Capital Estimation and Operational Risk Modelling Issues

Practical Techniques for the Management and Measurement of OR
Financial Stability Institute, Basel, 26 Oct 2006

Michael Kalkbrenner
Risk and Capital Management
Risk Analytics & Instruments

A Passion to Perform. Deutsche Bank

Agenda

- AMA at DB
- Data
- Modelling frequency and severity distributions
- Incorporation of insurance
- Modelling dependence
- Calculation and allocation of risk capital
- Model validation

Deutsche Bank

AMA model development at Deutsche Bank

Timeline

1999 Systematic collection of loss data

2000 Economic capital with LDA

- Top-down model: loss distribution at Group level, capital allocation with risk indicators
- Internal and external loss data
- Qualitative adjustment with Incentive Scheme

2001 AMA project

2002 Development of AMA model

2003 Implementation of prototype

2004 EC test calculations with AMA model

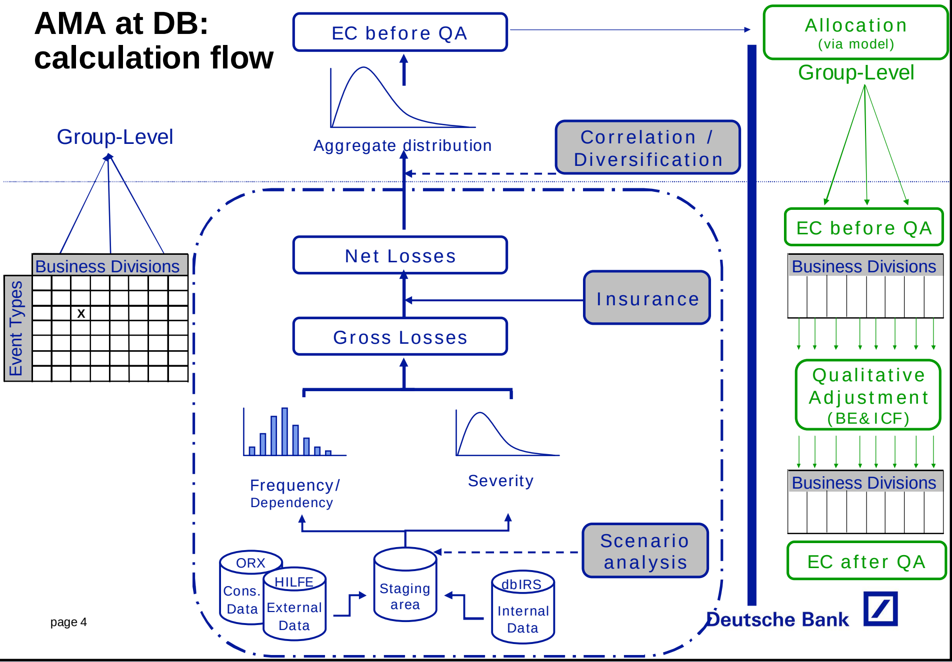
2005 Official EC calculation with AMA model (starting Q2 05)

2006 Implementation of production engine

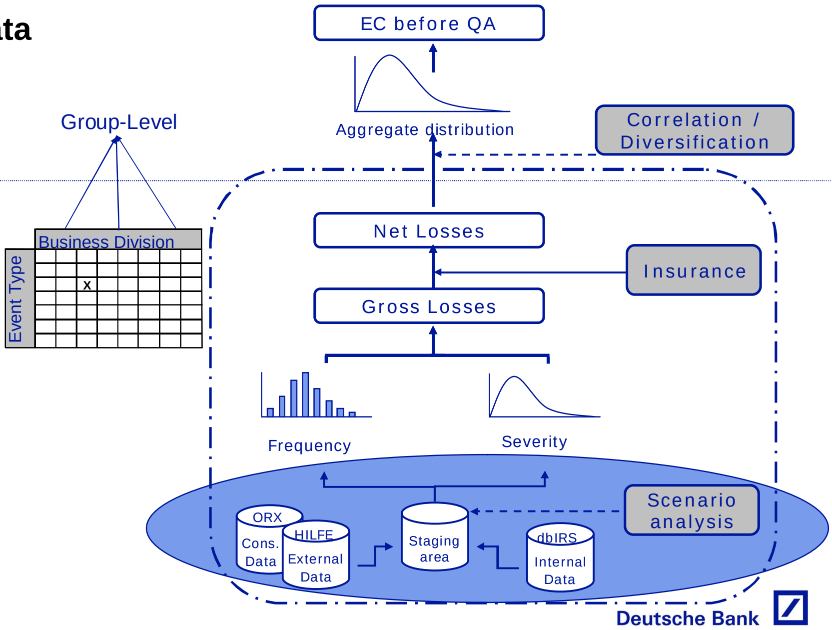
AMA application submitted in September

Regulatory approval: in-house visit starts Oct. 30

AMA at DB: calculation flow



Data



Data for modelling loss distributions

Data sources

- Internal loss data
- Consortium data
- Commercial loss database
- Scenarios

Internal loss data is the most important data source

- Each firm's operational losses are a reflection of its underlying operational risk exposure
- Internal losses are used for
  - modelling frequencies (exclusively)
  - modelling severities
  - estimating correlations

Motivation for using external data and scenarios

- Additional information on severity profile, in particular on risk of unexpected losses (tails of severity distributions)

Data classification

DB's Business Line / Event Type matrix

Basel Level 1	Internal Event Types	Business Lines						
		BL1	BL2	BL3	BL4	BL5	BL6	Group
Internal Fraud	Fraud							
External Fraud								
Damage to physical assets	Infrastructure							
Business disruption …								
Clients, Products, Business Practices	Clients, Products, Business Practices							
Execution, delivery, process management	Execution, delivery, process management							
Employment practices, workplace safety	Employment practices, workplace safety							

Deutsche Bank

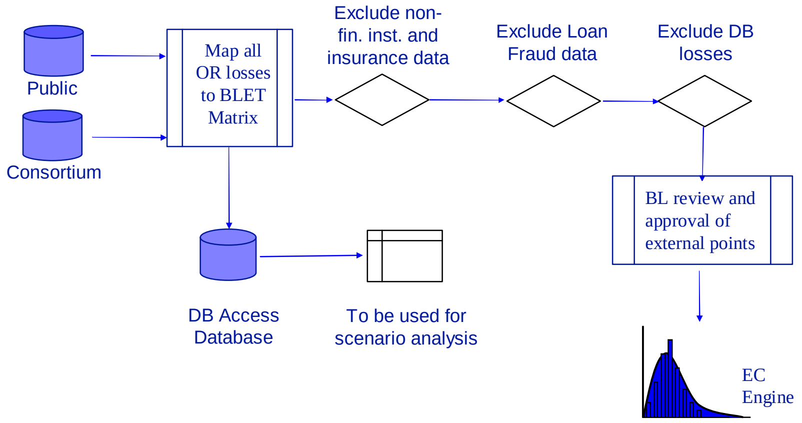
Data processing

Relevant Loss Data Process
- Identification of external losses relevant for Deutsche Bank

Weighting of loss data
- Split losses
- Old losses
- Scaling of external data and scenarios

Deutsche Bank

Creating a relevant loss data set



Scenarios are added as individual data points to relevant external losses

Treatment of split losses

Split losses

- consist of several components that are assigned to different business lines but have the same underlying cause
- are modelled as
  - an aggregate loss, i.e. loss amount is the sum of the components
  - in each BL/ET cell affected
  - but with reduced weight

Example

- Consider a penalty of 100m that has been split between
  - business line A (70m) and
  - business line B (30m)
- The total amount of 100m is assigned to both business lines with weights
  - 70% for business line A and
  - 30% for business line B

Reduced weights for old losses
Motivation

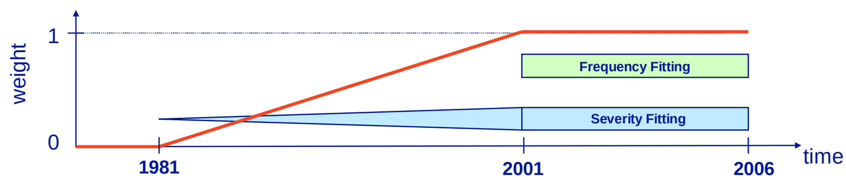
Issue
- Historic losses are less relevant for estimation of current risk profile
- Impact of a given loss should therefore phase-out over an appropriate time period
- However, extreme losses remain significant for comprehensive risk assessment

Strategy for use of old data
- Frequency: use only recent losses (data sufficient)
- Severity: reduce impact of older losses by reducing their weights (but keeping the original loss amounts) in severity fitting

Deutsche Bank

Reduced weights for old losses
Algorithm

- **Employ relevant time intervals**
  - Time periods: core period of 5 years
    time decay period of 20 years
  - Frequency fitting: core period losses only
  - Severity fitting: full weight for core period losses
    weight for earlier losses is linearly reduced to zero over 20 years



- **Rationale**
  - Core period of 5 years is in line with minimum regulatory data requirements
  - Time decay period of 20 years stabilizes calculation and takes into account infrequent occurrence of extreme losses

Deutsche Bank

Biased external loss data

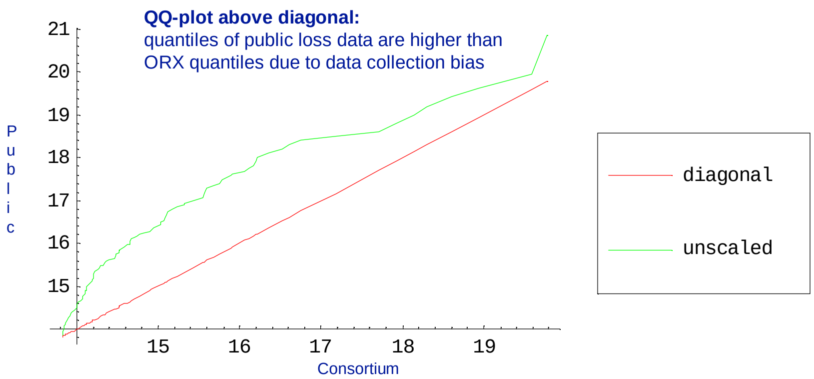
Scale Bias
- Operational risk is dependent on the size of the bank, i.e. the scale of operations
- The actual relationship between the size of the institution and the frequency and severity may be stronger or weaker depending on the particular OR category

Truncation Bias and Data Capture Bias
- Collection thresholds are not uniform for different data sets
- Data is often captured with a systematic bias. This problem is particularly pronounced with publicly available data: there exists a positive relationship between the loss amount and the probability that the loss is reported
- The disproportionate number of large losses could lead to an estimate that overstates a bank's exposure to operational risk

Scaling in AMA at DB
- No correction of Scale Bias since it is considered less relevant for severity modeling
- Correction of Truncation Bias and Data Capture Bias

Deutsche Bank

QQ-Plot: consortium versus public loss data



QQ-Plot
x-axis: log quantiles of consortium losses above 1m
y-axis: log quantiles of public loss data

Deutsche Bank

Scaling of public loss data

Assumption

- Consortium data and (unbiased) public loss data have the same risk profile, i.e. both reflect the generic OR profile of the finance industry

Scaling methodology

- Basic idea: adjust the probabilities (and not the size) of the public loss events in order to reflect the unbiased loss profile, i.e. increase the probability of small losses and decrease the probability of large losses
- Mathematical formalization is based on stochastic thresholds (Baud et al., 2002, and de Fontnouvelle et al., 2003): a loss is only entered into the public loss data base if its size is higher than the stochastic collection threshold

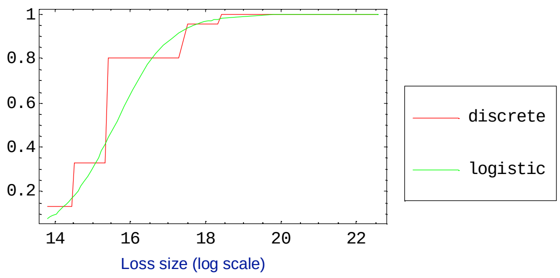
Baud, N., A. Frachot, and T. Roncalli (2002). Internal data, external data and consortium data for operational risk measurement: How to pool data properly? Group de Recherche Operationnelle, Credit Agricole, France.

de Fontnouvelle, P., V. DeJesus-Rueff, J. Jordan, and E. Rosengren (2003). Using Loss Data to Quantify Operational Risk. Federal Reserve Bank of Boston, Boston, MA.

Calibration of stochastic threshold

Methodology and calibration results

- At specified severity levels: match distribution functions (CDFs)
  - of public loss data and
  - of conditional consortium data X, i.e. conditional on X >H with stochastic threshold H
- The plot shows CDFs of a stochastic threshold with a discrete or logistic distribution applied to log losses:



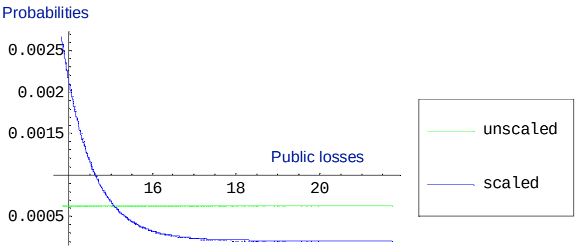
Risk and Capital Management

Probabilities and QQ-plot after scaling

Plot

x-axis: log size of losses in public loss data base

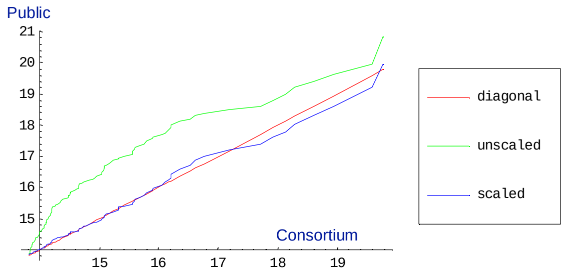
y-axis: unscaled and scaled probability of each loss



QQ-Plot

x-axis: log quantiles of consortium losses above 1m

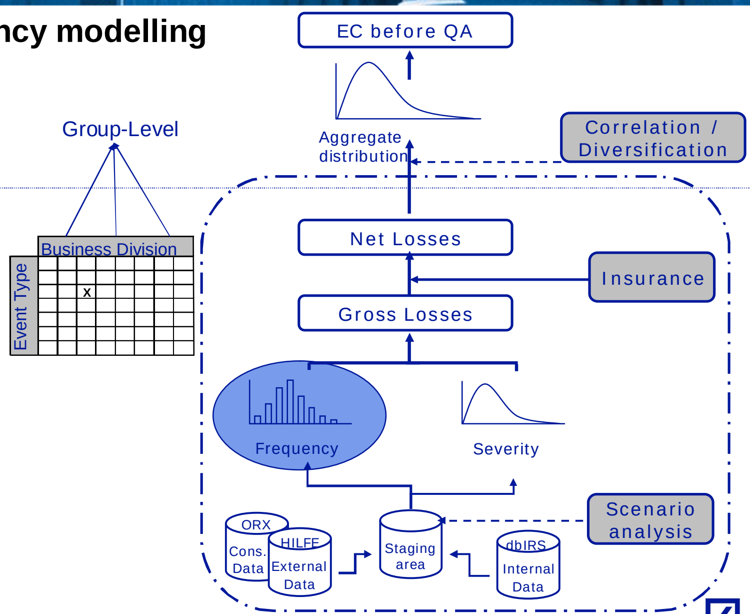
y-axis: log quantiles of unscaled and scaled public loss data



Deutsche Bank

Risk and Capital Management

Frequency modelling



Deutsche Bank

Frequencies in AMA at DB

Data

Only internal loss data is used for calibrating frequency distributions:

- Internal loss data reflects DB's loss profile most accurately
- Difficult to ensure completeness of external data (essential for application in frequency calibration)
- Lower data requirements in frequency modeling (compared to severity modeling)

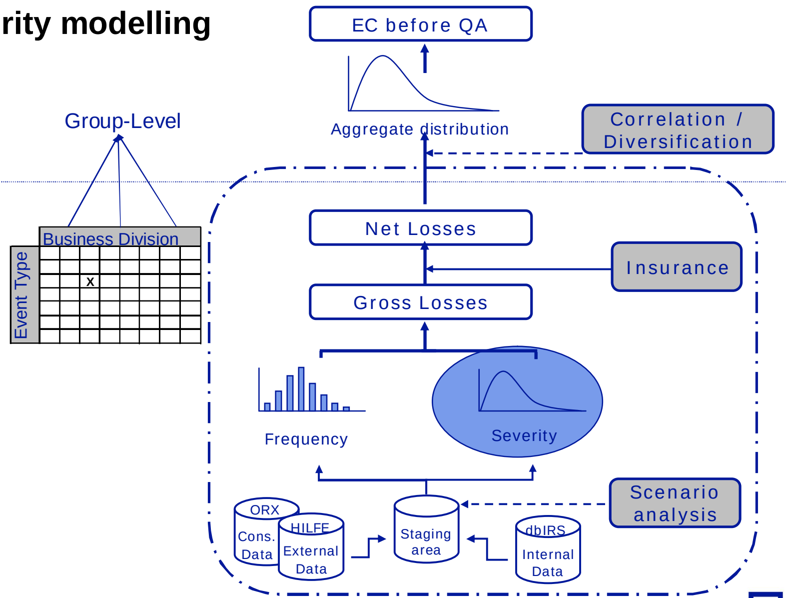
Implemented distributions

- Poisson (no dependence between occurrence of events in a cell)
- Negative Binomial (positive dependence)
- Selection algorithm based on statistical tests

Frequency distributions in official EC calculations

- Poisson in all cells
- Reason: negligible difference to combination of Poisson and Negative Binomial cells

Severity modelling



Modelling decisions

Range of distribution

- One distribution for the entire severity range
  or different distributions for small, medium and high losses?

Choice of distribution family

- Two-parametric distributions like lognormal, GPD
  or more flexible distribution families, i.e. three- or four-parametric,
  or even empirical distributions?
- One distribution family for all cells
  or selection of "best" distribution based on quality of fit?

Mixing internal and external data

- How much weight is given to internal and external data?
- How to combine internal and external data?

Severities in AMA at DB

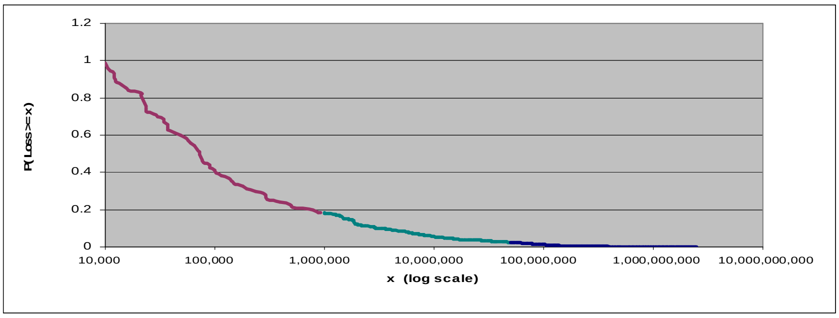
Range of distribution and choice of distribution family

- In many cells, data characteristics are different for small and big losses
- Different distributions for body and tail
  - Body: non-parametric (empirical) distribution
  - Tail: modified technique from Extreme Value Theory for tail modelling
- Empirical and parametric distributions are combined via a weighted sum applied to the cumulative distribution functions

Mixing internal and external data

- Internal data for calibrating body of distribution
- Internal and external data for calibrating tail

Core idea: piecewise defined severity distributions



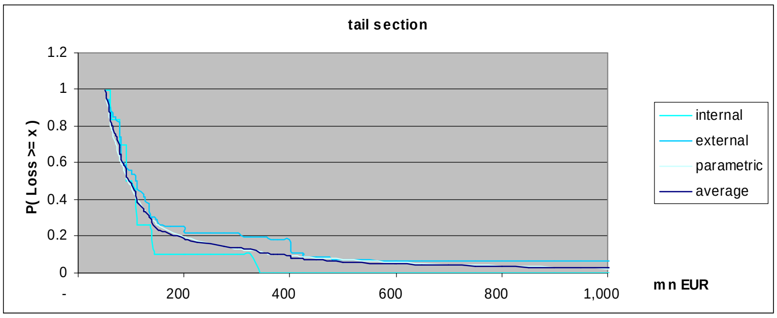
First section: given by empiric distribution of cell specific internal data

Mid section: given by weighted average of
- empiric distribution of cell specific internal data
- empiric distribution of cell specific external and scenario data

Tail section: given by weighted average of
- empiric distribution of cell specific internal data
- empiric distribution of cell specific external and scenario data
- parametric distribution calibrated on all data >= 50mn

Tail section: large losses

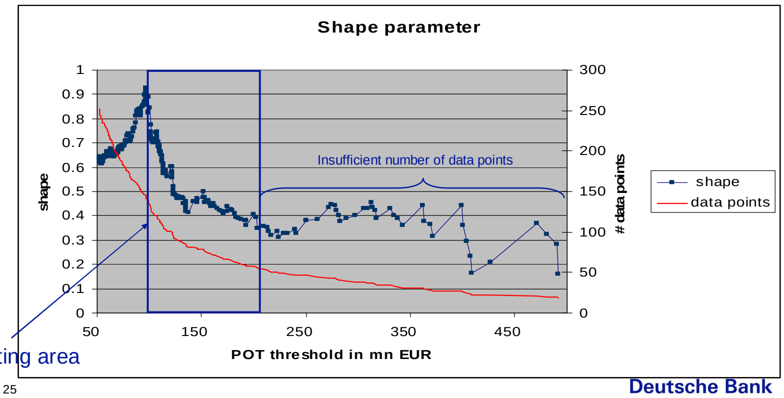
Internal and external losses supplemented by parametric distribution



Range: 50mn EUR – infinity

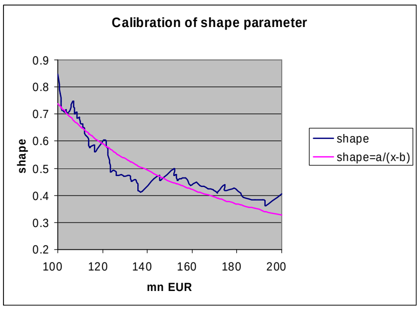
Calibration of the parametric tail distribution I/II

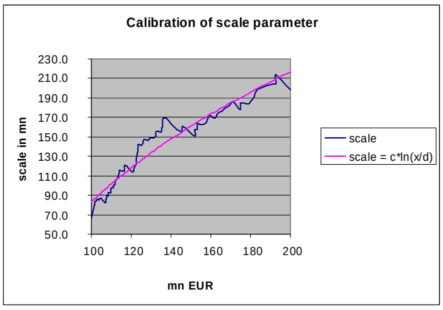
- Generalized Pareto distribution is calibrated at increasing thresholds via Peaks-over-Threshold method
- Shape parameter continues to fall whereas Extreme Value Theory demands shape to become constant
- The higher the shape the fatter the tail of the distribution



Fitting area

Calibration of parametric tail distribution II/II





Shape and scale parameter of GPD are functions of threshold *x*:

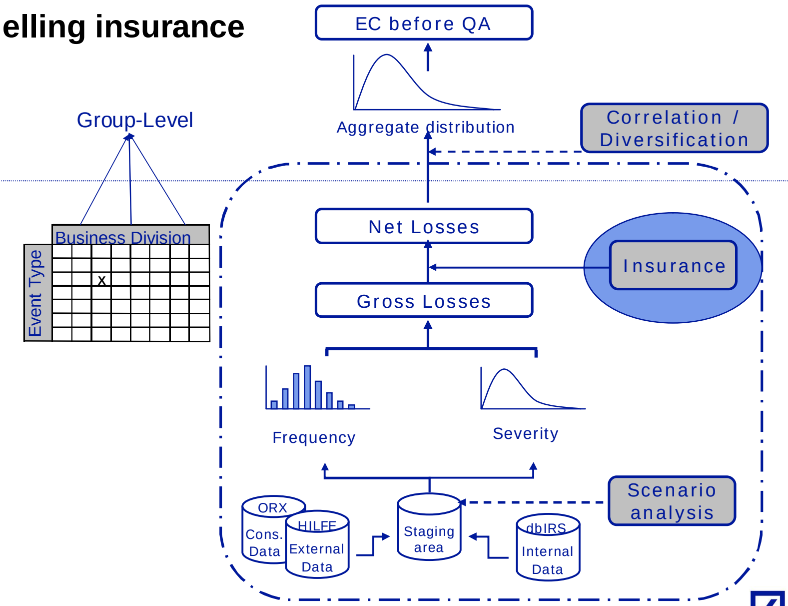
shape = a / (x – b)

*scale = c * LN(x / d)*

Parameters are calibrated such that distance between estimated shapes (scales) and shapes (scales) given by above function is minimized

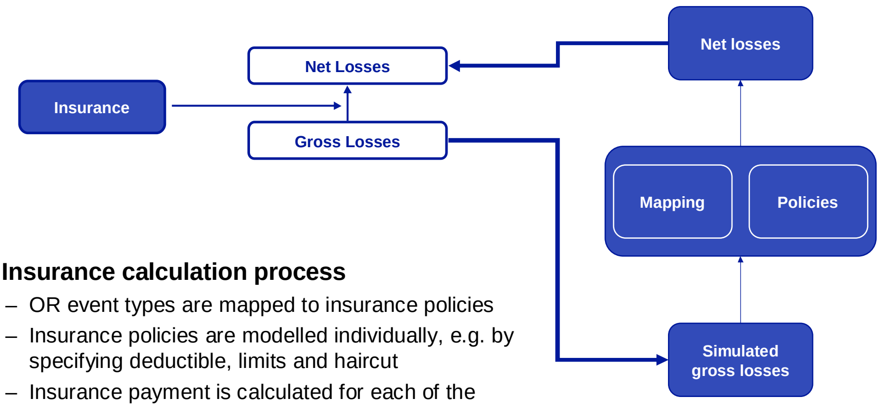
Generalized GPD used as parametric distribution for tail section: *F(x,a,b,c,d)* for x > 50mn

Modelling insurance



Deutsche Bank

Insurance in AMA at DB

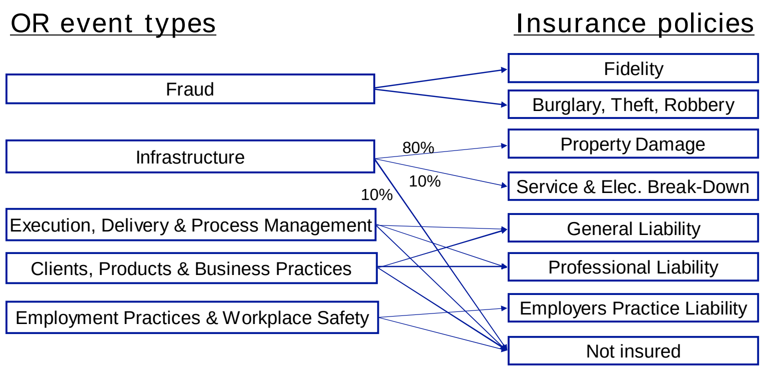


Insurance calculation process

- OR event types are mapped to insurance policies
- Insurance policies are modelled individually, e.g. by specifying deductible, limits and haircut
- Insurance payment is calculated for each of the simulated gross losses separately

Deutsche Bank

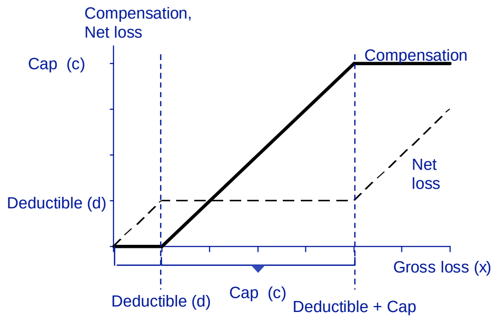
Insurance mapping



Deutsche Bank

Modelling insurance contracts

- Deductible: amount the bank has to cover by itself
- Cap: maximum amount compensated by the insurer



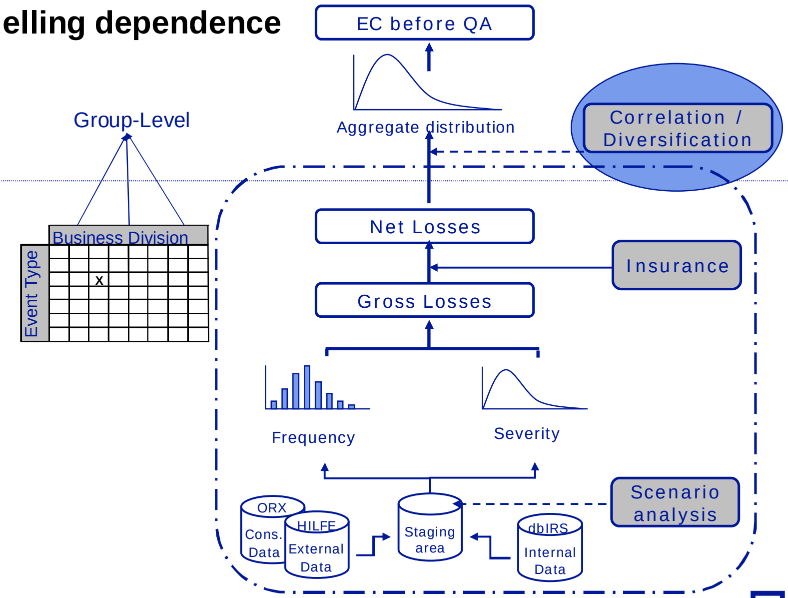
$$\min(c, \max(x - d, 0))$$

Additional features

- Aggregate caps
- Haircuts (regulatory requirements)

Deutsche Bank

Modelling dependence



Analyzing dependence

Dependence in a bottom-up LDA

- Within cells
  - Dependence between the occurrence of loss events
  - Dependence between the frequency distribution and the severity distribution
  - Dependence between the severity samples
- Between cells
  - Dependence between the frequency distributions
  - Dependence between the severity distributions

Statistical analyses performed at Deutsche Bank

- Based on internal loss data
- Identification of dependence between
  - occurrence of loss events within a cell => Frequency distribution not Poisson
  - frequency distributions in different cells => Copula applied to frequencies

Dependence in AMA at DB

Frequencies

- Gaussian copula applied to frequency distributions

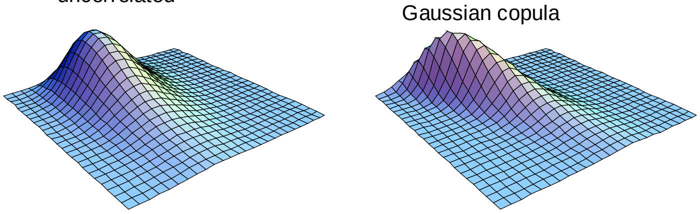
Severities

- Sum of split losses
- Severities of different loss events are independent

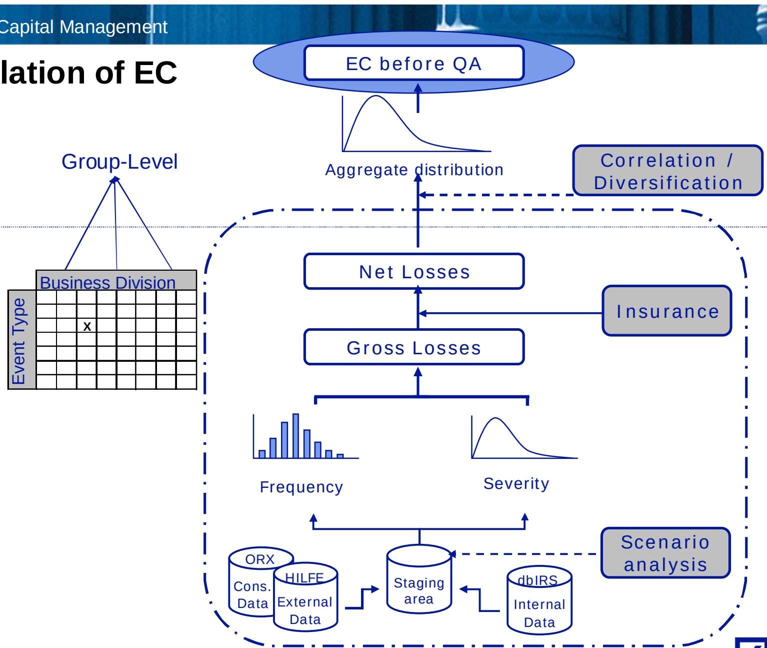
Example: Gaussian copula applied to a Poisson and a Negative Binomial distribution

uncorrelated

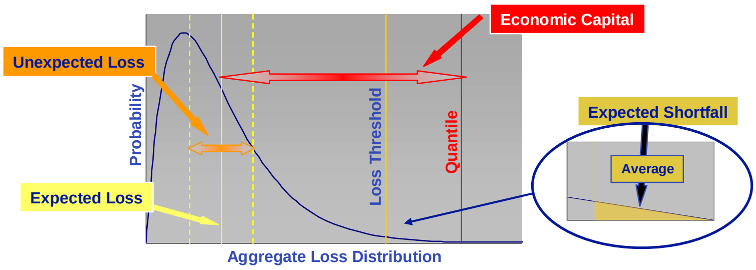
Correlation factor $\sqrt{0.5}$ in Gaussian copula



Calculation of EC



Calculation and allocation of EC



Aggregate loss distribution: Monte Carlo simulation

Economic Capital: 99.98% Quantile minus Expected Loss

Capital allocation

- Cell level: Expected Shortfall allocation
- Divisional level: Aggregation of EC in divisional cells plus proportional contributions of Group cells

Validation

- Basic properties of LDA model
  - Variance analysis
  - Loss distributions for heavy-tailed severities
- Sensitivity analysis of basic components of LDA models
  - Frequencies
  - Severities
  - Dependence
  - Insurance
- Impact analysis of stress scenarios
- Backtesting and benchmarking
  - Benchmarking the tail of the aggregate loss distribution against individual data points

Variance analysis
Cell level

- Variance analysis
  - does not provide information on quantiles of loss distribution
  - but: quantifies impact of frequencies and severities on volatility of aggregate losses
  - is independent of specific distribution assumptions
- Variance of aggregate losses (*F* and *S*: frequency and severity distribution):

$$E(F)\cdot Var(S)+Var(F)\cdot E(S)^2$$

Conclusion

- Importance of frequency distribution depends on relationship of $Var(F)/E(F)$ (frequency vol) and $Var(S)/E(S)^2$ (severity vol)
- In high impact cells, the volatility of severities dominates and the actual form of the frequency distribution is of minor importance:

$$E(F)\cdot Var(S)+Var(F)\cdot E(S)^2$$

Variance analysis
Group level

Frequency correlations

- Variance of loss distribution at Group level

$$\sum_{j=1}^{m} E(F_j)\cdot Var(S_j)+Var(F_j)\cdot E(S_j)^2+\sum_{j,k=1,j\neq k}^{m} Cov(F_j,F_k)\cdot E(S_j)\cdot E(S_k)$$

- Variance in the homogeneous model (*c:* homogeneous correlation coefficient)

$$m\cdot(E(F)\cdot Var(S)+Var(F)\cdot E(S)^2\cdot(c\cdot(m-1)+1))$$

Impact of frequency correlations depends on

- number of (relevant) cells *m* and
- relationship of $Var(F)/E(F)$ (frequency vol) and $Var(S)/E(S)^2$ (severity vol)

In general, the impact of frequency correlations is rather limited and less significant than the impact of correlations of severities or loss distributions

Loss distributions for heavy-tailed severities

Subexponential distributions

- Heavy-tailed: tail decays to 0 slower than any exponential Exp[a*x], a<0
- Tail of the sum of subexponential variables has the same order of magnitude as tail of the maximum:

$$\lim_{x\to\infty} \frac{P(X_1 + \ldots + X_n > x)}{P(\max(X_1, \ldots, X_n) > x)} = 1$$

Aggregate loss distributions of subexponential severities

- Let *F* be a frequency distribution
- *S* the distribution function of a subexponential severity
- *G* the distribution function of the aggregate loss distribution
- Under general conditions on *F* (satisfied by Poisson and Negative Binomial):

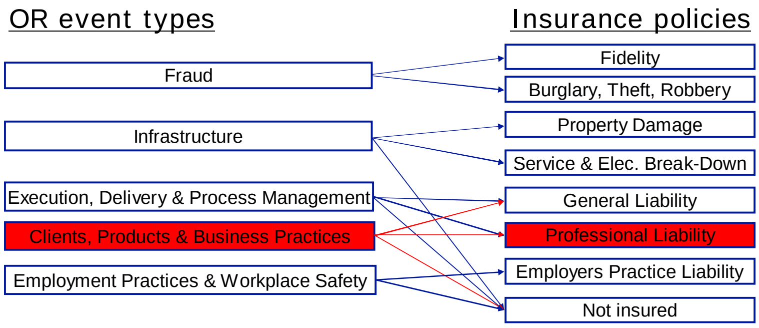
$$\lim_{x\to\infty} \frac{\overline{G}(x)}{\overline{S}(x)} = E(F), \qquad \text{where } \overline{S}(x) := 1 - S(x)$$

Sensitivity analysis of basic LDA components

Based on theoretical results and experience with Deutsche Bank's LDA model

- Frequency distributions
  - Mean of frequency distribution is important
  - Shape has limited impact on capital in cells with fat-tailed severities
  - Shape has limited impact on Group capital
- Severity distributions
  - Weights and techniques for combining different data sources are important
  - Significant impact of distribution assumptions for severity tails and tail probabilities
- Dependence
  - Impact depends on the level where dependence is modelled, e.g. frequencies, severities or aggregate losses
  - Limited impact of frequency correlations

Sensitivity analysis of insurance model



- Clients, Products & Business Practices consumes most of the capital
  - Impact of mapping percentages to insurance contracts
  - Most severe losses fall under Professional Liability: single limit of PL is particularly important
- Higher reduction (in percentage) for median (EL) than for high quantiles (EC and RC)
- Insurance may cause reallocation of capital between different event types

Stressing loss data

Add (remove) internal and/or external losses and analyze impact on capital

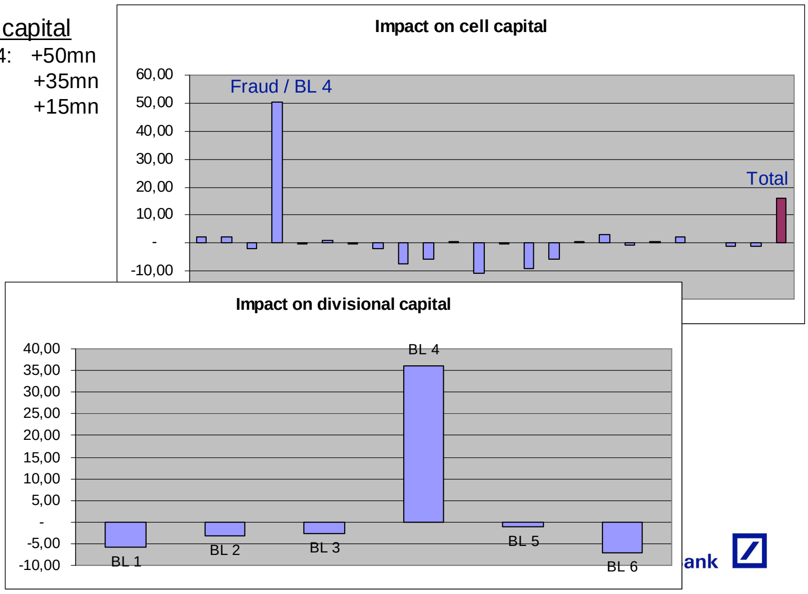
- Scenarios provided by business and ORM to quantify
  - potential future risks
  - impact of business strategies
  - risk reduction by OR management
- Scenarios specified by developers to analyze sensitivity of model

scenario	Description
time decay for losses	losses reveive full weight for 5 years, linear decreasing weight over 20y period for severity fitting. One 5 year period taken into account for frequencies
new ORX mapping	new orx mapping as proposed in EC WG 11.Aug.
AM UK sale (about 30% losses, mainly small losses in execution)	remove AM UK losses from severity and frequency fitting
AM UK sale (about 30% losses, mainly small losses in execution)	remove AM UK losses from frequency fitting
super gau GM/CF	additional losses (split 50 GM : 50 CF) in billion USD: internal 0.75, external 2.6, 2.2, 2.2, 1.5, 1.5, 1.5, 1, 1, 0.5, 0.5 losses were NOT mapped to group (default methodology) but assigned to GM Clients and CF Clients with weight of 50%
integration of scenarios	about 50 scenarios are integrated as OpVar data points (see ECWG presentation Nov 3rd) about 4 scenarios per BL and ET, no scenarios for ET clients Due to additional scenarios in Execution, cell specific modelling for Execution on external data was possible
5 % low impact loss removal in GM	remove 5% of internal GM losses, losses chosen equally spaced between 10K and 500K (large loss threshold)
5% low impact loss removal in all Divsions	remove 5% of internal losses in each division, losses chosen equally spaced between 10K and large loss thresholds
infrastructure + 500mn	infrastructure + 500mn
GM Fraud + 200mn	GM Fraud + 200mn
50 additional small events in CF Fraud	50 events from 10tsd - 50tsd (exponential step size) are added to CF execution internal Benchmark scenario 811 (integration of scenarios)

Stress scenario: add 200mn loss in a Fraud cell

Impact on capital

Fraud / BL 4: +50mn
BL 4: +35mn
Group: +15mn



ank

Backtesting and benchmarking

- Backtesting
  - Sequential testing of a model against reality to check the accuracy of the predictions
  - Backtesting is frequently used for the validation of market risk models
  - In credit and operational risk, the inherent shortage of loss data severely restricts the application of backtesting techniques to capital models
- Benchmarking
  - Comparison of a bank's operational risk capital charge against a bank's close peers
  - Comparison of the AMA capital charge against the BIA or TSA capital charges
  - Comparison of the LDA model outputs against adverse extreme, but realistic, scenarios

These tests help to provide assurance over the appropriateness of the level of capital but there are obvious limitations

Deutsche Bank

Benchmarking
Tail of aggregate loss distribution versus individual data points

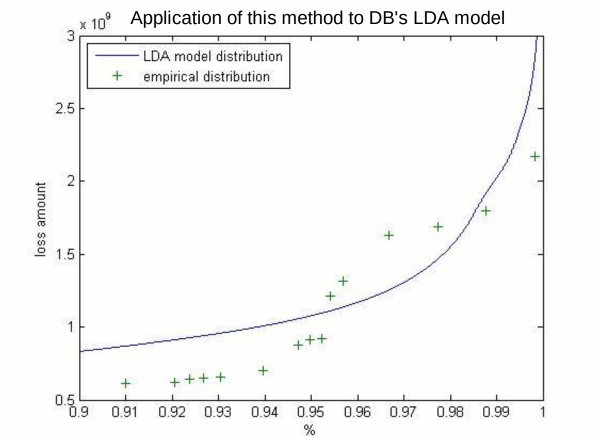
- Based on assumption that these tails have the same order of magnitude:
  - Tail of aggregate loss distribution calculated in a bottom-up LDA model
  - Tail of loss distribution directly specified at Group level
- Loss distribution specified at Group level:
  - Take all losses (across business lines and event types) above a high threshold, say 1m, for the specification of a severity distribution *S*
  - Calculate the bank's average annual loss frequency *n* above 1m
- Under the assumption that *S* is subexponential, identify

$$\alpha - quantiles \text{ of the loss distribution } S_1 + ... + S_n \text{ with}$$
$$\alpha - quantiles \text{ of the maximum distribution } \max(S_1,...,S_n) \text{ with}$$
$$1-((1-\alpha)/n) - quantiles \text{ of the severity distribution } S$$

Deutsche Bank

Benchmarking result

- 1-((1-alpha)/n) – quantiles of the severity distribution correspond to individual losses for appropriate alpha and n
- The amount of loss data provides a limit for the confidence level that can be derived directly from the data



Deutsche Bank

For more information

F. Aue and M. Kalkbrener (2006).
LDA at work. Deutsche Bank, Frankfurt.

michael.kalkbrener@db.com