
		0.542	0.00483273	0.995167274	-0.585246112	22
		0.544	0.01848855	0.981511448	-0.558193073	23
		0.546	0.03076446	0.969235536	-0.53453025	24
		0.548	0.04185006	0.958149944	-0.513713208	25
		0.55	0.05190254	0.948097464	-0.495303599	26
		0.552	0.06105331	0.93894669	-0.47894508	27
		0.554	0.06941294	0.930587064	-0.464345556	28
		0.556	0.07707509	0.92292491	-0.451263613	29
		0.558	0.0841196	0.915880401	-0.4394983	30
		0.56	0.09061488	0.909385119	-0.428881182	31
		0.562	0.09661986	0.903380137	-0.419270218	32
		0.564	0.10218558	0.897814419	-0.410544868	33
		0.566	0.10735639	0.892643612	-0.402602344	34
		0.568	0.11217105	0.88782895	-0.395354472	35
		0.57	0.11666255	0.883336651	0.388725385	36

Given a "y", to find a "p" that maximize Blue's Long-run reward

p*	y	z*	x*	B*(p,y)	R*(x,z)
0.3	0.019801658	0.132371012	0.867628988	-0.436312101	0.35952337

Reset   Compute p*

r = 4, b = 0.5, s = 0.3

By holding y constant, this worksheet plots $\hat{B}(p,y)$ as a function of p, and computes p that maximizes it. It implements the golden section search method. The value of y is a required input. This worksheet implements the algorithm in Chapter II, Section C, "Blue's Optimal Strategy."
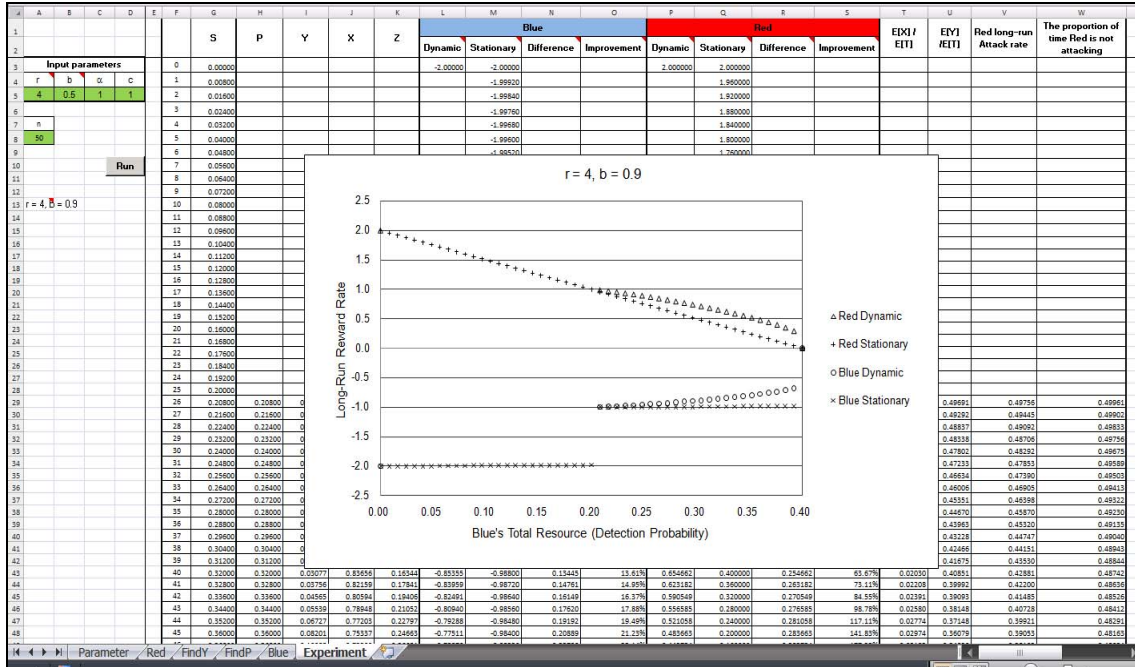
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6. Experiment

This worksheet shows --given r, b, α, c -- the comparison of dynamic and stationary allocations using different values of s. In addition to numbers, it shows a plot to compare the two strategies.

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