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心得報告

債券投資組合之績效評估方法

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債券投資組合之績效評估方法

前言

在評估績效時，首需計算超額報酬率，一般來說，以投資組合報酬率減去指標指數報酬率來衡量，稱之為算數法，此法雖簡單直覺，然易造成殘差與爭論；幾何法具有呈比例性、可轉換性、可連結性等特性，能產生一致的超額報酬率，預期未來將成為產業趨勢。

鑒於各資產管理帳戶所提供之績效貢獻方式不盡相同，有必要了解報酬率拆解方法，因為事後的報酬率貢獻分析（return attribution）有助經理人了解其績效來源，提供經理人未來調整投資組合之參考，亦方便委託人了解經理人之操作技巧。

至於委託人是否需要一套統一的績效評估系統來評估所有帳戶，吾等之研習心得為，績效評估須能反映經理人之投資決策流程，例一，若一多幣別資產管理帳戶，先由 currency overlay 經理人決定外匯部位，再由債券經理人負責債券投資決策，則該帳戶勢必將兩者之績效分開。續前例，若該資產管理公司之績效評估系統，採用指標指數使用的匯率當成外匯經理人之交易匯率（大部份商用系統是如此），則該系統在外匯管理績效部份，恐只能提供 currency allocation attribution；若該系統可以記錄外匯經理人之交易匯率，則可將外匯

管理績效區分為 currency allocation attribution 及 currency timeing（selection）兩部份。例二，若一多幣別資產管理帳戶之指標指數為 hedged index，且先由債券經理人決定 overweight 高殖利率之澳洲債券，underweight 低殖利率之日本債券，再由外匯經理人將 overweight 之澳幣部位 hedged to index（neutral），則該避險成本（即，無法充分獲得澳幣升值之機會成本）所造成的績效減項，應歸屬於債券經理人，而非外匯經理人，因為外匯經理人是基於 hedged to neutral 而被動做出避險決定，並非因看空澳幣而主動做出避險決定。

本報告章節安排如下，第一節介紹超額報酬率之衡量，第二節介紹資產配置法（Allocation Model），為股票型投資組合典型的績效評估方法，第三節介紹因素貢獻法（Factor Attribution Model），較適合評估債券型投資組合之績效，第四節則試圖整理出一個一般性的多幣別債券投資組合績效評估架構，第五節為結論與未來發展方向。

第一節、 超額報酬率之衡量

超額報酬率之衡量有兩種方法，一為算數超額報酬率（Arithmetic Excess Return），二為幾何超額報酬率（Geometric Excess Return）。

一、 算數超額報酬率

$$a = r - b \tag{1.1}$$

二、 幾何超額報酬率

$$g = (1+r) / (1+b) - 1 \tag{1.2}$$

其中，r 為投資組合報酬率，b 為指標指數報酬率。例如表 1－1 中投資組合之超額報酬率，若以算數法衡量為 2%，若以幾何法衡量則為 1.9%。

表 1－1、算數與幾何超額報酬率之比較

Portfolio	Benchmark
Start Value = 1,000 End Value = 1,070	Start Value = 1,000 End Value = 1,050
r = 7%	b = 5%
Portfolio added value = 70 – 50 = 20	
Arithmetic Excess Return, a = 7% - 5% = **2%** (= 20 / 1,000) Geometric Excess Return, g = 1.07 / 1.05 – 1 = **1.9%** (= 20 / 1,050)	

算數法之優點為簡單直覺，根據 Bacon C.（2008），目前全球使用算數或幾何超額報酬率者各半，然幾何法預期會逐漸變成產業標準，因其具有下列優勢：

1、　呈比例性（Proportionality）

表 1－2、幾何超額報酬率之呈比例性

Portfolio	Benchmark
Start Value = 1,000 End Value = 500	Start Value = 1,000 End Value = 250
r = -50%	b = -75%
Arithmetic Excess Return, a = -50% - (-75%) = **25%** Geometric Excess Return, g = 0.5 / 0.25 – 1 = **100%**	

表 1－2 顯示幾何超額報酬率正確反映投資組合績效是指標績效之 1 倍，並非只超出 25%。此外，下列關係式顯示：

$g = (1+r) / (1+b) - 1 = (r - b) / (1+b)$

當市場上漲時，b＞0，因此算數超額報酬率＞幾何超額報酬率，

當市場下跌時，b＜0，因此算數超額報酬率＜幾何超額報酬率，

是以，算數超額報酬率變異幅度較大，幾何超額報酬率較為平滑。

2、　可轉換性（Convertibility）

表 1－3、幾何超額報酬率之可轉換性

Portfolio (base currency: US$)	Benchmark (base currency: US$)
Start Value = \$1,000 = €1,000*1 (\$1= €1) End Value = \$1,284 = €1,070*1.2 (\$1= €1.2)	Start Value = \$1,000 End Value = \$1,155 = €1,050*1.1
r = 28.4%	b = 15.5%
Portfolio added value = \$1,284 - \$1,155 = \$129	
Arithmetic Excess Return, a = **12.9% = 28.4% - 15.5%** = \$129 / \$1,000 Geometric Excess Return, g = **11.2% = 1.284 / 1.155 - 1** = \$129 / \$1,155	

幾何超額報酬率最大的優點在於可轉換性，可將以基礎貨幣表示之報酬率（r、b）轉換成以當地貨幣表示之報酬率（r_L、b_L）與

匯率報酬率（c_r、c_b），因此，以基礎貨幣表示之幾何超額報酬率（g），可拆解成以當地貨幣表示之超額報酬率，來反映債券經理人之績效，以及匯率部分之超額報酬率，來反映外匯經理人之績效；然而以基礎貨幣表示之算數超額報酬率則在轉換與拆解後，產生無法歸屬績效之殘差項。

Portfolio return in base currency: $r = (1+r_L)*(1+c_r) - 1$

Benchmark return in base currency: $b = (1+b_L)*(1+c_b) - 1$

$$a = r - b = (1+r_L)*(1+c_r) - (1+b_L)*(1+c_b)$$

$$= (r_L - b_L) + (c_r - c_b) + \underbrace{(r_L*c_r - b_L*c_b)}_{\text{殘差項}}$$

$$g = (1+r)/(1+b) - 1 = \underbrace{(1+r_L)/(1+b_L)}_{\text{債券經理人之績效}} * \underbrace{(1+c_r)/(1+c_b)}_{\text{外匯經理人之績效}} - 1$$

3、　可連結性（Compoundability）

表 1－4、幾何超額報酬率之可連結性

投資組合年度報酬率(四季累積)：(1.07)*(1.07)*(1.07)*(1.07) – 1 = 31.1% 指標指數年度報酬率(四季累積)：(1.05)*(1.05)*(1.05)*(1.05) – 1 = 21.6%	
算數法	幾何法
單期超額報酬率 = 7% – 5% = 2% 年度超額報酬率 = 31.1% - 21.6% = 9.5%	單期超額報酬率 = 1.07 / 1.05 – 1 = 1.9% 年度超額報酬率 = 1.311 / 1.216 – 1 = 7.8%
<u>年度</u>超額報酬率與 <u>累積單期</u>超額報酬率**不一致** 1.02*1.02*1.02*1.02 – 1 = 8.2% ≠9.5%	<u>年度</u>超額報酬率與 <u>累積單期</u>超額報酬率**一致** 1.019*1.019*1.019*1.019 - 1 = 7.8%

由表 1－4 可知，不論單期或年度超額報酬率均宜以幾何法計算，以減少誤差。

$$(1+g) = (1+g_1)*(1+g_2)*\ldots*(1+g_n)$$

第二節、 資產配置法（**Allocation Model**）

資產配置法是股票型基金典型的績效評估方法，最早由 Brinson, Hood and Beebower（1986）、Brinson and Fachler（1985）提出，將績效來源拆解為資產配置（asset allocation）與證券選擇（security selection）兩大部份，分別介紹如下。

一、Brinson, Hood and Beebower（1986）－算數法

投資組合報酬率：$r = \sum_{i=1}^{N} w_i r_i = 8.3\%$

指標指數報酬率：$b = \sum_{i=1}^{N} W_i b_i = 6.4\%$

算數超額報酬率：$a = r - b = 1.9\%$ (2.1)

其投資組合報酬率之拆解方式如圖 2-1 所示，因此，績效來源包含指標指數報酬率（b）、資產配置報酬率（r_{AA}）、證券選擇報酬率（r_{SS}）、與交叉項（r_{inter}）：

$r = b + r_{AA} + r_{SS} + r_{inter}$ = 6.4% -1.2% + 3.0% + 0.1% = 8.3%

$r_{AA} = b_S - b$, in which semi-notional benchmark return $b_S = \sum w_i b_i$

= 5.2% - 6.4% = -1.2%

$r_{SS} = r_S - b$, in which semi-notional portfolio return $r_S = \sum W_i r_i$

= 9.4% - 6.4% = 3.0%

$r_{inter} = r - r_S - b_S + b$ = 8.3% - 9.4% - 5.2% + 6.4% = 0.1%

圖 2-1、算數報酬率之拆解－Brinson, Hood and Beebower（1986）

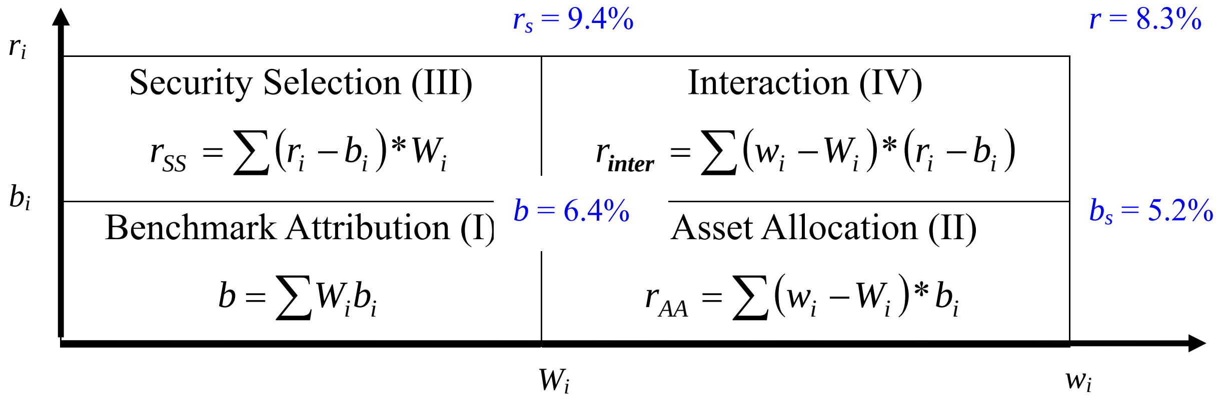


表 2-1、算數報酬率之拆解－Brinson, Hood and Beebower（1986）

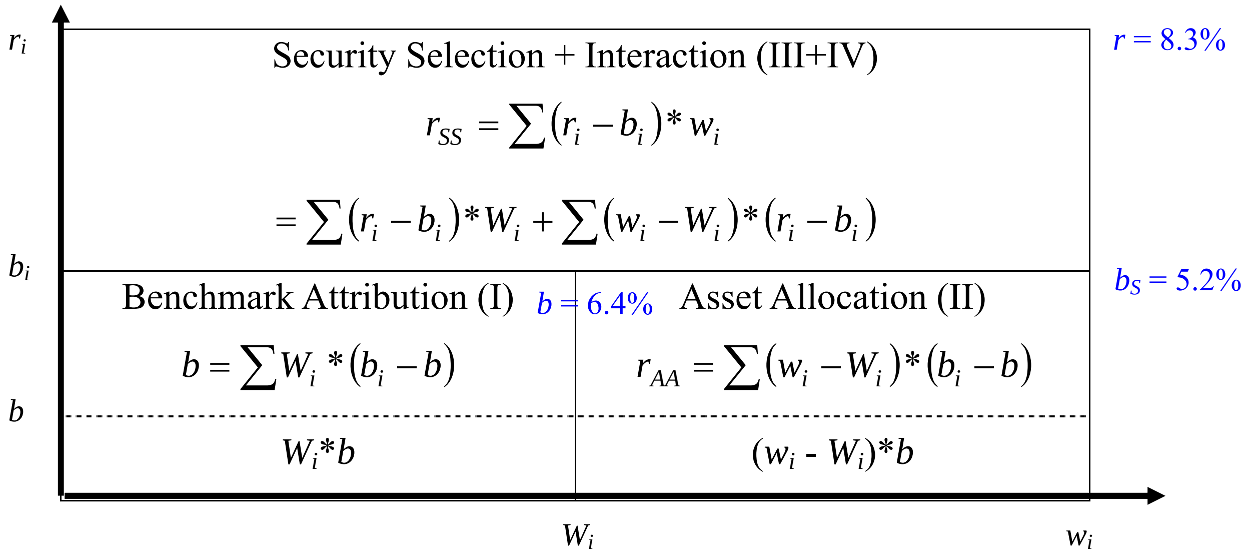
Equity (%)	Port Wgt w_i (1)	Bmk Wgt W_i (2)	Port Ret r_i (3)	Bmk Ret b_i (4)	Semi b_{Si} (5) = (1)*(4)	Semi r_{Si} (6) = (2)*(3)	AA $r_{AA\,i}$ (7) = [(1)-(2)]*(4)	SS $r_{SS\,i}$ (8) = [(3)-(4)]*(2)	Interaction $r_{inter\,i}$ (9) = [(1)-(2)]*[(3)-(4)]
UK	40	40	20	10	4.0	8.0	0.0	4.0	0.0
JP	30	20	-5	-4	-1.2	-1.0	-0.4	-0.2	-0.1
US	30	40	6	8	2.4	2.4	-0.8	-0.8	0.2
Total	100	100	r 8.3	b 6.4	b_S 5.2	r_S 9.4	$r_{AA} = b_S - b$ -1.2	$r_{SS} = r_S - b$ 3.0	$r_{inter} = r - r_S - b_S + b$ 0.1

缺點：此模型無法清楚分辨經理人是否勝過大盤，例如，若 overweight in negative market，則 return attribution 應為負值，反之，若 underweight in negative market，則應視為有正面績效。

二、Brinson and Fachler（1985）－算數法

為改善前述模型無法分辨經理人是否勝過大盤之缺點，又投資決策流程首先為資產配置，其次為證券選擇，交叉項實為殘差，不屬於投資決策，故將交叉項併入證券選擇，修正為圖 2-2。

圖 2-2、算數報酬率之拆解—Brinson and Fachler（1985）



$r_{AA} = b_S - b$, in which semi-notional benchmark return $b_S = \sum w_i b_i$

$= 5.2\% - 6.4\% = -1.2\%$

$r_{SS} = r - b_S = 8.3\% - 5.2\% = 3.1\%$

表 2-2、算數報酬率之拆解—Brinson and Fachler（1985）

Equity (%)	Port Wgt w_i (1)	Bmk Wgt W_i (2)	Port Return r_i (3)	Bmk Return b_i (4)	Semi b_{Si} $= w_i * b_i$ (5)	AA $r_{AA\,i}$ $= (w_i - W_i) * (b_i - b)$ (6)	SS + Interaction $r_{SS\,i}$ $= w_i * (r_i - b_i)$ (7)
UK	40	40	20	10	4.0	0.00	4.0
JP	30	20	-5	-4	-1.2	-1.04	-0.3
US	30	40	6	8	2.4	-0.16	-0.6
Total	100	100	r 8.3	b 6.4	b_S 5.2	r_{AA} $= b_S - b$ -1.2	r_{SS} $= r - b_S$ 3.1

例如，日股大盤跌，而投資組合 overweight 日股，故表 2-2（$r_{AA\,i} = -1.04\%$）較表 2-1（$r_{AA\,i} = -0.4\%$）更能突顯示日股負的資產配置績效。另日股與美股之投資組合報酬率比大盤差，故證券選擇方面皆為 negative。

三、Brinson and Fachler（1985）－幾何法

Bacon, C.（2008）將 Brinson and Fachler（1985）模型改為以幾何法衡量超額報酬率，結果如圖 2－3 與表 2－3 所示。

幾何超額報酬率：$g = \frac{1+r}{1+b} - 1 = (1.083/1.064) - 1 = 1.79\%$ (2.2)

幾何法相較於算數法之優點為，可將投資組合報酬率清楚地分解成資產配置與證券選擇績效，不會產生殘差項。

$$1+g = \frac{1+r}{1+b} = \frac{1+b_S}{1+b} * \frac{1+r}{1+b_S}$$

$$=> (1+g) = (1+r_{AA})*(1+r_{SS})$$

$$= (1 - 1.13\%)*(1 + 2.95\%)$$

$$r_{AA} = \frac{1+b_S}{1+b} - 1 \qquad \text{[Top-Down Approach]}$$

$$= \sum (w_i - W_i) * \left(\frac{1+b_i}{1+b} - 1 \right) \qquad \text{[Bottom-Up Approach]}$$

$$= (1+5.2\%) / (1+6.4\%) - 1 = -1.13\%$$

$$r_{SS} = \frac{1+r}{1+b_S} - 1 \qquad \text{[Top-Down Approach]}$$

$$= \sum w_i * \left(\frac{1+r_i}{1+b_i} - 1 \right) * \left(\frac{1+b_i}{1+b_S} \right) \qquad \text{[Bottom-Up Approach]}$$

$$= (1+8.3\%) / (1+5.2\%) - 1 = 2.95\%$$

圖 2－3、幾何報酬率之拆解－Brinson and Fachler（1985）

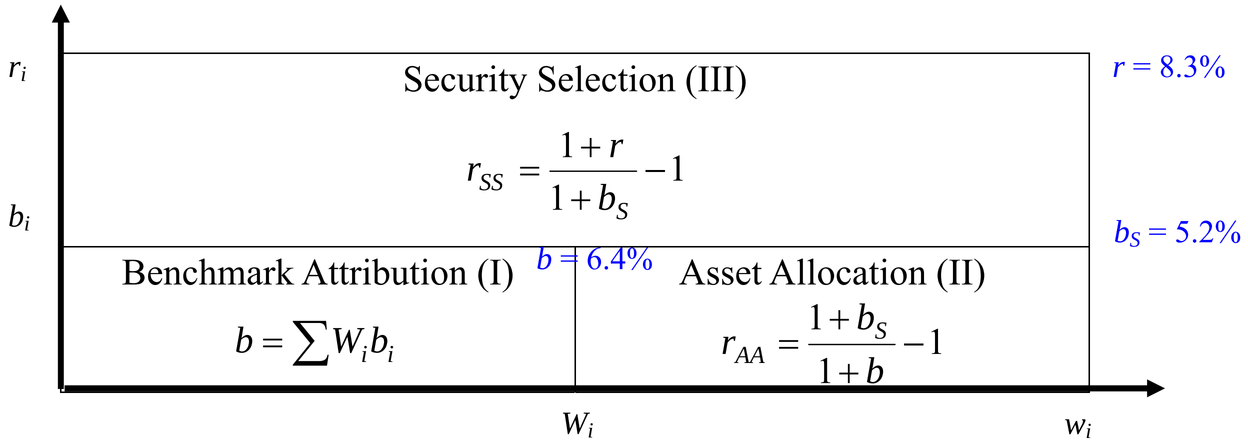


表 2－3、幾何報酬率之拆解－Brinson and Fachler（1985）

Equity (%)	Port Wgt w_i (1)	Bmk Wgt W_i (2)	Port Return r_i (3)	Bmk Return b_i (4)	Semi b_{Si} = w_i*b_i (5)	AA r_{AAi} = (w_i-W_i)* ((1+b_i)/(1+b)-1) (6)	SS r_{SSi} = w_i*((1+r_i)/(1+b_i)-1) *((1+b_i)/(1+b_S)) (7)
UK	40	40	20	10	4.0	0.00	3.80
JP	30	20	-5	-4	-1.2	-0.98	-0.28
US	30	40	6	8	2.4	-0.15	-0.57
Total	100	100	r 8.3	b 6.4	b_S 5.2	r_{AA} = (1+b_S)/(1+b)-1 -1.13	r_{SS} = (1+r)/(1+b_S)-1 2.95

第三節、 因素貢獻法（Factor Attribution Model）

某些文獻將分析股票型基金績效的資產配置法，沿用於評估股債平衡型基金或債券型基金，例如 Van Breukelen（2000）。惟因債券（尤其是公債）之績效表現主要受某些共同因子（如，殖利率曲線變化）影響，較少受個別債券因素影響，以致證券選擇部份之績效貢獻不大，故有越來越多文獻認為需要發展適合債券型基金的績效分析方法，例如 Campisi（2000）提出因素貢獻法，將績效來源拆解為息票收入報酬（income return）與價格報酬（price return）兩大部份，茲將各模型介紹如下。

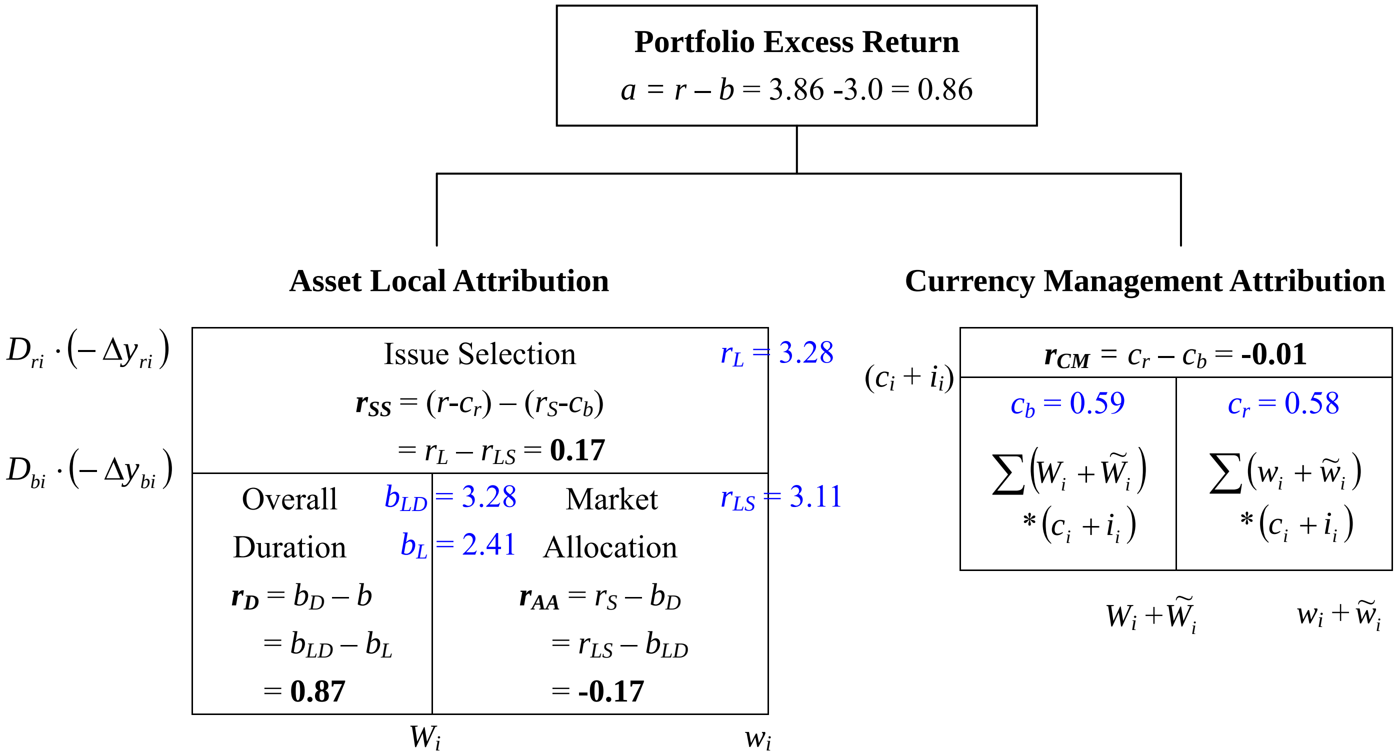
一、Van Breukelen（2000）－算數法

Van Breukelen 沿用 Brinson 之資產配置法，優點為可用同樣的架構拆解股票與債券報酬率，便於應用在股債平衡型基金，其中，債券報酬率以存續期間來衡量（比較圖 3-1 與圖 2-1 之左座標軸），

$$r = \frac{\Delta P}{P} = -MD \cdot \Delta y \text{，}$$

缺點為存續期間只能捕捉殖利率曲線水平移動。

圖 3-1、算數超額報酬率來源—Van Breukelen（2000）



$a = r - b = r_D + r_{AA} + r_{SS} + r_{CM}$ (Table 3-2)

Currency timing (selection) skill can not separately be observed.

$\widetilde{W}_i$: the weight of forward contracts in benchmark

$\widetilde{w}_i$: the weight of forward contracts in portfolio

Portfolio Local Return: $r_{Li} = i_i + D_{ri} * (-\Delta y_{ri})$ (3.1)

Benchmark Local Return: $b_{Li} = i_i + D_{bi} * (-\Delta y_{bi})$

Using the Karnosky and Singer (1994) definition,

Portfolio Base Currency Return with Forward Contracts:

$$r = \underbrace{\sum w_i * (r_{Li} - i_i)}_{asset\ \ local\ \ return} + \underbrace{\sum (w_i + \tilde{w}_i) * (c_i + i_i)}_{currency\ \ management} \qquad (3.2')$$

Portfolio Base Currency Return without Forward Contracts in this Chapter:

$$r = \sum w_i * (r_{Li} - i_i) + \sum w_i * (c_i + i_i) \qquad (3.2)$$

Similary, Benchmark Base Currency Return without Forward Contracts:

$$b = \sum W_i * (b_{Li} - i_i) + \sum W_i * (c_i + i_i) \qquad (3.3)$$

1. Top-Down Attribution Approach

Substituting equation (3.1) into equation (3.2) and (3.3), a top-down attribution can be derived as followings.

<u>Portfolio Base Currency Return</u>:

$$r = \sum \underbrace{w_i * D_{ri}}_{\substack{Weighted \\ Duration}} * (-\Delta y_{ri}) + \sum w_i * (c_i + i_i)$$

$$= r_L + c_r$$

= 3.28% + 0.58% = 3.86%

<u>Benchmark Base Currency Return</u>:

$$b = \sum \underbrace{W_i * D_{bi}}_{\substack{Weighted \\ Duration}} * (-\Delta y_{bi}) + \sum W_i * (c_i + i_i)$$

$$= b_L + c_b$$

= 2.41% + 0.59% = 3%

<u>Arithmetic Excess Return</u>:

$$a = r - b$$

$$= \underbrace{\sum w_i * D_{ri} * (-\Delta y_{ri}) - \sum W_i * D_{bi} * (-\Delta y_{bi})}_{Asset\ \ Local\ \ Return\ \ Attribution} + \underbrace{\sum w_i * (c_i + i_i) - \sum W_i * (c_i + i_i)}_{Currency\ \ Management\ \ Attribution}$$

= $r - b$ = 3.86% - 3% = 0.86%

2. Bottom-Up Attribution Approach

A bottom-up attribution can be derived by the following procedures (see Table 3-1 and Table 3-2).

Two asset reference funds:

Implied Portfolio Yield Changes: $\Delta y_{ri} = -\dfrac{r_{Li} - i_i}{D_{ri}}$

Implied Benchmark Yield Changes: $\Delta y_{bi} = -\dfrac{b_{Li} - i_i}{D_{bi}}$

Overall duration notional fund:

$$\begin{aligned} b_D &= \sum D_\beta * W_i * D_{bi} * (-\Delta y_{bi}) + \sum W_i * (c_i + i_i) \\ &= b_{LD} + c_b \end{aligned}$$

where $D_\beta = D_r / D_b$ = duration beta = 5.3 / 3.9 = 1.36

b_D = 3.28% + 0.59% = 3.87%

Duration-adjusted semi-notional fund:

$$\begin{aligned} r_S &= \sum w_i * D_{ri} * (-\Delta y_{bi}) + \sum W_i * (c_i + i_i) \\ &= r_{LS} + c_b \end{aligned} \tag{3.4}$$

r_S = 3.11% + 0.59% = 3.70%

<u>Asset Local Return Attribution</u>:

(I) Overall Duration Attribution:

Only when the overall duration is part of the investment decision process, the overall duration effect is measured as:

$$\begin{aligned} r_D = b_D - b &= \left[\sum D_\beta * W_i * D_{bi} * (-\Delta y_{bi}) + c_b\right] - \left[\sum W_i * D_{bi} * (-\Delta y_{bi}) + c_b\right] \\ &= b_{LD} - b_L \end{aligned}$$

r_D = 3.87% - 3% = 3.28% - 2.41% = 0.87%

(II) Market Allocation:

$$\begin{aligned} r_{AA} = r_S - b_D &= \left[\sum w_i * D_{ri} * (-\Delta y_{bi}) + c_b\right] - \left[\sum D_\beta * W_i * D_{bi} * (-\Delta y_{bi}) + c_b\right] \\ &= r_{LS} - b_{LD} \end{aligned}$$

Applying the Brinson and Fachler approach:

$$r_{AA} = r_S - b_D = r_{LS} - b_{LD}$$
$$= \sum \left(w_i * D_{ri} - D_\beta * W_i * D_{bi}\right) * \left(-\Delta y_{bi} + \Delta y_b\right)$$

r_{AA} = 3.70% - 3.87% = 3.11% - 3.28% = -0.17%

If the overall duration is not part of the investment decision process we can omit a step and move directly to:

$$r_S - b = \left(r_{LS} + c_b\right) - \left(b_L + c_b\right) = \sum w_i * D_{ri} * \left(-\Delta y_{bi}\right) - \sum W_i * D_{bi} * \left(-\Delta y_{bi}\right)$$
$$= \sum \left(w_i * D_{ri} - W_i * D_{bi}\right) * \left(-\Delta y_{bi} + \Delta y_b\right)$$

(III) Issue Selection:

$$r_{SS} = \left(r - c_r\right) - \left(r_S - c_b\right) = \sum w_i * D_{ri} * \left(-\Delta y_{ri}\right) - \sum w_i * D_{ri} * \left(-\Delta y_{bi}\right)$$
$$= \sum w_i * D_{ri} * \left(-\Delta y_{ri} + \Delta y_{bi}\right)$$
$$= r_L - r_{LS}$$

r_{SS} = 3.28% - 3.11% = 0.17%

Currency Management (without Forwards) Attribution:

$$r_{CM} = c_r - c_b = \sum w_i * \left(c_i + i_i\right) - \sum W_i * \left(c_i + i_i\right)$$
$$= \sum \left(w_i - W_i\right) * \left(c_i + i_i - c_b\right)$$

r_{CM} = 0.58% - 0.59% = -0.01%

表 3-1、算數報酬率之拆解－Van Breukelen（2000）

(%) Bond	Weight		Modified Duration		Local Return		Crncy Ret	Local Intrst Rate	Base Currency Return			
									Local Asset		Currency Mgt	
	Port w_i (1)	Bmk W_i (2)	Port D_{ri} (3)	Bmk D_{bi} (4)	Port r_{Li} (5)	Bmk b_{Li} (6)	c_i (7)	i_i (8)	Port r_L (9)	Bmk b_L (10)	Port c_r (11)	Bmk c_b (12)
UK	50	50	7.8	5	5.6	3.5	0	1.0	2.30	1.25	3.4	0.6
JP	20	10	1.0	2	0.5	0.5	0	0.1	0.08	0.04	1.1	1.1
US	30	40	4.0	3	3.2	3.0	0	0.2	0.90	1.12	-2.8	4.4
Total	100	100	D_r 5.3	D_b 3.9	$r = r_L+c_r$ 3.86	$b = b_L+c_b$ 3.0			r_L 3.28	b_L 2.41	$c_r = i_r$ 0.58	$c_b = i_b$ 0.59

註：$D_r = \sum w_i * D_{ri}$；$D_b = \sum W_i * D_{bi}$；$r = r_L + c_r$; $b = b_L + c_b$;

$$r_L = \sum w_i * (r_{Li} - i_i);\ \ b_L = \sum W_i * (b_{Li} - i_i);$$

$$c_r = \sum w_i * (c_i + i_i) = i_r;\ \ c_b = \sum W_i * (c_i + i_i) = i_b;$$

表 3-2、算數超額報酬率來源－Van Breukelen（2000）

(%) Bond	Weighted Duration		Implied Yield Change		Reference Asset Fund		Overall Duration	Market Allocation	Issue Selection	Crncy Mgt
	Port $w_i * D_{ri}$ (13)	Bmk $D_\beta * W_i * D_{bi}$ (14)	Port Δy_{ri} (15)	Bmk Δy_{bi} (16)	Port r_{LS} (17)	Bmk b_{LD} (18)	r_D (19)	r_{AA} (20)	r_{SS} (21)	r_{CM} (22)
UK	3.9	3.4	-0.59	-0.50	1.95	1.70		-0.06	0.35	0.00
JP	0.2	0.3	-0.40	-0.20	0.04	0.05		0.03	0.04	-0.05
US	1.2	1.6	-0.75	-0.93	1.12	1.52		-0.14	-0.22	0.04
Sub-total					r_{LS} 3.11	b_{LD} 3.28	$=b_{LD} - b_L$ 0.87	$=r_{LS} - b_{LD}$ -0.17	$=r_L - r_{LS}$ 0.17	
Total			Δy_r -0.62	Δy_b -0.62	r_S 3.70	b_D 3.87	$=b_D - b$ 0.87	$=r_S - b_D$ -0.17	0.17	$=c_r - c_b$ -0.01
Arithmetic Excess Return $a = r - b = r_D + r_{AA} + r_{SS} + r_{CM}$ = 0.87-0.17+0.17-0.01 = 0.86										

註：$\Delta y_{ri} = -\dfrac{r_{Li} - i_i}{D_{ri}}$；$\Delta y_{bi} = -\dfrac{b_{Li} - i_i}{D_{bi}}$；$\Delta y_r = -\dfrac{r_L - i_r}{D_r}$；$\Delta y_b = -\dfrac{b_L - i_b}{D_b}$；

$$r_S = r_{LS} + c_b = \sum w_i * D_{ri} * (-\Delta y_{bi}) + \sum W_i * (c_i + i_i);$$

$$b_D = b_{LD} + c_b = \sum D_\beta * W_i * D_{bi} * (-\Delta y_{bi}) + \sum W_i * (c_i + i_i);$$

$$r_D = b_D - b = b_{LD} - b_L;$$

$$r_{AA} = r_S - b_D = r_{LS} - b_{LD} = \sum (w_i * D_{ri} - D_\beta * W_i * D_{bi}) * (-\Delta y_{bi} + \Delta y_b);$$

$$r_{SS} = (r - c_r) - (r_S - c_b) = r_L - r_{LS} = \sum w_i * D_{ri} * (-\Delta y_{ri} + \Delta y_{bi});$$

$$r_{CM} = c_r - c_b = \sum (w_i - W_i) * (c_i + i_i - c_b)$$

表 3-2 之解析：

Overall Duration:
In Table 3-1, the portfolio duration (D_r = 5.3) is much larger than the benchmark duration (D_b = 3.9), since yields are falling and markets are rising, this is a positive effect, adding 0.87% of value.

Market Allocation:
The portfolio is overweight UK bonds (3.9 > 3.4), which underperformed the overall index slightly (0.59% < 0.62%), losing 0.06% of value.
The portfolio is underweight JP bonds (0.2 < 0.3), which underperformed the overall index slightly (0.40% < 0.62%), adding 0.03% of value.
The portfolio is underweight US bonds (1.2 < 1.6), which outperformed the overall index slightly (0.75% > 0.62%), losing 0.14% of value.

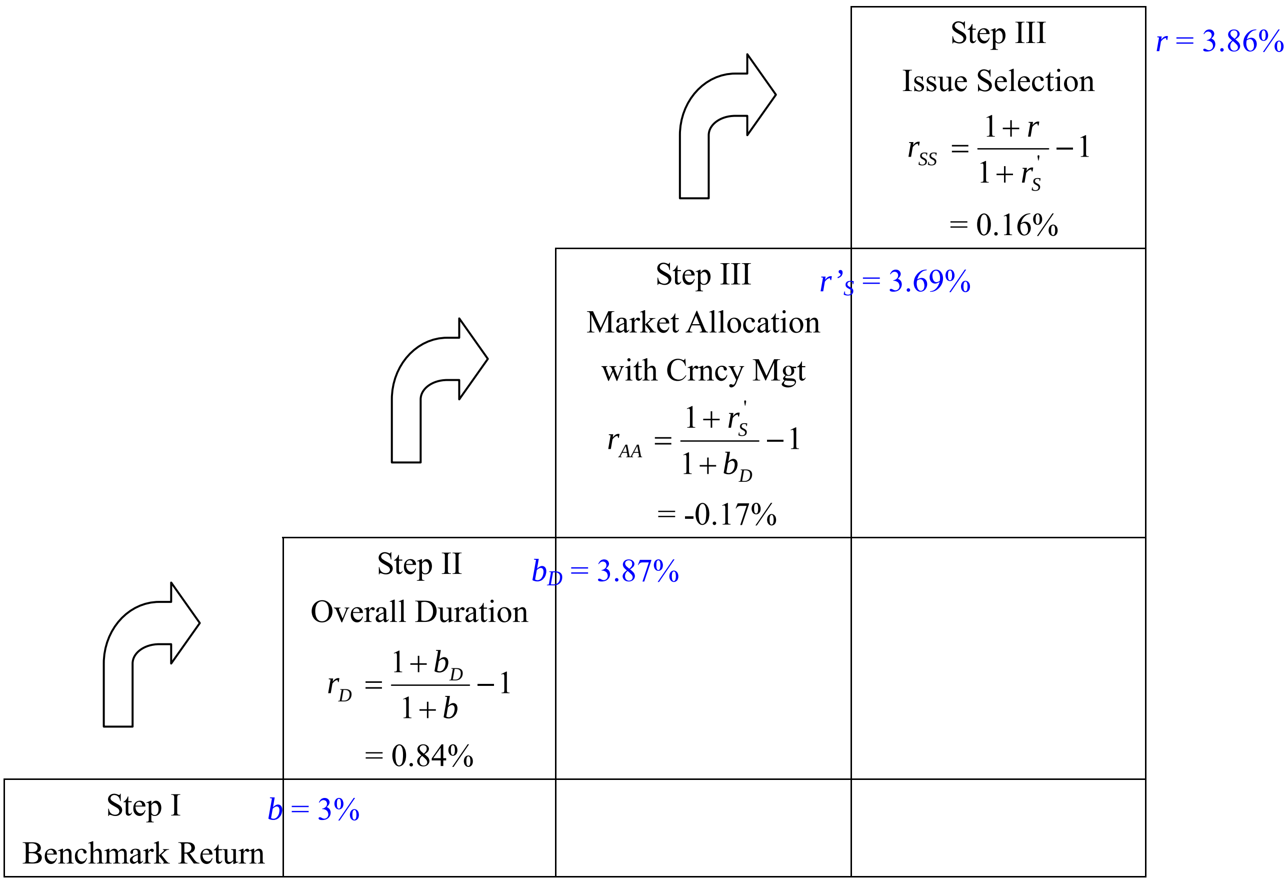
Issue Selection:
The portfolio outperformed in UK bonds, adding 0.35% of value, due to yield falling greater than benchmark (0.59% > 0.50%).
The portfolio outperformed in JP bonds, adding 0.04% of value, due to yield falling greater than benchmark (0.40% > 0.20%).
The portfolio underperformed in US bonds, losing 0.22% of value, due to yield falling smaller than benchmark (0.75% < 0.93%).

Currency Management:
In Table 3-1, the portfolio is overweight (20% > 10%) in low yielding JP interest rates, losing value, but this is almost offset by an underweight (30% < 40%) in low yielding US interest rates.

二、Van Breukelen（2000）－幾何法

Bacon, C.（2008）將 Van Breukelen（2000）模型改為以幾何法衡量超額報酬率，圖 3-2 顯示可依四步驟將幾何超額報酬率拆解出來。

圖 3-2、幾何超額報酬率來源－Van Breukelen（2000）



$$g = \frac{1+r}{1+b} - 1$$

$$= (1+r_D)*(1+r_{AA})*(1+r_{SS}) - 1 \quad = 0.83\% \qquad \text{(Table 3-3)}$$

$$= \left(\frac{1+b_D}{1+b}\right)*\left(\frac{1+r_S^{'}}{1+b_D}\right)*\left(\frac{1+r}{1+r_S^{'}}\right) - 1$$

Geometric Overall Duration Attribution:

$r_D = \frac{1+b_D}{1+b} - 1$ = 1.0387/1.03 – 1 = 0.84% [Top-Down Approach]

Geometric Market Allocation with Currency Management Attribution:

A revised reference fund: (compare equation (3.4) with equation (3.5))

$$\begin{aligned} r_S^{'} &= \sum w_i * D_{ri} * (-\Delta y_{bi}) + \sum w_i * (c_i + i_i) \\ &= r_{LS} + c_r \end{aligned} \tag{3.5}$$

$r_S^{'}$ = 3.11% + 0.58% = 3.69%

Total market allocation: $r_{AA} = \frac{1+r_S^{'}}{1+b_D} - 1$ = 1.0369/1.0387 – 1 = -0.17% [Top-Down]

Individual market allocation: $(w_i * D_{ri} - D_\beta * W_i * D_{bi}) * \frac{(-\Delta y_{bi} + \Delta y_b)}{1+b_D}$ [Bottom-UP]

Individual currency management: $(w_i - W_i) * \left(\frac{1+i_i}{1+c_b} - 1\right) * \frac{1+c_b}{1+b_D}$ [Bottom-UP]

Market allocation attribution and currency management attribution are combined in one step since portfolio weight (one part of the market allocation decision) determines the allocation to different interest rates.

Geometric Issue Selection Attribution:

Total issue selection: $r_{SS} = \frac{1+r}{1+r_S^{'}} - 1$ = 1.0386/1.0369 –1 = 0.16% [Top-Down]

Individual issue selection: $w_i * \left(\frac{1+r_{Li}}{1+D_{bi-Adj}} - 1\right) * \frac{1+D_{bi-Adj}}{1+r_S^{'}}$ [Bottom-UP]

where $D_{bi\text{-}Adj} = \frac{D_{ri}}{D_{bi}} * (b_{Li} - i_i) + i_i$ is Duration adjusted Benchmark

Geometric Excess Return in base currency:

$$\begin{aligned} g &= \frac{1+r}{1+b} - 1 \\ &= (1+r_D)*(1+r_{AA})*(1+r_{SS}) - 1 \\ &= \left(\frac{1+b_D}{1+b}\right) * \left(\frac{1+r_S^{'}}{1+b_D}\right) * \left(\frac{1+r}{1+r_S^{'}}\right) - 1 \end{aligned}$$

表 3-3、幾何超額報酬率來源－Van Breukelen (2000)

(%) Bond	Weighted Duration			Impl ΔYld	Reference Asset Fund		Overall Duration	Market Allocation	Crncy Mgt	Issue Selection
	Port w_i*D_{ri} (13)	Bmk D_β*W_i*D_{bi} (14)	Adj.Bmk $D_{bi\text{-}Adj}$ (15)	Bmk Δy_{bi} (16)	Port r_{LS} (17)	Bmk b_{LD} (18)	r_D (19)	r_{AAi} (20)	r_{CMi} (21)	r_{SSi} (22)
UK	3.9	3.4	4.90	-0.50	1.95	1.70		-0.06	0.00	0.34
JP	0.2	0.3	0.30	-0.20	0.04	0.05		0.03	-0.05	0.04
US	1.2	1.6	3.93	-0.93	1.12	1.52		-0.13	0.04	-0.21
Sum					r_{LS} 3.11	b_{LD} 3.28		-0.16	-0.01	0.16
Total				Δy_b -0.62	$r_S^{'}$ 3.69	b_D 3.87	$r_D = \frac{1+b_D}{1+b}-1$ 0.84	$r_{AA} = \frac{1+r_S^{'}}{1+b_D}-1$ -0.17		$r_{SS} = \frac{1+r}{1+r_S^{'}}-1$ 0.16
Geometric Excess Return $g = (1+r_D)*(1+r_{AA})*(1+r_{SS}) - 1 = 1.0084*0.9983*1.0016 - 1 = 0.83\%$										

註：$r_S^{'} = r_{LS} + c_r = \sum w_i * D_{ri} * (-\Delta y_{bi}) + \sum w_i * (c_i + i_i)$;

$$r_{AAi} = (w_i * D_{ri} - D_\beta * W_i * D_{bi}) * \frac{(-\Delta y_{bi} + \Delta y_b)}{1+b_D};$$

$$r_{CMi} = (w_i - W_i) * \left(\frac{1+i_i}{1+c_b} - 1\right) * \frac{1+c_b}{1+b_D};$$

$$r_{SSi} = w_i * \left(\frac{1+r_{Li}}{1+D_{bi-Adj}} - 1\right) * \frac{1+D_{bi-Adj}}{1+r_S^{'}}, \text{ where } D_{bi\text{-}Adj} = \frac{D_{ri}}{D_{bi}} * (b_{Li} - i_i) + i_i$$

三、Campisi（2000）－算數法

1、基本拆解

圖 3-3 顯示，債券總報酬率可拆解為息票收入（income return）與價格報酬（price return）兩大部份，而價格報酬可進一步解析成源自於三項風險因子：無風險債券殖利率曲線變動（Treasury Effect）、風險性債券殖利率曲線變動（Spread Effect）、殘差項（Selection Effect or Residuals）等，茲以數學符號表達如下。

Total Return = Income Return + Price Return

Income Return = Annual Coupon Rate / Beginning Market Price

Price Return = Effect of Yield Changes

= Treasury Effect + Spread Effect + Selection Effect

In the **single-currency** portfolio, Campisi adapts equation (3.1) replacing local interest rates with income return and adding spread effects as follows:

Portfolio Return: $r_i = I_{ri} + D_{ri} * \left(-\Delta y^t\right) + D_{ri} * \left(-\Delta ys_i\right) + \varepsilon_i$

Benchmark Return: $b_i = I_{bi} + D_{bi} * \left(-\Delta y^t\right) + D_{bi} * \left(-\Delta ys_i\right)$

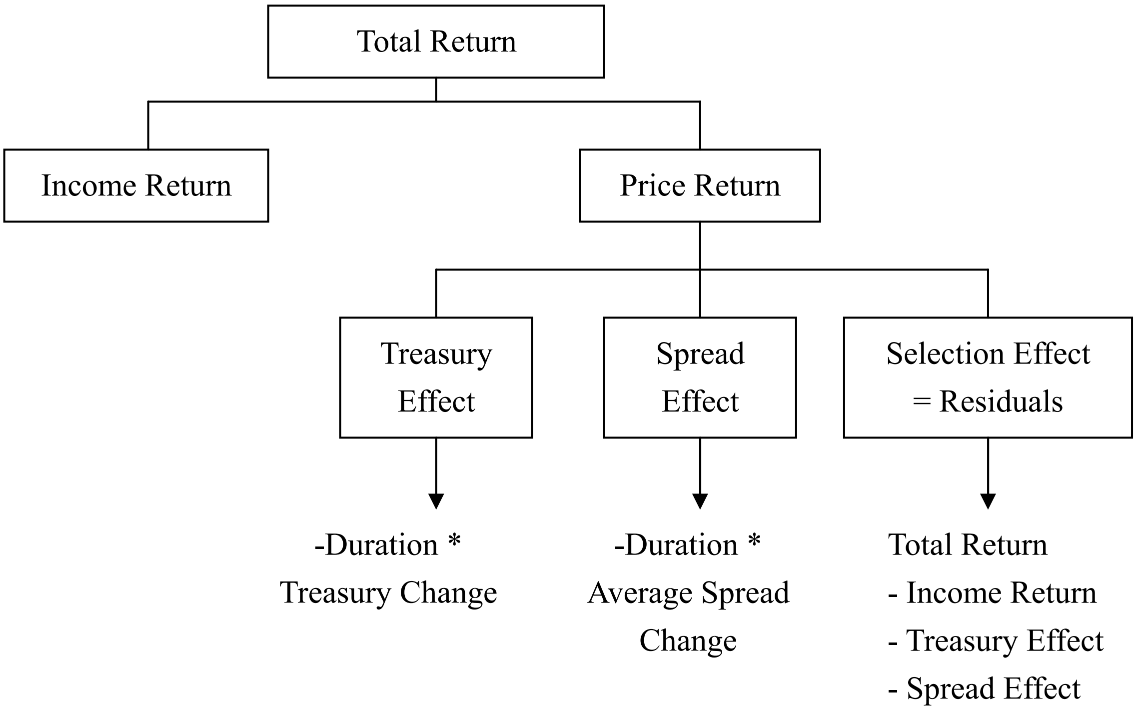
(P.S. There should be no residual returns in the benchmark.)

where I_i is income return in sector *i*

Δy^t is change in Treasury interest rates at duration bucket D^t

Δys_i is change in benchmark spreads in sector *i*

圖 3-3、算數報酬率之基本拆解－Campisi（2000）



Arithmetic Excess Return

$a = r - b = ER_I + ER_T + ER_{YS} + r_{SS}$

(Table 3-4 ~ 3-6)

Arithmetic Excess Return

$a = r - b = ER_I + ER_T + ER_{YS} + r_{SS}$ = 0.08% - 1.92% + 0.02% + 0.02% = -1.80%

Income Excess Return:

$ER_I = I_r - I_b$ = 0.52% - 0.45% = 0.08%

Treasury Effect:

Along the Treasury yield curve, the price return in each duration bucket is: $D^t * \left(-\Delta y^t\right)$

Portfolio Treasury Effect:

In portfolio, each sector return due to changes of UST curve is: $w_i * \left[D^t * \left(-\Delta y^t\right)\right]_{Dri}$,

where t is the Treasury duration bucket corresponds to the portfolio duration in sector i (D_{ri}).

Total portfolio Treasury return is:

$$T_r = \sum w_i * T_{ri} = \sum w_i * \left[D^t * \left(-\Delta y^t\right)\right]_{Dri}$$

Benchmark Treasury Effect:

In benchmark, each sector return due to changes of UST curve is: $W_i * \left[D^t * \left(-\Delta y^t\right)\right]_{Dbi}$

Total benchmark Treasury return is:

$$T_b = \sum W_i * T_{bi} = \sum W_i * \left[D^t * \left(-\Delta y^t\right)\right]_{Dbi}$$

Therefore, the added value from the Treasury Effect is:

$ER_T = T_r - T_b$ = 4.19% - 6.11% = -1.92%

Spread Effect:

The benchmark yield spread changes against UST curve in sector i can be estimated as:

$\Delta ys_i = -\dfrac{b_i - I_{bi} - D_{bi} * \left(-\Delta y^t\big|_{Dbi}\right)}{D_{bi}}$, where t is the Treasury duration bucket corresponds to the benchmark duration in sector i.

Benchmark Spread Effect:

In benchmark, each sector return due to benchmark yield spread changes is: $D_{bi} * \left(-\Delta ys_i\right)$

The contribution from benchmark yield spread changes in sector *i* is: $W_i * D_{bi} * (-\Delta ys_i)$

Total contribution from benchmark yield spread changes is:

$$YS_b = \sum W_i * YS_{bi} = \sum W_i * D_{bi} * (-\Delta ys_i)$$

Portfolio Spread Effect:

In portfolio, each sector return due to benchmark yield spread changes is: $D_{ri} * (-\Delta ys_i)$

The contribution from benchmark yield spread changes in sector *i* is: $w_i * D_{ri} * (-\Delta ys_i)$

Total contribution from benchmark yield spread changes is:

$$YS_r = \sum w_i * YS_{ri} = \sum w_i * D_{ri} * (-\Delta ys_i)$$

Therefore, the added value from the Spread Effect is:

$ER_{YS} = YS_r - YS_b = 0.17\% - 0.15\% = 0.02\%$

Selection Effect (or Residuals):

Any contribution to return not derived from income, treasury or spread effect must be issue selection:

$$r_{SS} = \sum w_i * r_{SSi} = \sum w_i * (r_i - I_{ri} - T_{ri} - YS_{ri}).$$

表 3-4、美國公債殖利率曲線變化－Bull Flattening

Duration Bucket (year) D^t	Yield Change (%) Δy^t	Price Effect (%) $D^t * (-\Delta y^t)$
3.60	-1.00	+3.60
4.00	-1.05	+4.20
4.30	-1.10	+4.73
4.75	-1.20	+5.70
4.88	-1.25	+6.09
5.25	-1.35	+7.09

表 3-5、算數超額報酬率來源－Campisi（2000）－基本拆解

(%) US Bond	Weight		Modified Duration		Local=Base Return		Income Return		Treasury Effect		Spread Effect			Select Effect
	Port w_i (1)	Bmk W_i (2)	Port D_{ri} (3)	Bmk D_{bi} (4)	Port r_i (5)	Bmk b_i (6)	Port I_{ri} (7)	Bmk I_{bi} (8)	Port T_{ri} (9)	Bmk T_{bi} (10)	Δys_i (11)	Port YS_{ri} (12)	Bmk YS_{bi} (13)	Port r_{SSi} (14)
UST	20	50	4.75	4.75	6.0	6.0	0.30	0.30	5.70	5.70				
Corp	65	40	3.60	5.25	4.4	8.0	0.52	0.57	3.60	7.09	-0.07	0.23	0.34	0.05
HY	15	10	4.30	4.00	5.6	5.0	0.82	0.71	4.73	4.20	-0.02	0.10	0.09	-0.05
Sum	100	100	D_r 3.94	D_b 4.88	r 4.9	b 6.7	I_r 0.52	I_b 0.45	T_r 4.19	T_b 6.11		YS_r 0.17	YS_b 0.15	r_{SS} 0.02
Total							$ER_I = I_r - I_b$		$ER_T = T_r - T_b$		$ER_{YS} = YS_r - YS_b$			
Arithmetic Excess Return $a = r - b = ER_I + ER_T + ER_{YS} + r_{SS}$ = 0.08% - 1.92% + 0.02% + 0.02% = -1.80%														

註：$D_r = \sum w_i * D_{ri}$； $D_b = \sum W_i * D_{bi}$； $r = \sum w_i * r_i$； $b_L = \sum W_i * b_i$；

$I_r = \sum w_i * I_{ri}$； $I_b = \sum W_i * I_{bi}$；

$$T_r = \sum w_i * \left[D^t * \left(-\Delta y^t\right)\right]_{Dri} = 0.2*5.7\% + 0.65*3.6\% + 0.15*4.73\% = 4.19\%;$$

$$T_b = \sum W_i * \left[D^t * \left(-\Delta y^t\right)\right]_{Dbi} = 0.5*5.7\% + 0.4*7.09\% + 0.1*4.20\% = 6.11\%;$$

$$\Delta ys_i = -\frac{b_i - I_{bi} - D_{bi} * \left(-\Delta y^t \big|_{Dbi}\right)}{D_{bi}}, \text{ e.g. } \Delta ys_{Corp} = -(8\% - 0.57\% - 5.25*1.35\%) / 5.25;$$

$$YS_r = \sum w_i * YS_{ri} = \sum w_i * D_{ri} * \left(-\Delta ys_i\right) = 0.65*3.6*0.07\% + 0.15*4.3*0.02\% = 0.17\%;$$

$$YS_b = \sum W_i * YS_{bi} = \sum W_i * D_{bi} * \left(-\Delta ys_i\right) = 0.4*5.25*0.07\% + 0.1*4.0*0.02\% = 0.15\%;$$

$$r_{SS} = \sum w_i * r_{SSi} = \sum w_i * \left(r_i - I_{ri} - T_{ri} - YS_{ri}\right)$$

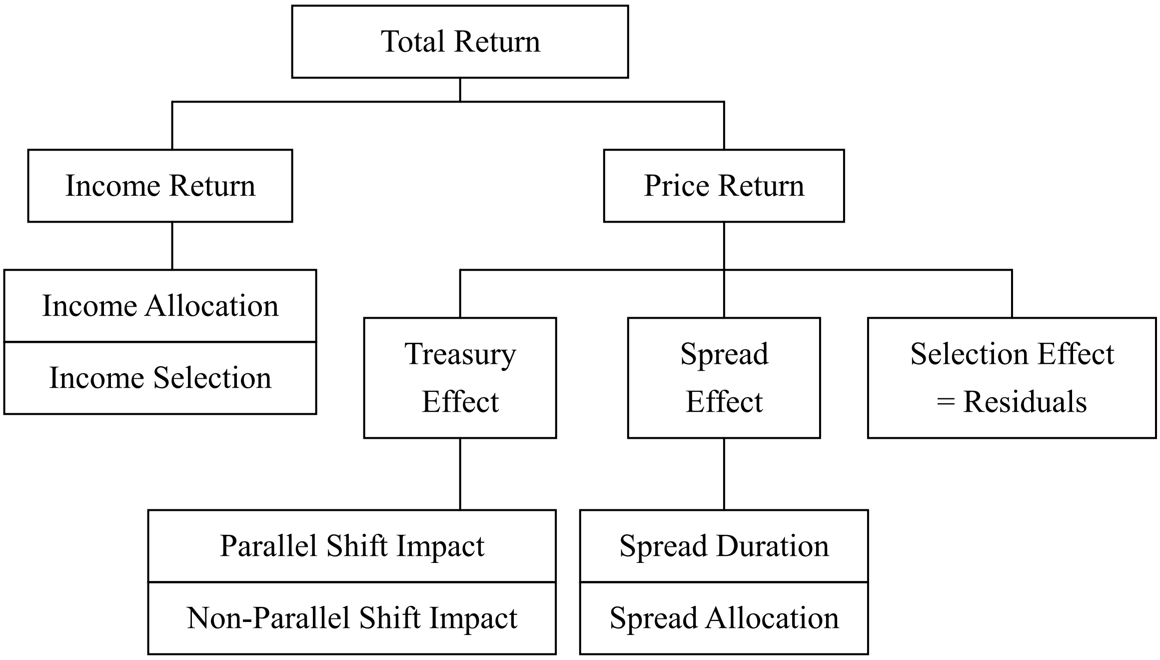
$$= 0.65*(4.4\% - 0.52\% - 3.60\% - 0.23\%) + 0.15*(5.6\% - 0.82\% - 4.73\% - 0.10\%) = 0.02\%.$$

In this example, the portfolio duration (D_r = 3.94) is much smaller than benchmark duration (D_b = 4.88), which leads to a huge negative Treasury Effect when UST curve was bull-flattening. In addition, the Corp and the HY yield spreads over Treasury tightened, which leads to a positive Spread Effect.

2、詳細拆解

基本拆解仍以存續期間來概估各項風險因子變動對債券報酬率之影響，缺點為只捕捉殖利率曲線水平移動，若欲了解殖利率曲線非水平移動之影響，可進一步拆解 Campisi 模型如圖 3-4，資料續表 3-4。

圖 3-4、算數報酬率之詳細拆解—Campisi（2000）



<u>Income Return:</u>

(1) Income Allocation:

$$I_{AA} = \sum (w_i - W_i) * (I_{bi} - I_b)$$

= (0.2-0.5)*(0.3%-0.45%)+(0.65-0.4)*(0.57%-0.45%)+(0.15-0.1)*(0.71%-0.45%)
= 0.09%

(2) Income Selection:

$$I_{SS} = \sum w_i * (I_{ri} - I_{bi})$$

= 0.2*(0.3%-0.3%) + 0.65*(0.52%-0.57%) + 0.15*(0.82%-0.71%) = -0.01%

Treasury Effect:

Interpolate the yield change at the benchmark duration along the UST curve:

$\Delta y^t \big|_{Db}$ = -1.25%

For example, from Table 3-4, the benchmark duration is 4.88 year which corresponds to a Treasury yield change of –1.25%.

(1) Duration /or Parallel Shift Impact:

$Dur_{Parallel} = (D_r - D_b) * -\Delta y^t \big|_{Db}$

= (3.94 - 4.88)*1.25% = -1.18%

(2) Non-parallel Shift Impact:

$Dur_{Non\text{-}Parallel} = \sum w_i * D_{ri} * \left[\left(-\Delta y^t \big|_{Dri}\right) - \left(-\Delta y^t \big|_{Db}\right)\right]$

= 0.2*4.75*(1.20%-1.25%) + 0.65*3.6*(1%-1.25%) + 0.15*4.3*(1.1%-1.25%)
= -0.74%

Spread Effect:

Total benchmark spread change: $\Delta ys_b = \dfrac{-(b - I_b - T_b)}{D_b}$

= - (6.7%-0.45%-6.11%) / 4.88 = -0.029%

(1) Spread Duration: the overall spread change at portfolio duration vs. benchmark duration

$Spd_{Dur} = (D_r - D_b) * (-\Delta ys_b)$

= (3.94 – 4.88) * 0.029% = -0.027%

(2) Spread Allocation:

$Spd_{AA} = \sum (w_i * D_{ri} - W_i * D_{bi}) * [(-\Delta ys_i) - (-\Delta ys_b)]$

= (0.2*4.75 – 0.5*4.75)*(0.0% – 0.03%) + (0.65*3.6 – 0.4*5.25)*(0.07% – 0.03%)
+ (0.15*4.3 – 0.1*4)*(0.02% – 0.03%) = 0.05%

Selection Effect:

$r_{SS} = \sum w_i * r_{SSi} = \sum w_i * (r_i - I_{ri} - T_{ri} - YS_{ri})$

表 3-6、算數超額報酬率來源－Campisi（2000）－詳細拆解

Income Return		Treasury Effect		Spread Effect			Select Effect
Allocation (7)	Selection (8)	Duration (9)	Non-Parallel (10)	Bmk Spd Change (11)	Spread Duration (12)	Spread Allocation (13)	Port $w_i * r_{SSi}$ (14)
0.04	0.00		-0.05			0.04	
0.03	-0.03		-0.59			0.01	0.03
0.01	0.02		-0.10			0.00	-0.01
I_{AA} 0.09	I_{SS} -0.01	$Dur_{Parallel}$ -1.18	$Dur_{Non\text{-}Parallel}$ -0.74	Δys_b -0.029	Spd_{Dur} -0.027	Spd_{AA} 0.05	r_{SS} 0.02
$ER_I = I_{AA} + I_{SS}$		$ER_T = Dur_{Parallel} + Dur_{Non\text{-}Parallel}$		$ER_{YS} = Spd_{Dur} + Spd_{AA}$			
Arithmetic Excess Return $a = r - b = ER_I + ER_T + ER_{YS} + r_{SS}$ = 0.08% - 1.92% + 0.02% + 0.02% = -1.80%							

Interpreting the detailed results is that the portfolio was short duration and lost 118 bps for a parallel shift down in Treasury curve, and another 74 bps given that the Treasury curve was flattening. Structurally, the portfolio benefited from slightly higher income and a tightening in credit spreads plus a slight contribution from issue selection.

3、Campisi 模型與標準 Brinson 模型之比較

For comparison, attribution effects using the standard Brinson model are calculated in Table 3-8. The misleading conclusion from the Brinson model would be significantly poor selection in corporate bonds (-2.34%), while it is 0.03% in the Campisi model.

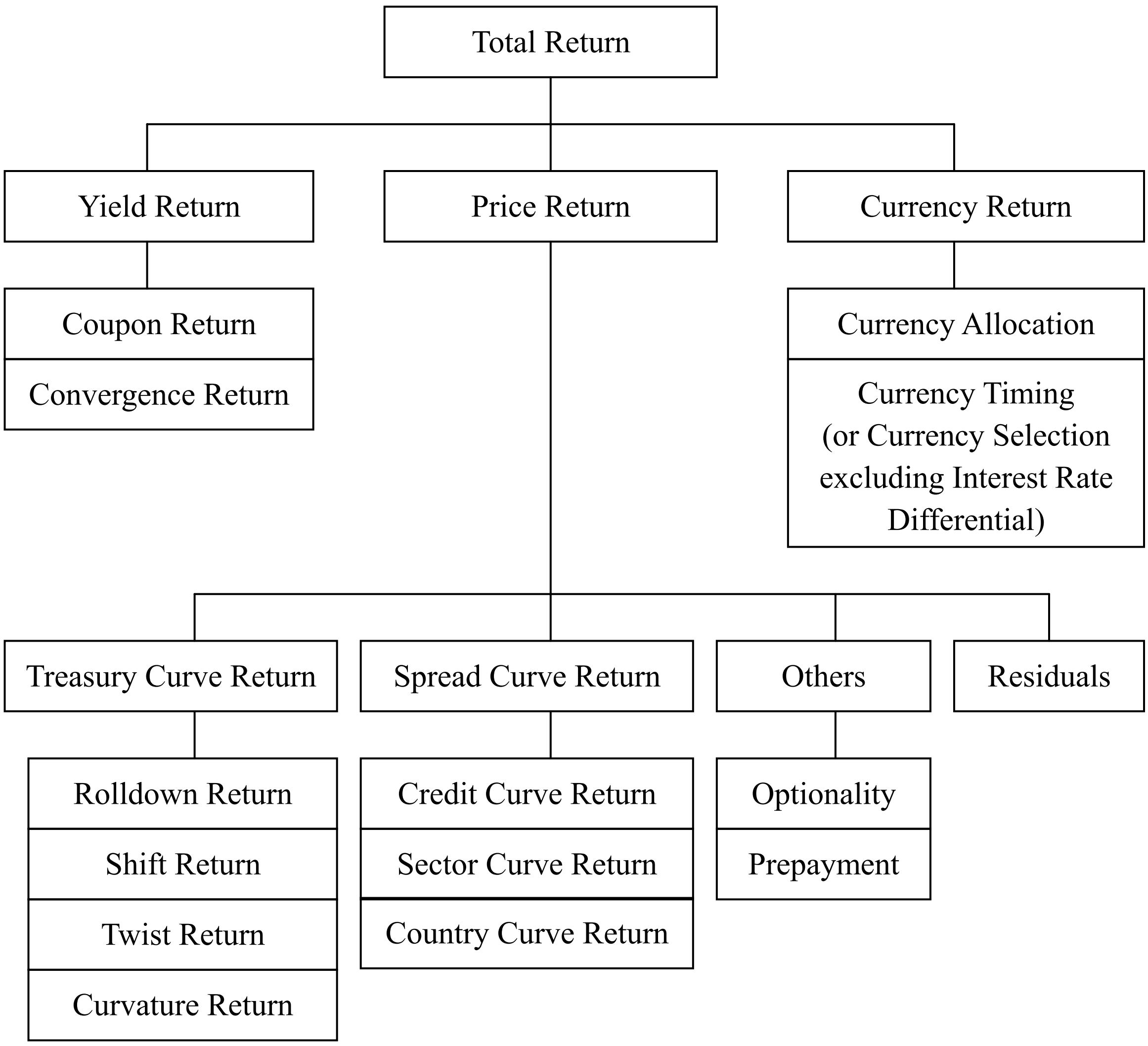
表 3-7、算數超額報酬率來源－標準 Brinson 模型

(%) US Bond	Weight		Local=Base Return		Asset Allocation	Security Selection
	Port w_i (1)	Bmk W_i (2)	Port r_i (3)	Bmk b_i (4)	$(w_i - W_i)*$ $(b_i - b)$ (5)	$w_i *$ $(r_i - b_i)$ (6)
UST	20	50	6.0	6.0	0.21	0.00
Corp	65	40	4.4	8.0	0.33	-2.34
HY	15	10	5.6	5.0	-0.09	0.09
Total	100	100	r 4.9	b 6.7	r_{AA} 0.45	r_{SS} -2.25

第四節、 多幣別債券投資組合績效評估－一般性架構

Campisi（2000）之因素貢獻法為純債券型基金之績效評估奠定了基礎，本節參考 Colin（2005）、Campisi and Spaulding ed.（2007）與 Bacon（2008）等實務書籍，試圖以一個一般性的架構來拆解多幣別債券投資組合之報酬率。

圖 4-1、多幣別債券投資組合報酬率拆解－一般性架構



首先，將以基礎貨幣標示的債券報酬率（R）分解成以當地貨幣標示的債券報酬率（R_L）與匯率報酬率（R_C）：

$$(1+R)=(1+R_L)*(1+R_C)$$

I. Currency Return

Country manager's performance

$$R_C=\frac{S^t}{S^{t-1}}-1=\frac{S^t}{F^{t-1}}*\frac{F^{t-1}}{S^{t-1}}-1=(1+f)*(1+d)-1$$

Currency manager's performance

where $f=\frac{S^t}{F^{t-1}}-1$ is benchmark forward currency return

$d=\frac{F^{t-1}}{S^{t-1}}-1$ is interest rate differentials or forward premium

Typically there is a separate country and currency allocation in tandem within the investment decision process. If the "country manager" decides to overweight Japanese equities that will inevitably create a long position in JPY; the "currency manager" wishing to keep a neutral JPY position may decide to hedge the exposed JPY position using Forwards. The process of maintaining a neutral currency position is described as "hedged to neutral".

If the base currency of the portfolio is GBP, the currency manager will sell JPY and buy GBP. In effect, the currency manager is borrowing JPY to buy GBP – there is a "cost or benefit" attached to this depending on the interest rate differentials between the two currencies at the time. In this case, borrowing low yield currency (JPY) and buy high yield currency (GBP) is a benefit, and it is a cost vice versa. This hedging cost/benefit inherent in a "hedged to neutral" process (i.e. a natural consequence due to macros) should not belong to the currency manager.

Apart from hedging exposed positions caused by country managers or achieving hedged positions implied by hedged benchmarks (i.e. "passive" currency management), the currency manager may be seeking to generate "active" currency positions by Forwards or other derivatives that implicitly include interest rate differentials. In other words, taking a currency allocation "bet" must be exposed to the cost (or benefit) of these interest rate differentials. Therefore, the "forward currency return" rather than the "spot currency return" must be used to measure currency allocation effects. Crucially, this hedging cost/benefit should be borne by the country allocator not the currency manager.

其次，設債券價格為利率與時間之函數（$P(y,t)$），則利用 Taylor 展開式可拆解出以當地貨幣標示的債券報酬率來源：

$$R_L = \frac{\Delta P}{P} = \frac{1}{P} \cdot \left(\frac{\partial P}{\partial t} \cdot \Delta t + \frac{\partial P}{\partial y} \cdot \Delta y \right) = \underbrace{y \cdot \Delta t}_{Yield\ Return} + \underbrace{\left(-MD \cdot \Delta y + \frac{1}{2} \cdot C \cdot (\Delta y)^2 \right)}_{\textbf{Price Return}}$$

其中，$\Delta y = y_{t+1} - y_t$。

II. Yield Return

Yield Return = Coupon Return + Convergence Return（溢折價攤銷）

$$R_{Yield} = R_{Coupon} + R_{Convergence}$$

$$y_t \cdot \Delta t = Cpn \cdot \Delta t + (y_t - Cpn) \cdot \Delta t$$

III. Price Return

Price Return = Curve Return + Spread Return + Others + Residuals

III-1. Treasury Curve Return (Reference Curve Return)

$$R_{Curve} = R_{Shift} + R_{Twist} + R_{Curvature} + R_{Rolldown}$$

$$r_{yc} = -MD \cdot \left[\Delta y_{\mathbf{shift}} + \Delta y_{\mathbf{twist}} + \Delta y_{\mathbf{curvature}} + \Delta y_{\mathbf{roll}} \right] + \frac{1}{2} \cdot C \cdot (\Delta y)^2$$

而拆解殖利率曲線變動三因子之方法，常見者包括：

1. No Model Method
2. Polynomial Term Structure Models (Cubic Spline Fitting

Method)

3. Nelson-Siegel Term Structure Models (Parsimonious Fitting Method)
4. Principal Component Analysis (Multivariate Method)

1. No Model Method

$$\Delta y_{shift} = \frac{1}{4}\left(\Delta y_2 + \Delta y_5 + \Delta y_{10} + \Delta y_{30}\right)$$

$$\Delta y_{twist} = \Delta y_{30} - \Delta y_2$$

$$\Delta y_{curvature} = \Delta y - \Delta y_{shift} - \Delta y_{twist} - \Delta y_{roll}$$

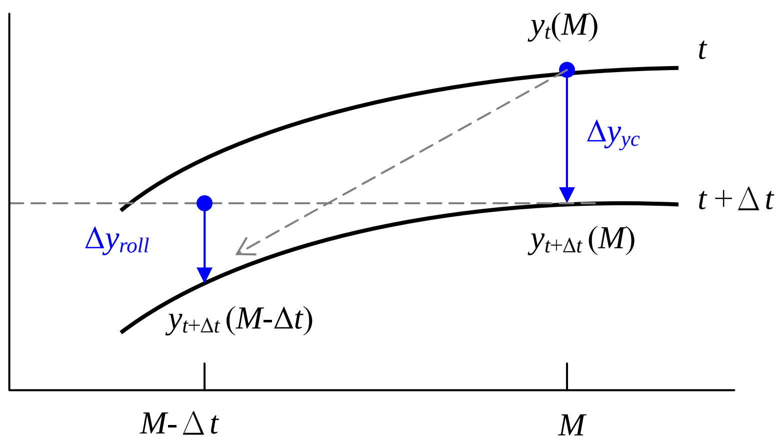
where

$$\Delta y = y_{t+\Delta t}(M\text{-}\Delta t) - y_t(M) = \Delta y_{roll} + \Delta y_{yc}$$

$$\Delta y_{roll} = y_{t+\Delta t}(M\text{-}\Delta t) - y_{t+\Delta t}(M)$$

$$\Delta y_{yc} = y_{t+\Delta t}(M) - y_t(M)$$

In an upward sloping curve, roll return will always > 0. If curve shifts up and a loss arises, this will show up in appropriate attribution. Roll return cannot be attributed to yield or to yield curve movements. On the contrary, in a downward sloping curve, a negative roll return would occur. Therefore, we should separate roll return from yield curve return.



2. Polynomial Term Structure Models (Cubic Spline Fitting Method)[1]

Fitting a second-order polynomial function to the yield curve:

$$y(m) = a_0 + a_1 m + a_2 m^2$$

Depending on the value of parameters, *a*, the function can model a straight line, a slanted line or a parabola. If the curve changes shape, the parameter values will vary over time.
A twist point can be added to this function by rescaling the maturity variable *m* by an amount *S*, so that

$$y(m) = a_0 + a_1(m - S) + a_2(m - S)^2$$

With this modification, a curve rotation about maturity *S* will be expressed entirely as twist without any parallel movement.
Yield contributions from sub-component movements can then be calculated as follows:

$$\Delta y_{shift} = a_0^{t+1} - a_0^{t}$$

$$\Delta y_{twist} = \left(a_1^{t+1} - a_1^{t}\right)\cdot(m - S)$$

$$\Delta y_{curvature} = \left(a_2^{t+1} - a_2^{t}\right)\cdot(m - S)^2$$

Advantages:
The great advantage is that it is straightforward to identify the meaning of various terms.

Disadvantages:
(1) The curve is not well behaved at the long end, since the yield rises as the square of the maturity. The yield does not therefore tend to a constant value at high maturities.
(2) Even if two curves are quite similar at nearby dates, the polynomial coefficients can differ widely between the two, and this can give rise to spurious attribution returns.
(3) The placement of the twist point is crucial.

[1] 參見：賀蘭芝，民國 98 年 7 月，「債券利率曲線配適與投資之應用」，PIMCO「2009 Client Conference」心得報告，出國地點：美國加州。

3. Nelson-Siegel Term Structure Models (Parsimonious Fitting Method)

Nelson-Siegel (1987) fit market forward rates to a parsimonious function:

$$f(m) = \beta_0 + \beta_1 \exp\left(-\frac{m}{\tau}\right) + \beta_2\left[\left(\frac{m}{\tau}\right)\exp\left(\frac{m}{\tau}\right)\right]$$

where $f(m)$ is the forward rate for maturity m, and τ is a scale length actor.
Market yields are then calculated from:

$$y(m) = \frac{1}{m}\int_0^m f(\tau)d\tau$$

which gives

$$y(m) = \beta_0 + (\beta_1 + \beta_2)\cdot\left(\frac{\tau}{m}\right)\cdot\left[1-\exp\left(-\frac{m}{\tau}\right)\right] - \beta_2 \exp\left(-\frac{m}{\tau}\right) \quad (*)$$

This has the convenient feature that $\begin{cases} y(m) \to (\beta_0 + \beta_1) & as\ m \to 0 \\ y(m) \to \beta_0 & as\ m \to \infty \end{cases}$,

i.e., the curve becomes asymptotically flat as *m* becomes large.

Advantages:
These functions behave very much like real yield curves, and is therefore, a good choice to use for attribution analysis.

Notes:

(1) It is necessary for τ to be fixed, so that changes in its value cannot affect the curve shape. Nelson and Siegel note that there is a tradeoff in choosing a suitable value:
Small value of τ corresponds to rapid decay in the regressor that can fit curvature at low maturities well, while cannot fit excessive curvature over longer maturity ranges.
Large value of τ produces slow decay in the regressor that can fit curvature over longer maturity ranges, but cannot follow extreme curvature at short maturities.

(2) The precise value of τ will depend on the characteristics of the yield curve. Based on research and some worked examples, a value of τ of around 30% of the highest maturity appears to be a suitable starting point. [τ = 30% * Max Maturity]

(3) Alternatively, since the form of the equation is unchanged under a change in variable, $\frac{m}{\tau} \to m^{'}$, no information is lost if we set τ = 1 and move any maturity rescaling into the raw data, so equation (*) becomes: [τ = 1]

$$y(m) = \beta_0 + (\beta_1 + \beta_2)\frac{[1-\exp(-m)]}{m} - \beta_2 \exp(-m) \qquad (**)$$

Thoughout the remainder of the report, equation (**) will be used.

(4) More complex Nelson-Siegel type function may fit to real world yield curve data better. Bolder and Streliski (1999) from the Royal Bank of Canada describes a five-parameter Nelson-Siegel model and other sophisticated approaches.

To include a curve twist point, we again introduce a maturity scale length, *S*, which may be interpreted as the maturity about which a yield curve twist is occurring. Then we can set

$$y_0(m) = \beta_0 + \beta_1 \exp(-S),$$
$$if \begin{cases} S = 0 \to y_0 = \beta_0 + \beta_1 \,(asymp\ yld\ as\ m \to 0\ ) \\ S \sim \infty \to y_0 = \beta_0 \,(asymp\ yld\ as\ m \sim \infty) \end{cases}$$

$$y_1(m) = \beta_1\left[\frac{1-\exp(-m)}{m} - \exp(-S)\right]$$

$$y_2(m) = \beta_2\left[\frac{1-\exp(-m)}{m} - \exp(-m)\right], \text{ independent of } S$$

Yield contributions from shift, twist and curvature movements may then be calculated as:

$$\Delta y_{shift} = \left[\beta_0^{t+1} + \beta_1^{t+1}\exp(-S)\right] - \left[\beta_0^{t} + \beta_1^{t}\exp(-S)\right]$$

$$\Delta y_{twist} = \left(\beta_1^{t+1} - \beta_1^{t}\right)\left[\frac{1-\exp(-m)}{m} - \exp(-S)\right]$$

$$\Delta y_{curvature} = \left(\beta_2^{t+1} - \beta_2^{t}\right)\left[\frac{1-\exp(-m)}{m} - \exp(-m)\right]$$

4. Principal Component Analysis (Multivariate Method)[2]

The vector of yield change at each maturity:
$\Delta \boldsymbol{Y} = [\Delta y_1, \Delta y_2, \Delta y_3, \ldots, \Delta y_m]$

Yield change vector can be described by a sum of other vectors:

$$\Delta \boldsymbol{Y} = \sum_{i=1}^{m} w_k \boldsymbol{X}_k$$

where each $\boldsymbol{X}_k$ is orthonormal. Each orthonormal vector describes a direction in which the curve can move.

Some portfolio managers immunize yield curve risk by PCA method, i.e., immunize against movements in each direction. If it is the way the portfolio risk had been hedged, one could use the principal component decomposition for attribution analysis. The sum of each movement at each maturity would add up to the actual yield movement for that maturity.

Advantages:
If a portfolio has been hedged using PCA, then it makes sense to produce an attribution analysis based on the same breakdown of curve movements.

Disadvantages:
This approach is not so clearly understandable for presentation purpose, particularly the individual curve movements do not have a straightforward interpretation.

[2] 參見：賀蘭芝，民國 93 年 11 月，「市場風險衡量方法」，BIS「風險管理進階研討會」心得報告，出國地點：瑞士琉森。

III-2. Spread Return

$$R_{Spread} = -MD \cdot \Delta y_{CS}$$

表 4-1、風險溢酬曲線報酬率貢獻度分析

Swap Curve	Credit Curve	Sector Curve
Swap Spread Return Issue Specific Return	Credit Spread Return	Sector Return

1. Swap Curve Attribution
   Swap curve attribution measures the effect that changes in the spread between a country's Treasury curve and the swap curve. There is only one swap curve per country.

2. Credit Curve Attribution
   Credit curve attribution measures the effects on return of yield curves rated below AAA.
   With careful structuring, it is possible to immunize the portfolio return from any changes of Treasury curve, and leaves all the profit from credit movements. For investors who follow this strategy, detailed credit attribution is of great interest.

3. Sector Curve Attribution
   Sector yield curves are assigned to market sectors such as healthcare, banks, and retail. Given a bond, only one sector curve can be associated.
   If attribution is carried out in this manner, one would show results in terms of broad yield curve movements, then decompose credit spread returns by sector.
   Sector curve attribution is performed in exactly the same way as swap or single credit curve attribution. The only difference is that there are multiple sector curves for each country.

4. Country Curve Attribution
   For emerging market bonds, country effect is an important source of return.
   For example, a USD-denominated bond issued by an Asian country into the US market or international markets will carry a risk

premium compared to the exact same bond issued by US issuer. The higher yield is informally known as the Yankee premium. In general, a Eurobond is a security that is denominated in a currency other than that of the country in which it is issued.
Consider a BBB-rated Thai Yankee bond that is trading at 100 bps premium to its USD-issued counterparty, but lies on the Thai USD curve. In this case, the country spread is 100 bps and the liquidity spread is 0. Without the ability to measure country effect, the 100 bps will be attributed to liquidity or issue-specific factors, which is misleading.

III-3. Other Returns

1. Outright option: the optionality return can be separated as

$$r = y \cdot \Delta t + \left(-MD \cdot \Delta y + \frac{1}{2} \cdot C \cdot (\Delta y)^2 \right) + \underbrace{\left(Delta \cdot dS + \frac{1}{2} \cdot \Gamma \cdot (dS)^2 + v \cdot d\sigma + \theta \cdot dt \right)}_{\textit{Option Return}}$$

where S indicates underlying asset price.

2. Embedded option: such as prepayment option in MBS

Using Static Cash Flow Yield (SCFY) method,

$y = y_{UST}$ + Static Spread

where y_{UST} is the WAL- or duration-matched Treasury Note yield.

Alternatively, using OAS Monte Carlo simulation method,

$y = SR_{UST}$ + Option Cost + OAS

where SR_{UST} is the spot rates on the Treasury Curve.

This gives

Δy - ΔSR_{UST} - Δy_{OPT} = ΔOAS

第五節、 結論與未來發展方向

管理一投資組合，後檯之績效分析，對於中檯之風險管理、與前檯之投資調整具有反餽作用，好的分析系統有助經理人了解其績效來源，提供經理人未來調整投資組合之參考，能增進投資組合報酬率，因此績效貢獻分析系統（attribution systems）與風控系統（risk management）及投資組合最適化系統（portfolio optimization software）同等重要。

績效貢獻分析須能反映經理人之投資決策流程，本報告介紹了幾種不同的拆解模型，相信規模較大的知名資產管理公司皆有內部自行開發的系統，以反映自家的投資決策結構。

對委託人而言，以一套系統來分析所有帳戶之績效，或許可供內部參考，但宜否以該分析結果，進一步干涉經理人之操作，則有待商榷。若需外購績效評估系統，建議宜注意能否提供下列功能：

1、 殖利率曲線配適功能，得以精確地衡量債券報酬率，而非只有存續期間概算法。
2、 風險控管工具，如風險值（Value-at-Risk）。
3、 情境分析工具，如蒙地卡羅模擬模型（Monte-Carlo Simulation）。
4、 投資組合最適化求解系統。
5、 與前、中檯系統整合之可能性。

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