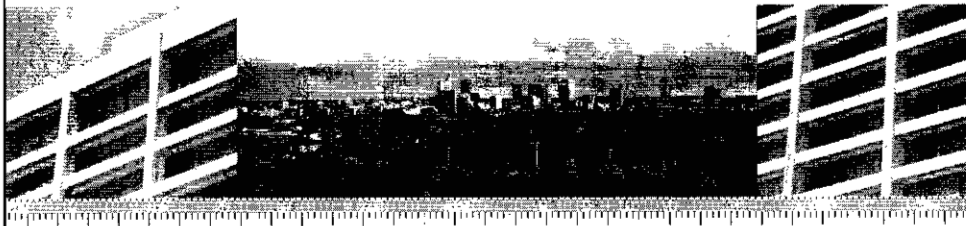


Financial Market-Based Indicators for Monetary Policy and Financial Stability

Dr. Franziska Schobert
Deutsche Bundesbank



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- I Risk-Neutral Density Functions

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Why are Inflation Expectations Important?

Monetary policy has to be forward-looking because of time lags of the transmission mechanism

As inflation expectations influence contracting-parties' behaviour at labour, goods, and financial markets, inflation expectations become important for future inflation.

Short-run inflation expectations: Information on potential first & second round effects

Long-run inflation expectations: Information on monetary policy's credibility

Monetary Policy and Zero-Bound Constraint

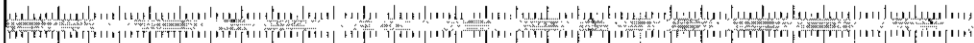


The lower bound of nominal interest rates is typically close to zero

Anchoring of inflation expectations at the Eurosystem's definition of price stability "inflation below but close to 2%"

Fisher identity

Real interest rate \approx nominal rate – expected inflation



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Inflation Expectations & Communication



Introductory Statement of the ECB president on 2 April 2009:

...The Governing Council will continue to ensure a firm anchoring of medium-term inflation expectations. Such anchoring is indispensable to supporting sustainable growth and employment and contributes to financial stability.

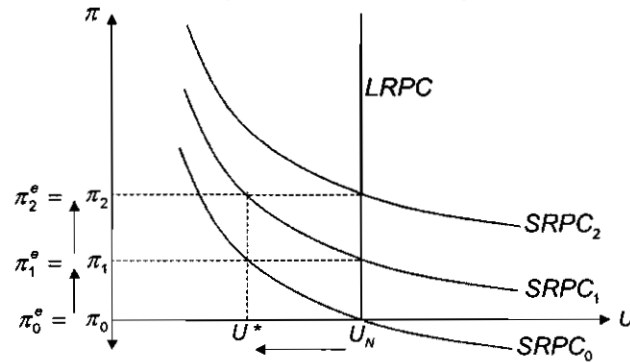
...Looking ahead, base effects stemming from past energy price effects will play a significant role in the shorter-term dynamics of the HICP. Accordingly, we expect to see headline annual inflation rates declining further in the coming months and temporarily reaching negative levels around mid-year. Thereafter, annual inflation rates should increase again. Such short-term movements are, however, not relevant from a monetary policy perspective. Looking further ahead, over the policy-relevant horizon, annual HICP inflation is expected to remain below 2% in 2010, reflecting mainly ongoing sluggish demand in the euro area and elsewhere. Available indicators of inflation expectations over the medium to longer term remain firmly anchored in line with the Governing Council's aim of keeping inflation rates at levels below, but close to, 2% over the medium term.



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Theoretical Considerations

The role of inflation expectations in a Phillips-curve setting



See also: Barro/ Gordon (1983)

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Inflation Expectations

Measures based on

1. Financial market indicators

- BEIR: break-even inflation rate based on inflation-indexed bonds
- Inflation swaps

2. Surveys

- Consensus Economic Expert Forecast
- SPF (Survey of Professional Forecasters)

-

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Break-Even Inflation Rate

The yield gap between nominal bonds and inflation-indexed (i.e. real) bonds (each maturing in j years) is termed break-even inflation rate (BEIR)

$$BEIR_t^j = \left(\frac{1 + i_t^j}{1 + r_t^j} - 1 \right) \times 100$$

BEIR approximately quantifies average expected inflation over a certain time period from now on

Example

Data as of 27 February 2009:

Yield (ytm) on inflation-indexed government bond Germany 1.5% 2016: 1.699%

Ytm on nominal government bond Germany 4.0% 2016: 2.867%

Average expected rate of inflation (yoy) between 2009 and 2016: 1.148%

BEIR Curve

Background: Growing market of inflation-indexed government bonds in the euro area

BEIR curve: difference between the real zero-coupon term structure and a corresponding nominal term structure.

Implied forward rates derived from the term structure can be used to compute **implied forward BEIR curve**

Caveats: both curves include possible inflation risk and liquidity premia

Bundesbank Monthly Report, Euro real terms structures and break-even inflation rates, pp. 36-37, August 2007

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Forward BEIR

"Forward-BEIRs" can be calculated by using BEIRs of different maturities

$$BEIR_t^{t+h,j} = \left[\frac{\left(\frac{1 + \frac{i_t^{t+h+j}}{100}}{1 + \frac{i_t^{t+h+j}}{100}} \right)^{t+h+j}}{\left(\frac{1 + \frac{i_t^{t+h}}{100}}{1 + \frac{i_t^{t+h}}{100}} \right)^{t+h}} - 1 \right] \times 100$$

!The "forward-BEIR" gives expected inflation over j years, starting in period $t+h$

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Example: Forward BEIR

Data as of 27 February 2009:

Ytm on inflation-indexed government bond Germany 1.5% 2016:	1.699%
Ytm on nominal government bond Germany 4.0% 2016:	2.867%
Ytm on inflation-indexed gov. bond France OATi 3.0% 2012:	1.605%
Ytm on nominal government bond France OAT 5.0% 2012:	2.071%
Average expected rate of inflation (yoy) between 2012 and 2016:	1.669%

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BEIRs: Pros & Cons

Advantages of BEIRs

- Data availability up to ultra-high frequencies
- Revealing expectations "backed by money"

Shortcomings of BEIRs

- Maturities of nominal and real bonds do not necessarily match each other exactly
- Shortage of real bonds...
 - ...hampers calculating forward-BEIRs
 - ...makes calculation of BEIR-curves demanding

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BEIRs: Pros & Cons

I...

- I Professional calculation necessitates algorithms for calculating zero-rates instead of ytm
- I BEIRs are subject to various premia
 - I Liquidity premia
 - I Inflation risk premia
- I Real bonds are linked to price indices which are potentially subject to seasonalities
 - I Increasing bias in BEIR as real bonds are approaching maturity
 - I Potential bias due to bond selection

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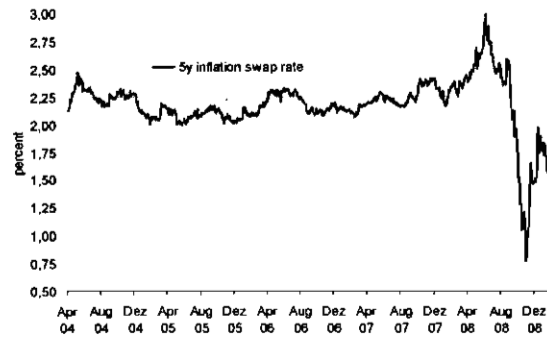
Inflation Swaps

- I Non-standardised, OTC-contract between an "inflation-buyer" and an "inflation-seller"
- I "Inflation-buyer" periodically pays a fixed rate ("inflation swap rate") to the "inflation-seller"
- I "Inflation-seller" periodically pays a variable rate to the "inflation-buyer", which corresponds to the inflation rate
- I The "inflation swap rate" is directly observable, providing a straightforward measure of inflation expectations until the swap matures

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5y Inflation Swap Rates

Evolution of 5y inflation swap rates for the Euro-area



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Inflation Swaps: Pros & Cons

Advantages of inflation swaps

- Full year maturity of swap-contracts makes calculated inflation expectations robust against seasonalities in respective price indices
- Data availability up to ultra-high frequencies

Shortcomings of inflation swaps

Potential biases due to...

- ...costs for structuring the contract
- ...lack of liquidity (liquidity premium)
- ...credit risk premium
- ...inflation risk premium

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Premia in BEIRs

Direction of changes in inflation expectations is important, but inflation expectations also convey a level-dimension.

⇒ Risk premia are not directly observable, however, time-varying risk premia distort interpretation of financial market data-based expectations

⇒ Potential solutions

! Taking differences

- between BEIR or swap rate and survey-based expectations
- between actual BEIR and model-based (theoretic) BEIR

! Identifying factors that determine respective premia

Seasonality in BEIRs

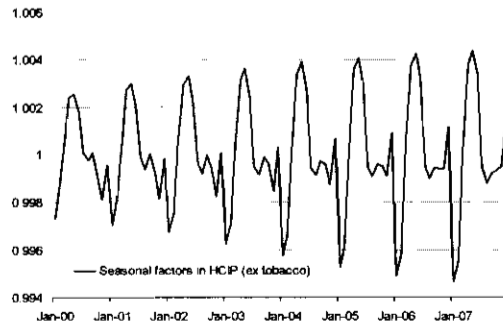
! German and French inflation-linked government bonds refer to HICP (ex tobacco)

! HICP (ex tobacco) exhibits seasonal factors, causing...

- ! ...systematic biases of BEIR
- ! ...dependence of BEIR to bonds used for calculations

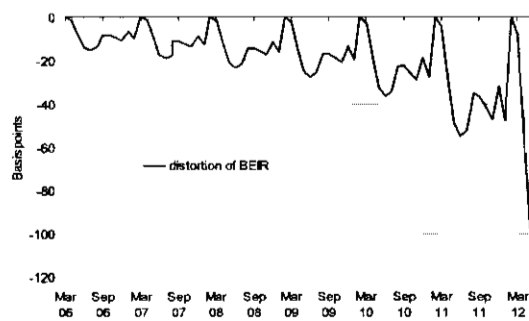
! Time-varying seasonal factors hamper calculation of BEIR

Seasonality Factors



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Distortions of BEIR



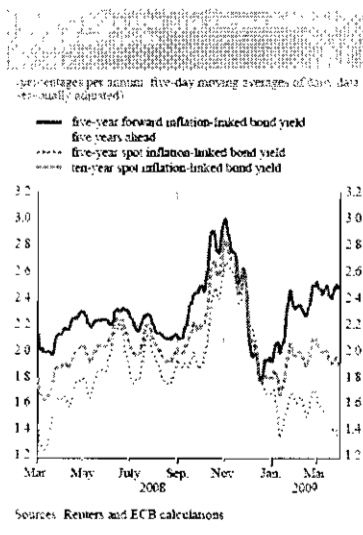
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Correcting for Seasonalities in BEIRs

Correcting for seasonalities has to take current seasonal factor and seasonality at maturity into account!

Hence, specific features (date of maturity, time to maturity) of the bonds used for calculating BEIRs become important

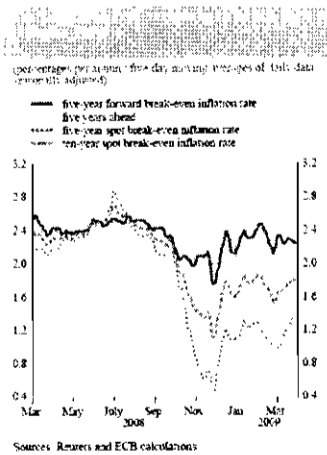
$$P_t^{sa} = P_t^{nsa} \times \frac{SF_{T-t}}{SF_{t-t}}$$



„Yields on long-term inflation-linked government bonds in the euro area decreased visibly in March.

Real five and ten-year spot yields declined by 45 and 30 basis points respectively to levels of 1.3% and 1.8%.

Overall, this seems to reveal primarily a further deterioration in investors' perceptions of the macroeconomic outlook.“



„In combination with the fact that nominal long-term yields did not change much, this led to a sharp increase in five and ten-year spot break-even inflation rates, which rose by about 45 and 20 basis points respectively. On 1 April, they stood at levels of 1.4% and 1.8% respectively. At the same time, five-year forward rates five years ahead, a measure of longer-term inflation expectations and related risk premia, remained broadly stable at 2.3%. The gap between the break-even inflation rates derived from bonds and those derived from swaps fluctuated significantly in the course of March, which suggests that the inflation expectations and related risk premia extracted from these debt instruments should still be interpreted with caution.“

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Why are Interest Rate Expectations Important?



Market's interest rate expectations affect the lending and borrowing rates facing firms and consumers and so play an important role in the transmission mechanism of monetary policy to the real economy

Information about the market's perception of current and future economic developments, which policymakers might also want to incorporate into their own view of the outlook.

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Reference Rates in the Euro Area



IEURIBOR (euro interbank offered rate): Up to 64 major banks daily report until 10:45 am their 1 to 12 month-interest rates to Bridge Telerate, which publishes the average rate at 11:00 am in the information systems.

IEONIA (euro overnight index average): The same banks daily report their overnight rates to the ECB until 18:45 pm, which calculates a volume-weighted average rate and publishes it via Bridge Telerate between 18:45 and 19:00 pm.

IEurepo index for secured cash market

IEONIA swap index for money market derivatives

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EONIA Swaps

In an EONIA swap, two parties agree to exchange the difference between the interest accrued at an agreed fixed interest rate for a fixed period on an agreed notional amount and interest accrued on the same amount by compounding EONIA daily over the term of the swap.

The "fixed leg" of this agreement is referred to as the EONIA swap rate. Hence the EONIA swap rates reflect the expected average EONIA rate over the maturity of the swap contract.

Nature of the swap arrangement limits the credit risks since no principle amounts are exchanged; the swap rates therefore price a more exact indication of the term premium the market adds due to increasing uncertainty for future interest rates.

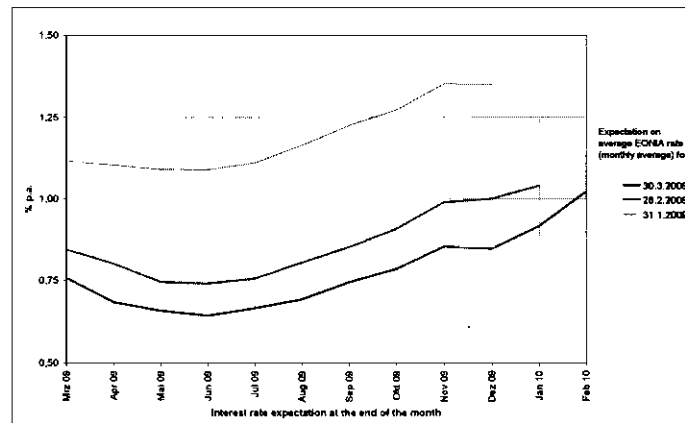
Traded maturities: 1-3 weeks, 1-12 months, 15, 18, 21, and 24 months.

Implied Forward Rates

$$f_t^{i,j} = \left[\frac{1 + r_{i+j,t} \cdot \frac{m_{i+j}}{1200}}{1 + r_{i,t} \cdot \frac{m_i}{1200}} - 1 \right] \cdot \frac{1200}{m_{i+j} - m_i}$$

Where $f_t^{i,j}$ represents the implied forward rate at time t for the i -month interest rate in j months. The rates $r_{i+j,t}$ and $r_{i,t}$ represents the spot rates with maturity i and $i-j$ at time t , respectively; m refers to the maturity of the EONIA swap.

Expectations on EONIA-Rates



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Theories of the Term Structure

Theory	Assumes . . .	Predicts . . .	Evaluation . . .
Segmented markets	Maturities are not substitutable; shorter maturities are preferred to longer maturities	Yields on different maturities are determined in separate markets	Explains shapes of the yield curve but not why short-term and long-term rates move together
Expectations	Maturities are perfect substitutes	Yield on an n -period bond equals the average of yields on one-period bonds over the next n periods of the yield curve	Explains why short-term and long-term rates move together but not the usual upward slope
Preferred habitat	Maturities are substitutable but not perfectly	Yield on an n -period bond equals the average of yields on one-period bonds over the next n periods plus a term premium	Explains both the shapes of the yield curve and why short-term and long-term rates move together

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Expectations Theory - In General

For an investment of \$1

i_t = today's interest rate on a one-period bond

i_{t+1}^e = interest rate on a one-period bond expected for next period

i_{2t} = today's interest rate on the two-period bond

Expectations Theory - In General

Expected return over the two periods from investing \$1 in the
two-period bond and holding it for the two periods

$$(1 + i_{2t})(1 + i_{2t}) - 1$$

$$= 1 + 2i_{2t} + (i_{2t})^2 - 1$$

$$= 2i_{2t} + (i_{2t})^2$$

Since $(i_{2t})^2$ is very small

the expected return for holding the two-period bond for two periods is

$$2i_{2t}$$

Expectations Theory - In General

If two one-period bonds are bought with the \$1 investment

$$(1 + i_t)(1 + i_{t+1}^e) - 1$$

$$1 + i_t + i_{t+1}^e + i_t(i_{t+1}^e) - 1$$

$$i_t + i_{t+1}^e + i_t(i_{t+1}^e)$$

$i_t(i_{t+1}^e)$ is extremely small

Simplifying we get

$$i_t + i_{t+1}^e$$

Expectations Theory - In General

Both bonds will be held only if the expected returns are equal

$$2i_{2t} = i_t + i_{t+1}^e$$

$$i_{2t} = \frac{i_t + i_{t+1}^e}{2}$$

The two-period rate must equal the average of the two one-period rates

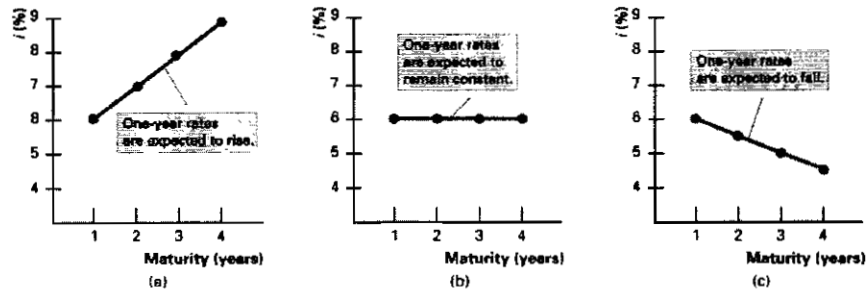
For bonds with longer maturities

$$i_{nt} = \frac{i_t + i_{t+1}^e + i_{t+2}^e + \dots + i_{t+(n-1)}^e}{n}$$

The n -period interest rate equals the average of the one-period interest rates expected to occur over the n -period life of the bond

Using the Yield Curve to Predict Interest Rates:

According to the Expectations Theory:



However: According to the Preferred Habitat Theory, longer term rates include term premia!

Liquidity Premium Theory

$$i_{nt} = \frac{i_t + i_{t+1}^e + i_{t+2}^e + \dots + i_{t+(n-1)}^e}{n} + l_{nt}$$

where l_{nt} is the liquidity premium for the n -period bond at time t

l_{nt} is always positive

Rises with the term to maturity

Liquidity premium (or preferred habitat) theory can explain the facts that:

- Interest rates on bonds of different maturities move together over time
- Yield curves usually slope upward

Liquidity Premium Theory and Expectations Theory

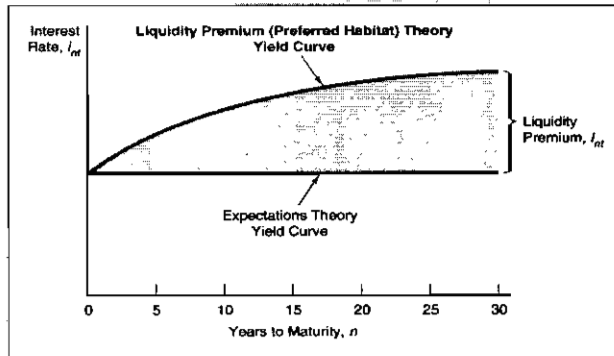
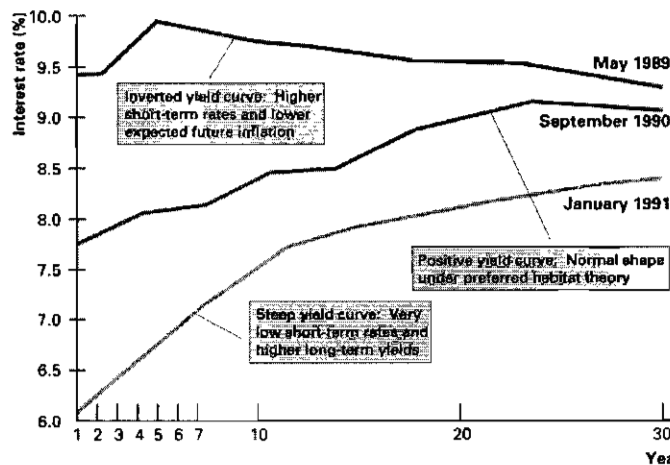


FIGURE 5 The Relationship Between the Liquidity Premium (Preferred Habitat) and Expectations Theory

Interpreting the Yield Curve



German Term Structure

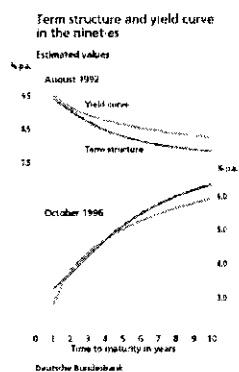
!Term structure of interest rates in the bond market shows the relation between the interest rates and maturities of zero coupon bonds without a default risk.

!Reinvestment of payment flows at the prevailing interest rate, not at a constant yield-to-maturity!

!For the purpose of the estimation, an assumption is made about the functional relation between interest rates and residual maturities.

!Estimation approach: similar to Nelson/ Siegel/ Svensson

Term Structure vs Yield Curve

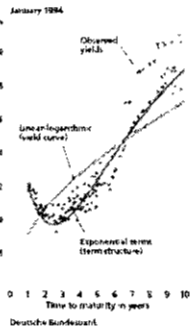


In contrast to the implied assumption of when calculating yields to maturity that all payment flows of a coupon bond carry the same rate of return (namely the yield to maturity),

the estimation of the term structure of interest rates assumes a different rate of return for each payment flow of a coupon bond at the interest rate corresponding to the current market conditions on the respective payment date.

Estimation Approach

Comparison of different estimation approaches

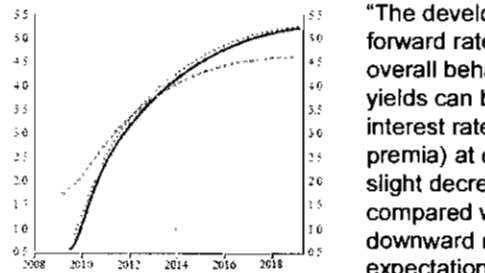


$$z(T, \beta) = \beta_0 + \beta_1 \left(\frac{1 - \exp(-T/\tau_1)}{T/\tau_1} \right) + \beta_2 \left(\frac{1 - \exp(-T/\tau_1)}{T/\tau_1} \exp(-\frac{T}{\tau_1}) \right) + \beta_3 \left(\frac{1 - \exp(-T/\tau_2)}{T/\tau_2} \exp(-\frac{T}{\tau_2}) \right)$$

Where $z(T, \beta)$ denotes the interest rate for the maturity T as a function of the parameter vector β .

Source: ECB, EuroHCTS (underlying data) and Fitch Ratings (ratings)

— 1 April 2009
- - - 27 February 2009
- - - 31 December 2008



Sources: ECB, EuroHCTS (underlying data) and Fitch Ratings (ratings).
Notes: The implied forward yield curve, which is derived from the term structure of interest rates observed in the market, reflects market expectations of future levels for short-term interest rates. The method used to calculate these implied forward yield curves is outlined in the "Euro area yield curve" section of the ECB's website. The data used in the estimate are euro area AAA-rated government bond yields.

"The development of the term structure of forward rates in the euro area shows how the overall behaviour of euro area long-term bond yields can be decomposed into changes in interest rate expectations (and related risk premia) at different horizons (see Chart 14). The slight decrease of long-term bond yields as compared with end-February is the result of a downward revision of short-term interest rate expectations over horizons of up to six months (down by 15 to 20 basis points), combined with less significant decreases of longer-term interest rate expectations."

Interpreting Forward Rates

“The shape of the forward rate curve directly shows the expected future course of (spot) interest rates. This is interesting from a monetary policy point of view, since it allows a better separation of expectations over the short, medium and long term than the term structure does.”...

„However, the objections raised against an overly strict interpretation of the term structure in the sense of the expectations theory apply even more forcefully to the forward rate curve; in the first place, the existence of risk and forward premiums which vary over time should be mentioned, as they can heavily affect the implied forward rates. Since corresponding empirical studies have generally been unable to reject the existence of such time-variable premiums, the forward rate curve should be interpreted with particular caution.“

Estimating the term structure of interest rates, Bundesbank Monthly Report,
Oct. 1997, p. 66

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Time-Varying Risk Premia

An estimation approach:

Regress the *ex-post* forecast errors onto macroeconomic and financial market information (e.g. cyclical macro and financial indicators because investor' risk aversion may vary with the business cycle) available *ex ante*.

See whether some part of the return is predictable.

Adjust market forward rates *ex ante* to form risk-adjusted forward rates.

Compare regression-based estimates with estimates based on survey expectations of interest rates (e.g. derived from Consensus Economics data) or from forward premia derived from an affine term structure model.

Measuring monetary policy expectations from financial market instruments,
ECB Working Paper No. 978, Dec 08, chapter 4

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Instruments used to analyse market expectations: risk-neutral density functions,
Deutsche Bundesbank Monthly Report, Oct. 2001, p. 31-47

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Options

- I Contracts that give the purchaser the right to buy or sell the underlying financial instruments at a specified price (**exercise price/ strike price**), within a specific period of time (**term to expiration**)
- I The seller (**writer**) of the option is obligated to buy or sell the financial instrument to the purchaser if the owner of the option exercises the right to sell or buy.
- I The owner pays an amount for the right to buy or sell (**premium**)
- I Two types of contracts:
 - I **American option**: can be exercised at any time up to expiration date
 - I **European option**: can only be exercised on the expiration date

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Factors affecting the Option Premium

1. Higher strike price \Rightarrow lower premium on call options and higher premium on put options
2. Greater term to expiration \Rightarrow higher premiums for both call and put options
3. Greater price volatility of underlying instrument \Rightarrow higher premiums for both call and put options



Remember: „Heads, I win; tails, I don't lose too badly.“

For more details see: Black-Scholes model

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Black-Scholes-Model: Assumptions

The price of the underlying instrument S_t follows a geometric Brownian motion (W_t is a Wiener process with constant drift μ and volatility σ).

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

It is possible to short sell the underlying stock.

There are no arbitrage opportunities.

Trading in the stock is continuous.

There are no transaction costs or taxes.

All securities are perfectly divisible.

It is possible to borrow and lend cash at a constant risk free interest rate.

The stock does not pay a dividend (extensions possible).

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Black-Scholes-Model: Equation

In the model, there is a unique price for any derivative of the stock. In particular, a European call option, which gives the right to buy one share at price K after T years, has the price C given by:

$$C = S_t \Phi(d_1) - Ke^{-rT} \Phi(d_2)$$

Where

$$d_1 = \frac{\ln(S_t / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}} \quad d_2 = \frac{\ln(S_t / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

S is the current price of the stock, r is the continuously compounded risk-free interest rate, and σ is the constant stock's volatility and Φ is the standard normal cumulative distribution function.

Black-Scholes-Model: The Greeks

Delta: $\delta C / \delta S$

Gamma: $\delta^2 C / \delta S^2$

Vega: $\delta C / \delta \sigma$

Theta: $-\delta C / \delta t$

Rho: $\delta C / \delta r$

Implied Volatility

In standard option price models, the option premium for European options can be calculated as a function of contractually specified variables (duration and strike price), data obtainable directly from the market (interest rates and the spot price of the underlying asset) and the expected variance of the underlying asset, which is not directly observable.

Under a given set of parameters, the price of an option in currency units corresponds to exactly one volatility value, which means that one variable may be unambiguously derived from another.

Implied Volatility cont'd

Implied volatility is that particular volatility which – using the standard calculation method as a basis– is compatible with the observed market price of the option. It measures the expected price dispersion of the underlying instrument during the option's duration.

The distinct mutual convertibility of option premium and implied volatility using the standard option pricing model led, in the special case of OTC foreign exchange options, to the convention of negotiating implied volatilities directly instead of via option premiums. In OTC trading, therefore, quotations are given directly in units of implied volatility, or "vols", which then imply a specific option premium.

Interpreting Implied Volatilities

Measure of symmetrical percentage fluctuation margins of the future financial market price, e.g. exchange rate

In addition to the information specific to the forward rate, the implied volatility is a measure of the average future dispersion.

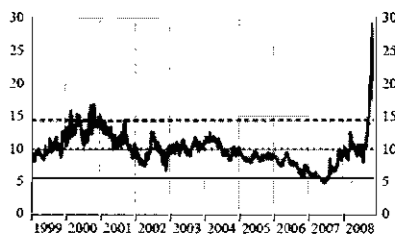
However

No account is taken on a possible asymmetry, e.g. appreciation or depreciation risk in case of the exchange rate as well as the probability assessment of extreme exchange rate fluctuations.

Chart 1.21 Implied volatility of the USD/EUR exchange rate and its historical range

(Jan 1999 - Nov 2008 percentage)

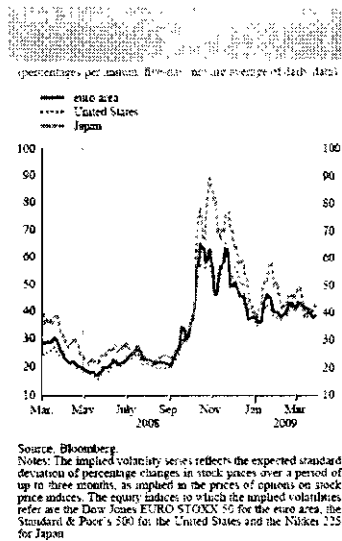
— implied volatility of the USD-EUR exchange rate
- - - - average of the USD-EUR implied volatility since January 1999
- - - - upper 95% confidence interval band
- - - - lower 95% confidence interval band



Source: Bloomberg.

Note: The horizontal lines display the mean value along with the 95% confidence band for the EUR/USD implied volatility. The average and the standard deviations are based on the values of the implied volatility between 1 January 1999 and 27 November 2008.

"In November, exchange rate uncertainty, measured by realised volatilities, reached values close to the historical peaks of 1985. Implied volatilities also rose and posted record values for the euro era. Looking ahead, implied volatilities for the one and three month horizons have risen significantly, and by more than longer-term volatilities. It appears, therefore, that markets expect that short-term swings in foreign exchange rate levels may continue (see Chart 1.21). As longer-term expectations for volatility are close to historical averages, however, swings are expected to dampen significantly over a one-year horizon."



“Near-term stock market uncertainty, as measured by option-implied volatility, decreased moderately on a global basis. At the end of the period under review, uncertainty was still high by historical standards, but well below the peaks reached in the autumn of 2008 (see Chart 19). Moreover, the expected volatility for the two-year horizon of stocks included in the Dow Jones EURO STOXX 50 index also decreased only little, suggesting that market participants’ uncertainty about stock market developments remained rather high, also over the medium term.”

Risk Reversal

A combination of the parallel purchase of an out-of-the-money call option and the sale of an out-of-the-money put option.

Both options expire on the same date and have strike prices which are equidistant in percentage terms from the forward rate at the time the agreement is concluded.

The market price of the risk reversal can be used to determine whether market players’ assessments of the appreciation and depreciation potential of the exchange rate are symmetrical.

Interpreting “Risk Reversal”

If market participants consider it equally likely that the exchange rate could move by a specific percentage in either direction, the risks incurred at both positions cancel each other out, leaving the risk reversal price at zero.

By contrast, if the players on the foreign exchange market estimate the potential loss incurred by a put option as a result of the exchange rate moving below the strike price as being higher than the potential profit incurred by a call option as a result of the exchange rate moving above the strike price, the risk reversal has a negative value.

Strangle

A combination of an out-of-the-money call option and an out-of-the-money put option, with both being held by the bearer.

As in the risk reversal, both options expire on the same date and have strike prices which are equidistant in percentage terms from the forward rate at the time the agreement is concluded.

The quotation of the strangle may serve as an indicator of extreme exchange rate fluctuations compared with the log-normal distribution.

Implied Volatilities & Strangles

Implied volatility in an at-the-money call or put option is a standardised parameter which, in the context of log-normal distributed price changes, adequately describes the average deviation of the financial market price from its mean movement.

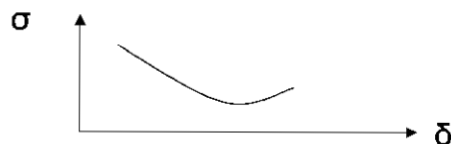
However, if one tries to describe the total relative dispersion of the price changes in financial market data by this variable only, the implied volatility of an at-the-money call option, for instance, is inadequate because in reality extreme volatilities of financial market prices can be observed more frequently than might be suggested by the log-normal distribution underlying the implied volatility measure.

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Volatility Smile

Implied volatility increases the more remote the strike price, as established in the option, is from the forward rate. In other words, it is observed that the implied volatility of options that are either in-the-money or out-of-the-money is generally higher than the implied volatility of at-the-money options.

The inference is that market participants expect a fluctuation margin of the financial market price above that which is compatible with the implied volatility of an at-the-money call option.



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Interpreting “Strangle”



Because the options considered in the context of the strangle are out-of-the money at the time of purchase (i. e. in the case of call options, the forward rate is below the strike price), the presupposition is that exchange rates will have changed markedly by the exercise date.

The willingness to pay for the strangle therefore increases with the risk perceived by market players of an exceptional exchange rate development up to maturity, with the result that the uncertainty assessment based on the observed implied volatility may be appropriately supplemented by the information which can be derived from the price quotations for this instrument.

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Implied Risk Neutral Density



Computational methods allow the calculation of an implied risk-neutral density (RND) of the expected changes in the value of the underlying asset, which reflects

- Implied volatility in the width
- The price of risk reversal in the skewness
- The quotation of the strangle in the fatness of the tails

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Computation from Option Data



Parametric model: Mixture of log-normal

Theoretical price of a call option:

$$C(S, K, T) = e^{-rT} \int_K^{\infty} (S - K) f(S) dS$$

- S Price of the underlying asset
- K Exercise price
- T Time to maturity
- f(S) Risk neutral density function
- r Risk-free interest rate

Specification as a mixture of log-normal densities:

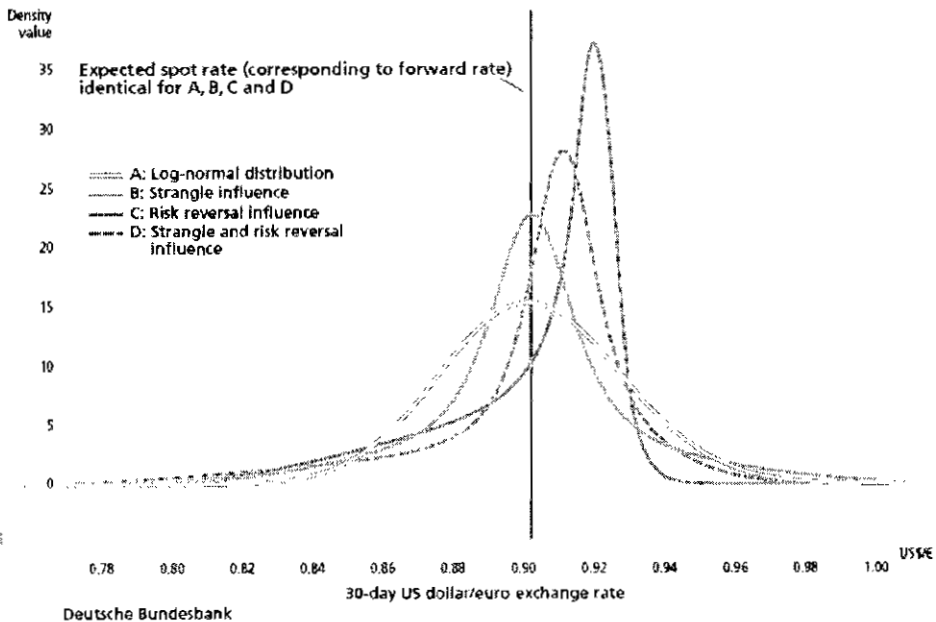
$$f(S) = w \text{LogN}(a_1, b_1, S) + (1-w) \text{LogN}(a_2, b_2, S)$$

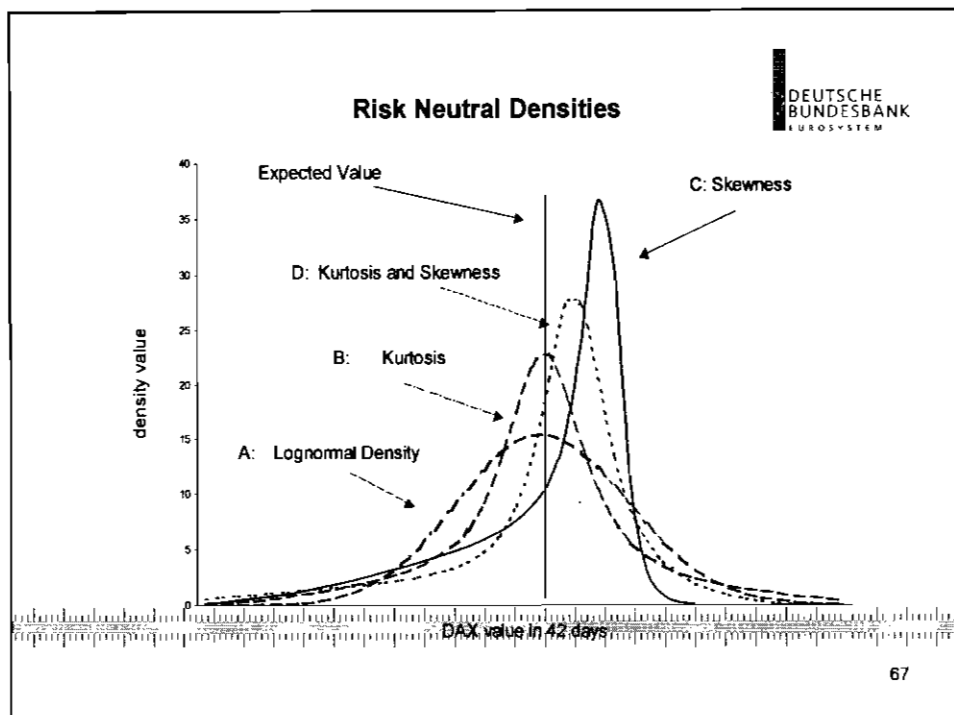
- w Weights of log-normal ($0 < w < 1$)
- a_i, b_i Location and distribution parameter of log-normal density

Jondeau, E. et al. (2007) Financial Modeling Under Non-Gaussian Distribution, Springer, chapter 11.3

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Risk-neutral density functions





Limitations

Relative unavailability of options with various strike prices and the consequent need to resort to an interpolation procedure.

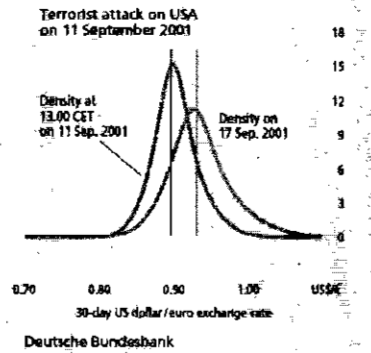
Prices for “wing options” are not available or contain high liquidity premiums.

Problems caused by “technical” market tensions.

Risk premiums

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Exchange Rate Expectations



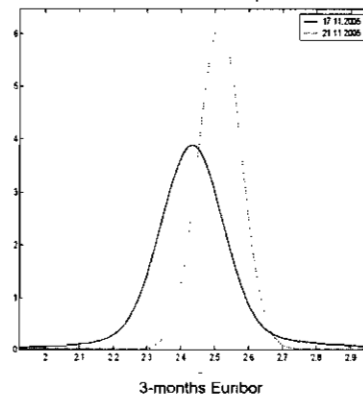
Influence on market players' exchange rate expectations of the impact of the terrorist attacks in New York and Washington on 11 September 2001.

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Interest Rate Expectations

Density value for 19.12.05 derived from data on options on Euribor-Futures

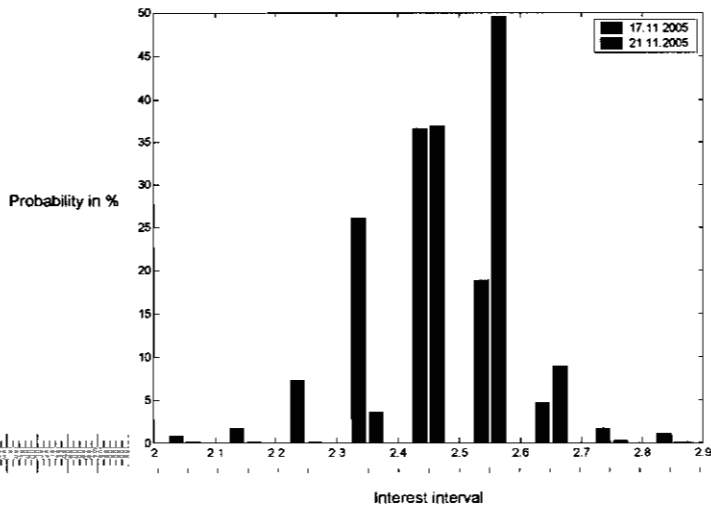
- I Distribution of expectations are calculated from estimated RNDs (from options on Euribor-Futures) for 3-months Euribor on 19.12.2005 (spot)
- I 17.11.2005: one day before ECB president announces an interest rate increase in a speech: 3M-Euribor (spot): 2.35%



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Interest Rate Expectations cont'd

Probability that 3-month-Euribor on 19.12.05 lies in the interval...



Stock Price Expectations

