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## 出席 2004 年第 22 屆 國際模態分析研討會報告書

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主辦機關:

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內容摘要: 本文為參加由實驗力學學會(Society of Experimental Mechanics, SEM)主辦之第 22 屆國際模態分析研討會(International Modal Analysis Conference, IMAC)報告書,說明參加研討會過程。會中有專題演講,約二百五十篇論文發表論文,著重在振動模態測試(experimental testing, ET)或實驗模態分析(experimental modal analysis, EMA)核心技術相關主題之應用與發展,另外,今年度大會主題為「連結測試到設計」(Linking Test to Design),最能顯示 EMA 發展之方向性、重要性、應用性及價值性,文中就筆者對 EMA 技術領域之瞭解與認知,就本年度發表論文作簡短之重點心得摘要以及建議。

本文電子檔已上傳至出國報告資訊網

## 摘要

本文為參加由實驗力學學會(Society of Experimental Mechanics, SEM)主辦之第 22 屆國際模態分析研討會(International Modal Analysis Conference, IMAC)報告書，說明參加研討會過程。會中有專題演講，約二百五十篇論文發表論文，著重在振動模態測試(experimental testing, ET)或實驗模態分(experimental modal analysis, EMA)核心技術相關主題之應用與發展，另外，今年度大會主題為「連結測試到設計」(Linking Test to Design)，最能顯示 EMA 發展之方向性、重要性、應用性及價值性，文中就筆者對 EMA 技術領域之瞭解與認知，就本年度發表論文作簡短之重點心得摘要以及建議。

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## 一、目的

本次出國出席參加第 22 屆國際模態分析研討會(International Modal Analysis Conference, IMAC)，主要發表學術研究論文，題目：Prediction of Harmonic Force Acting on Cantilever Beam (作用於懸臂樑簡諧外力預測)，藉由出席會議與各國振動領域人士有學術交流機會，並獲取振動實驗模態分析領域之新知與發展動態，做為未來國內學術研究發展之參考。

## 二、參加會議過程

第 22 屆國際模態分析研討會(International Modal Analysis Conference, IMAC)於 2004 年 1 月 26 日至 1 月 29 日在美國密西根州 Dearborn 舉行，該地位於底特律市郊區，為福特汽車大本營，與今年度大會主題「連結測試到設計」(Linking Test to Design)對汽車、航太工業發展有莫大之關係。本年度研討會和往年一樣，均安排與實驗模態分析相關之會前訓練課程，除了大會專題演講之外，分三天半同時進行七個場次超過 250 篇之論文發表，並且有連續兩天超過 30 家之振動噪音量測設備廠商參展，據悉接近五百人註冊參加此次研討會。

研討會首日準時於 8 點整由大會主席 Dr. Wicks 致歡迎詞揭開序幕，主辦單位之實驗力學學會(Society of Experimental Mechanics, SEM)理事長 Dr. Allemang 也應邀致詞後，隨即進行別開生面的大會專題演講，由兩大汽車廠包括日本三菱及美國通用公司，分別由其研發主管就其公司針對與大會主題「連結測試到設計」相關之研發情形做專題報告，此次安排頗有讓兩家公司互別苗頭之意，事實也達到非常好的良性競爭效果。三菱公司 Dr. Ando 之報告顯得準備充分，強調 CAD/CAE 及實驗測試結合的重要，也提出“Knowledge-base design”之研發概念，也舉出 information + theory + experience = innovation，也就是資訊、理論及經驗是創新之泉源。通用公司 Mr. Aase 則說明 virtual reality (VR)在該公司已有深入之應用，也指出 CAE 與實驗共同資料庫之建立以及雙方人員相互間之密切配合的重要關鍵。由兩位研發主管的報告，可以確知 CAE 及 EMA 的整合技術對汽車工業研究發展的重要與貢獻。

本次會議共有約 250 篇論文，共有 43 個主題，每個主題有 3-8 篇論文分成三天半進行發表報告，同時也分成七個場次同時報告，每一篇論文報告含討論有 30 分鐘，就發表論文內容之綜合分析如下：

1. 與本次大會主題「連結測試到設計」相關有 8 個主題，包括：模型更新與關聯

性(model updating and correlation)3 個、汽車工業、土木結構、工程實例、阻尼測試模型及試驗技術各 1 個主題。

2. 與模態測試(experimental testing, ET)或實驗模態分析(experimental modal analysis, EMA)核心技術相關之主題，包括：模態測試、模態參數辨識(modal parameter identification)、信號處理(signal processing)各 2 個主題，及系統辨識(system identification)、轉換器(transducer)各 1 個主題。
3. CAE/EMA 連結之整合分析技術，包括：不確定性量化分析與模型認證(uncertainty quantification & model validation) 3 個主題，試驗技術、解析方法各 2 個主題，及有限元素法 1 個主題。
4. 就模態分析之應用領域來分，包括：土木結構、汽車工業、飛機/太空、微機電系統(MEMS)、運動器材/樂器、轉動機械等。
5. 就模態分析之其他應用來看，包括：破壞偵測(damage detection)有 3 個主題，及主動控制、結構聲場耦合(vibro-acoustic)、聲學/噪音、健康診斷(health monitoring)等各 1 個主題。

研討會中為給予與會學生及新加入模態分析領域工程師之入門介紹，以促使模態分析技術之落實，及更方便使其對研討會進階論文發表之了解，也安排 3 個完全免費之額外主題，包括：

1. Basics of Linking Test to Design (6 小時)
2. Basics of Modal Analysis for the New/Young Engineers (2.5 小時)
3. Modal Topics – Basics You Need to Know (6 小時)

大會也依慣例安排儀器設備廠商參展，由於 IMAC 是振動噪音領域極其重要的研討會，集聚世界各地尤其是美國本土的產、官、學各界人士，因此參展的廠商不僅踴躍超過 30 家，也都有很大的排場，同時參展的時間、場地安排也考慮週到，在研討會第二天開展，大會就特別安排論文發表中場休息的 coffee break 在展覽會場延長達 1 小時，因此幾乎所有參加研討會的人都會到展示場。另外，從大會第一天起，都有各大公司贊助歡迎餐敘，以提供與會人士互相交流的社交機會。

筆者也抽空由大會提供免費入場參觀「亨利福特博物館」，館中蒐集了世界上第一部汽車、甘乃迪總統座車、未來車、及各類型之陸地車輛真是應有盡有，同時館內也正在舉行航空百年特展，展示項目都非常有創意，值得參考借鏡。

### 三、與會心得

IMAC 研討會可以說是振動實驗量測分析最重要之學術研討會，尤其在主要核心技术 ET 及 EMA 的發展與應用，參加 IMAC 的與會人士仍以美國本土人士居多，更特別的是產業界及官方實驗室人士比例相當高，可能超過一半，足見 EMA 技術在美國產業界的蓬勃發展與應用。就筆者在 IMAC 研討會的綜合觀察，提出幾點分析：

1. 在官方實驗室，就要特別提到美國 Los Alamos 國家實驗室，該實驗室至少組織了 10 個主題場次之論文發表，可以說是主導 EMA 發展的主要機構之一，因為他們也是提供研究計畫經費預算的單位。
2. 由振動噪音儀器設備廠商參展踴躍及展示內容的盛況，可以看出廠商對 IMAC 研討會的重視，同時也深知與會人士對振動噪音儀器設備應用的廣大商機，相較起來台灣在振動噪音實驗檢測這方面的普遍性就遠比不上美、日等國之實務的應用。
3. 大會所安排兩大汽車廠研發主管針對 CAE/EMA 於汽車工業的應用與發展之專題演講，也讓與會人士見識到分析與實驗技術結合對產業技術的提升，此點仍有待國內汽機車業界的體認與努力。
4. 研討會中論文發表時間達 30 分鐘，大部分報告者都能充分發揮，讓聽眾更深入瞭解其研究內容，同時也有充裕時間於雙向溝通，達到真正研討目的，以筆者發表的論文為例，提問題討論的狀況，比起國內學術研討會情形顯然熱烈的多。
5. 研討會所安排如 modal basics 系列基礎入門講題，係以教育與會學生及新進入模態分析之產業界人士的推廣教育，此點非常值得我們借鏡，因為唯有適當的教育才更能使得 CAE/EMA 之價值獲得體認與瞭解。
6. 研討會也有舉行學生論文競賽，值得一提的是，這些學生論文都是 Los Alamos 國家實驗室經費支持，係由學生於暑假期間到該實驗室實習研究完成之成果報告，此點值得國內國家實驗室或研究中心借鏡參考。

就 IMAC 研討會的學術論文內容而言，以本次大會主題「連結測試到設計」最能顯示 EMA 發展之方向性、重要性、應用性及價值性，茲就筆者對 EMA 技術領域之瞭解與認知，就本年度發表論文作簡短之重點心得摘要：

- 1 傳統 EMA 方法，係以衝擊錘(hammer)或振動激振器(shaker)為驅動器(actuator)，而以加速度計(accelerometer)為感測器(sensor)，進行振動實驗量測及分析，由於是相當成熟的技術，同時尤其以 PC-base 之儀器設備為主流，因此大部分之應用研究論文也都以此方法為發展基礎。

2. SLDV (scanning laser displacement vibrometer)掃瞄雷射位移計量測系統是相當受矚目的非接觸振動量測技術，可大大提升傳統 EMA 量測速度，最近也有了三軸向 SLDV 的商業產品，實為 EMA 之一大利器，缺點就是太貴。
3. Out-only operational modal analysis (OMA)即僅量測輸出信號之作業狀態下模態分析是近年來 EMA 技術的一大突破，可以不必在結構靜止狀態下，就能進行 EMA，也有某些公司完成商業軟體之開發，不過也有輸入必須為白噪音(white noise)假設上的缺失，也因此有著很大的研究發展空間。
4. OMA 之兩大主流方法為頻率域之 FDD (frequency domain decomposition)及時間域之 SSI (sub-space identification)仍不斷有創新的改進方式值得注意。
5. 有多篇論文闡述有關 CAE/EMA 整合技術於實務之工業產品開發設計應用，藉著實驗 EMA 及分析技術如 FEM 之整合，更使得相得益彰，國內在這方面似乎多在 EMA 及 CAE 之獨立使用，在二者之整合分析連結到產品的設計開發尤待推廣，而且應是產業界 R&D 之利器。
6. V&V (verification and validation)驗證與認證是 Los Alamos 國家實驗室所組織的三個場次論文發表主題，V&V 似乎是該實驗室在結構設計的作業流程之一，同時也是一長期發展計畫項目。驗證方面，包括：程式(code)及計算(calculation)之驗證；認證方面，包括：解析正確的方程式、符合目的、精確度等，該實驗室已發展了值得參考的作業流程。
7. EMA 應用領域涵蓋機械、土木、汽車、航太等工業，以專題演講所著重的汽車工業為例，從零組件之設計到整車之分析，除了 CAE 外，都需要對應實驗之驗證與認證，EMA 技術就扮演極重要角色，國內之應用發展，仍有待學術界將此技術導入工業界之應用。
8. 另一值得注意的發展，如前述僅量測輸出信號之作業狀態下模態分析技術之發展，特別對於大型結構如土木橋樑、建築、飛機等，應用振動特性為基礎的破壞檢測(damage detection)、健康監測(health monitoring)及診斷(diagnosis)之相關研究依舊相當熱門，所謂 vibration-base method 是主要的非破壞檢驗技術之一環，對於國內老舊、新建橋樑或建築物之檢測與診斷不致是一個可行的、有效的方法。

#### 四、建議

1. 本研討會已邁入第 22 屆為振動與噪音領域主要會議之一，主辦單位為隸屬於國際理論與應用力學聯盟(IUTAM)之實驗力學學會(Society of Experimental Mechanics,



SEM)，國內相關領域之學者相當多，不過今年度來自台灣僅筆者一人出席發表論文，與鄰國日本、韓國、甚至中國大陸相較顯得人單勢薄，國內有兩個主要之相關學會，振動與噪音學會以及音響學會，應可適度組團參與以促進學術交流活動，未來可爭取主辦此會議，也需要政府機構在經費上之協助，可提升國內振動與噪音領域研究之重視。

2. 由專題演講兩大汽車廠研發主管的報告及大會安排的模態分析基礎演講主題，可看出主辦單位已有深刻體認，模態分析工程教育及落實於工業界應用的重要性，如何落實振動領域包括分析與實驗量測之教學與研究，並促進產業對振動問題於產品設計開發之分析能力，有待學術單位與工業界之共同努力。
3. 此研討會每篇論文發表時間有 30 分鐘，同時進行 7 場次，另外，對出席發表論文掌控得宜，一般研討會開天窗的情形少很多，各場次嚴格遵守大會時間，使得與會者都能選擇合適場次聆聽論文發表，也有充分討論與雙向溝通的機會，此點是國內一般學術研討會所不及之處，如何改善國內學術研討會發表報告之品質，有待各方配合與努力。
4. 工業界常面臨振動與噪音問題，尤其是產品之開發與設計更需考慮，另外環保意識的高漲對環境噪音與振動品質之要求，新近高鐵所引發之振動問題，以及運轉中或規劃興建中之捷運或地鐵系統，在在顯示國內對振動與噪音技術之需求益形重要。由 IMAC 研討會也可發現各國尤其美國這方面技術於工業界的落實，國內從事相關研究者也不少，然而振動噪音在工業界之應用與發展顯然著墨太少，此點值得國內各界深思。

## 五、攜回資料名稱及內容

- 1 2004 年第 22 屆國際模態分析研討會論文光碟。
- 2 2004 年第 22 屆國際模態分析研討會大會手冊及摘要集。

## 附錄一

## Prediction of Harmonic Force Acting on Cantilever Beam

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### Nomenclatures

$a(x, t) = \hat{a}_i(t)$	predicted acceleration response of beam
$\hat{a}(x, t) = \hat{a}_i(t)$	measured acceleration response of beam
$A$	beam acceleration amplitude
$A_b$	the cross sectional area of beam
$b_b$	beam width
$C_b$	damping coefficient of beam
$E_b$	Young's Modulus of beam
$F(x, t)$	force function acting on beam
$F_j$	the $j$ -th impact force amplitude
$f_n$	the $n$ -th natural frequency of beam
$f$	harmonic excitation frequency
$I_b$	cross sectional moment of inertia of the beam
$L_b$	beam length
$N$	number of modes
$N_t$	number of time data points
$Q_t$	objective function
$q_n(t)$	modal coordinate
$t_b$	beam thickness
$w(x, t)$	beam lateral displacement
$x$	the location of the $j$ -th harmonic force in $x$ -coordinate
$x_j$	the location of the $j$ -th accelerometer sensor in $x$ -coordinate
$\omega_s = 2\pi f_s$	excitation frequency
$\omega_n = 2\pi f_n$	the $n$ -th natural frequency of beam
$\rho_b$	beam density
$\zeta_n$	the $n$ -th modal damping ratio of beam
$\phi_n(x)$	the $n$ -th displacement mode shape of beam
$\phi_n = \phi_n(x_j)$	the $n$ -th hammer mode shape function of beam at the $j$ -th location of the hammer actuator

## ABSTRACT

This paper presents the force prediction for a cantilever beam subjected to harmonic excitation. With the assumption of the structural modal parameters known a priori, the acceleration response of the beam due to the harmonic excitation is also measurable and used as the input for the prediction model. The force prediction algorithm can be developed to determine the harmonic force amplitude and its location, simultaneously. The beam response excited by the harmonic force is first derived. The optimization problem to determine the harmonic force amplitude and location is then formulated. The objective function can be defined as the mean square errors between the predicted and measured acceleration response, while the design variables are identified as the force amplitude and its location number associated with the structural mode shape. Theoretical simulation is presented to demonstrate the feasibility and correctness of the developed force prediction algorithm. Experimental verification is also carried out to validate the prediction model. Results show that the harmonic force amplitude and its location can be reasonably predicted. The developed methodology can be easily extended to other structures or applied by using different kinds of sensing devices for harmonic force prediction.

**Keywords:** force prediction, harmonic force, cantilever beam, optimization

## I. Introduction

Force prediction, identification or determination has drawn much attention for engineering design and applications. Stevens [1] gave an overview for those early-related works. Wang [2] not only provided with an extensive review about the subject but also attempted to develop a general approach in force prediction problem for arbitrary structures. The general idea of force prediction model was presented. He addressed several concerns in force prediction such as structural modeling techniques, solution methods of response estimation and types of sensors used in measuring response. The general optimization methods in determining the impact and harmonic forces were developed. This paper will modify Wang's approach [2] in predicting the unknown harmonic force acting on a cantilever beam.

Lots of mechanical components are in harmonic excitation conditions, in particular, for rotating machineries. Such harmonically excited forces, for examples due to imbalance or hydraulic flow, may not practically be measured but are interested and crucial for structural design or diagnosis. Verhoeven [3] constructed synthesized transfer functions from theoretical modal analysis to estimate the excitation forces of rotating machines. Vyas and Wicks [4] adapted the similar approach to determine the turbine blade forces. Karlsson [5] presented the prediction of complex amplitudes of harmonic force by assuming the force spatial distribution available a priori.

In force prediction, the force location is also of great interest. The pattern match technique is generally adopted to search the unknown force location. Moller [6] tentatively gave the spatial shape and position of harmonic point load to match the load location. Wu et al. [7] identified the impact force location by comparing the structural response among possible candidate locations. Similarly, Choi and Chang [8] determined the impact force time history and location in two separate solution loops. Doyle and his coworkers [9-11] solved for the time history and location of impact forces separately. Recently, Wang [2] developed an optimization method in predicting the unknown impact and harmonic forces acting on arbitrary structures. The force contents including the force amplitude and its location can be determined simultaneously. Wang and Chiu [12] followed Wang's formulation [2] and experimentally showed the determination of amplitude and location of the unknown impact force acting on a simply supported beam in one loop of solution. This paper slightly modifies Wang's method [2] to develop the prediction model for unknown harmonic force. Section II details the beam response analysis and the development of harmonic prediction model. Sections III and IV describe the implementation of prediction program and the experimental work, respectively. Section V shows both the theoretical and experimental prediction results, respectively, and demonstrates the feasibility of the developed force prediction model.

## II. Theoretical Analysis:

### 2.1 Beam Response analysis

Consider a uniform cantilever beam as shown in Figure 1(a). The system equation of motion for lateral vibration analysis can be written [13]

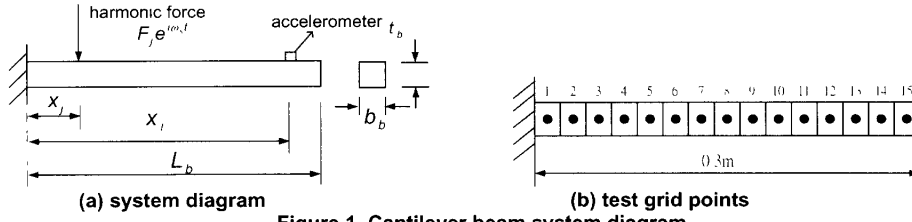


Figure 1. Cantilever beam system diagram

$$E_b I_b \frac{\partial^4 w(x,t)}{\partial x^4} + C_b \frac{\partial w(x,t)}{\partial t} + \rho_b A_b \frac{\partial^2 w(x,t)}{\partial t^2} = F(x,t) \quad (1)$$

If the applied force is harmonic acting at  $x = x_j$ , the force function can be expressed:

$$F(x,t) = F_j \delta(x - x_j) e^{i\omega_s t} \quad (2)$$

where Delta function  $\delta(x - x_j)$  represents the harmonic force location. From expansion theorem, the beam response can be assumed

$$w(x,t) = \sum_{n=1}^{\infty} \phi_n(x) q_n(t) = \sum_{n=1}^{\infty} \phi_n(x) Q_n e^{i\omega_s t} \quad (3)$$

By the substitution of Equation (3) into Equation (1), the beam displacement at  $x = x_i$  can be derived

$$w(x_i,t) = \sum_{n=1}^{\infty} \phi_n(x_i) Q_n e^{i\omega_s t} = e^{i\omega_s t} \sum_{k=1}^{\infty} \frac{F_j \phi_n(x_j) \phi_n(x_i)}{(\omega_n^2 - \omega_s^2) + i(2\xi_n \omega_n \omega_s)} \quad (4)$$

One can observe that beam displacement is functions of modal parameters, i.e.  $\omega_n$ ,  $\xi_n$  and  $\phi_n$ , as well as the harmonic force amplitude  $F_j$ , excitation frequency  $\omega_s$  and its location  $x_j$ . The beam acceleration can also be obtained

$$a(x_i,t) = a_i(t) = A e^{i\omega_s t} \quad (5)$$

where

$$A = -\omega_s^2 \sum_{n=1}^{\infty} \frac{F_j \phi_n(x_j) \phi_n(x_i)}{(\omega_n^2 - \omega_s^2) + i(2\xi_n \omega_n \omega_s)} \quad (6)$$

It is noted that in numerical simulation only  $N$  modes are included to calculate the beam acceleration

## 2.2 Development of Harmonic Force Prediction Model

The conceptual diagram for force prediction model is depicted in Figure 2. For a structure subjected to unknown force, the sensor can detect the structural response as the input to the prediction model. Once the system modal parameters are also known, the force contents, including force amplitude and its location can be determined. This work deals with the prediction of unknown harmonic force acting on the cantilever beam. The accelerometer is employed to measure the beam response as the input to the prediction model. The system modal parameters, including natural frequencies, damping ratios and mode shapes, that can be determined theoretically or experimentally are also assumed known. The force prediction model will be developed to determine the force amplitude and its location, simultaneously. The optimization problem to predict the unknown harmonic force is formulated as follows:

**Objective function:**

$$Q_t = \sum_{r=1}^{N_t} [a_i(t_r) - \hat{a}_i(t_r)]^2 = \sum_{r=1}^{N_t} \left[ -\omega_s^2 \sum_{n=1}^{\infty} \frac{F_j \phi_n(x_j) \phi_n(x_i)}{(\omega_n^2 - \omega_s^2) + i(2\xi_n \omega_n \omega_s)} e^{i\omega_s t_r} - \hat{a}_i(t_r) \right]^2 \quad (7)$$

**Design variables:**

$$F_j, j \quad (8)$$

When  $j = 1$ ,  $\phi_n(x_j)$  equal to  $\phi_n(x_1)$ ,  $n = 1, 2, \dots, N$ , and etc. The objective function  $Q_t$  is defined as the sum of square errors between the measured acceleration  $\hat{a}_i(t_r)$  and the predicted acceleration  $a_i(t_r)$  over the time range

from  $t_1$  to  $t_N$ . As shown in Equation (5), the predicted acceleration  $a_i(t_r)$  is functions of structural modal parameters and force contents. Structural modal parameter can be known. The unknown force contents are the force amplitude  $F_j$  and its location  $x_j$ , while the force excitation frequency  $\omega_s$  can be easily detected. The design variables can then be identified as  $F_j$  and  $j$ . The index  $j$  related to the location  $x_j$  will result in  $\phi_n(x_j), n = 1, 2, \dots, N$ . By the resolution of the optimization problem, the unknown harmonic force amplitude and its location index  $j$  can be determined simultaneously. The objective of the optimization problem is, therefore, to find  $F_j$  and  $j$  so as to minimize the sum of square errors between  $\hat{a}_i(t_r)$  and  $a_i(t_r)$ .

### III. Development of Prediction Program

The force prediction program is developed by Compaq Visual FORTRAN [14]. The optimization subroutine DBCPOL [15], which adopts direct search complex algorithm to solve general optimization problem with multiple design variables, is used to solve for the design variables, i.e. the force amplitude  $F_j$  and its location index  $j$ . The force prediction program flow chart is shown in Figure 3. There are two program options. Option (I) uses theoretically determined modal parameters and the specified harmonic force to generate the theoretical beam acceleration response to represent the measured response  $\hat{a}_i(t_r)$  for the verification of the force prediction model. Option (II) deals with the experimental validation. Program will read in experimentally measured beam acceleration  $\hat{a}_i(t_r)$  to predict the unknown applied harmonic force contents. Both theoretical and experimental prediction results will be presented in Section V.

### IV. Experimental Setup

Table 1 shows the beam dimensions and its material properties. Conventional modal testing is carried out to obtain the beam modal parameters and validated with theoretical modal analysis. The test grid points on the beam are shown in Figure 1(b). The first four natural frequencies of bending modes and the corresponding damping ratios are listed in Table 2. The first four bending mode shapes are shown in Figure 4. As one can observe, the modal parameters agree well between theoretical and experimental analysis.

Figure 5 shows the experimental layout for harmonic force prediction. The harmonic force is stimulated by the mini shaker (BK4810). Different force levels and excitation frequencies can be controlled by the signal generator (BK3016). The accelerometer (PCB352A10) is applied to measure the beam response  $\hat{a}_i(t_r)$  due to the harmonic force excitation. The force amplitude  $F_j$  can be monitored through the force transducer (BK8200) connected to the dual channel analyzer (BK3550). The force prediction program is operated off-line to determine the force amplitude  $F_j$  and location index  $j$  with the input of  $\hat{a}_i(t_r)$  and the determined structural modal parameters.

## V. Results and Discussions

### 5.1 Theoretical prediction results – Option (I)

This section presents the theoretical prediction results by program Option (I). The measured acceleration is replaced by the theoretically generated response to validate the developed prediction model. Figure 6 shows the prediction results for different force amplitudes and locations. The excitation frequency is  $f_1 < f_s = 30\text{Hz} < f_2$ , between the first and second natural frequencies. Figure 6(a) shows the force amplitude prediction results. The horizontal axis is the number of iteration in solving the optimization problem. The vertical axis represents the force amplitudes. The legend (6,2) in Figure 6(a) denotes  $i=6, j=2$ , i.e. the force location at position 2 and the accelerometer at position 6. The harmonic forces are applied at position 2, 5, 10 and 15, respectively. The horizontal dash line in Figure 6(a) indicates the applied harmonic force amplitude. One can observe that the predicted force amplitudes converge to the actual values for the four cases. Figure 6(b) shows the predicted force location results. The vertical axis represents the position index  $j$ . As one can see, the force position index  $j$  also converges to the actual applied force location very well. Figure 7 shows the similar prediction results to Figure 6 except that the excitation frequency is  $f_s = 14.01\text{Hz} \approx f_1$ , i.e. close to the first natural frequency. Both harmonic force amplitude and its location index converge to the actual values very well too. The force prediction model works

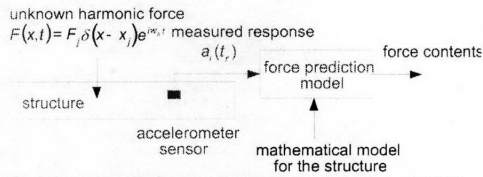


Figure 2. Conceptual diagram for force prediction

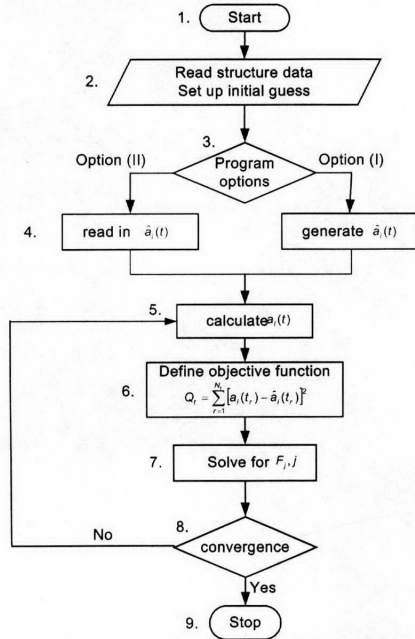


Figure 3. Force prediction program flowchart

Table 1. Beam dimensions and material properties

Material	Steel
Length ( $L_b$ )	0.3 m
Width ( $b_b$ )	0.0394 m
Thickness ( $t_b$ )	0.0016 m
Density ( $\rho_b$ )	7870 kg/m <sup>3</sup>
Young's Modulus ( $E_b$ )	207×10 <sup>9</sup> N/m <sup>2</sup>
Poisson ratio ( $\nu_b$ )	0.292

well for different force amplitudes and force locations as well as different excitation frequencies.

It is also interested to know the effect of sensor locations. Figures 8 and 9 show the theoretical prediction results considering different sensor location at positions 1, 5, 10 and 12 for off-resonance excitation ( $f_1 < f_s = 30\text{Hz} < f_2$ ) and on-resonance excitation ( $f_s = 14.01\text{Hz} \approx f_1$ ), respectively. Figure 8(a) shows the force amplitudes converge to the applied force levels very well except for the case  $(i,j)=(10,15)$  with triangle symbol. In Figure 8(b), the predicted force location is also exactly correct except the case  $(i,j)=(10,15)$  converging to  $j=14$ . This can be the cause that the

Table 2. Natural frequencies and damping ratios of cantilever beam

Mode	Experimental (Hz)	Theoretical (Hz)	Error (%)	Damping ratio (%)
1	14.01	14.728	-4.87	1.3478
2	90.39	92.298	-2.06	1.123
3	253.68	258.43	-1.83	0.455
4	497.07	506.46	-1.85	0.3957

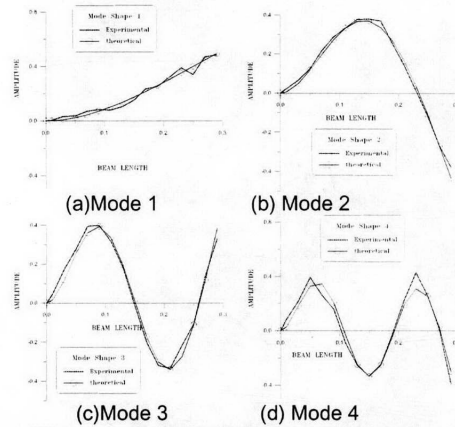


Figure 4. mode shapes of cantilever beam

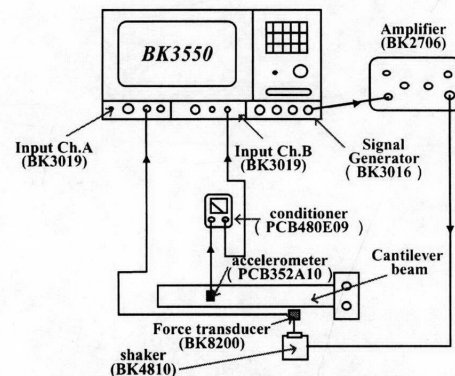


Figure 5. Experimental setup for harmonic force prediction

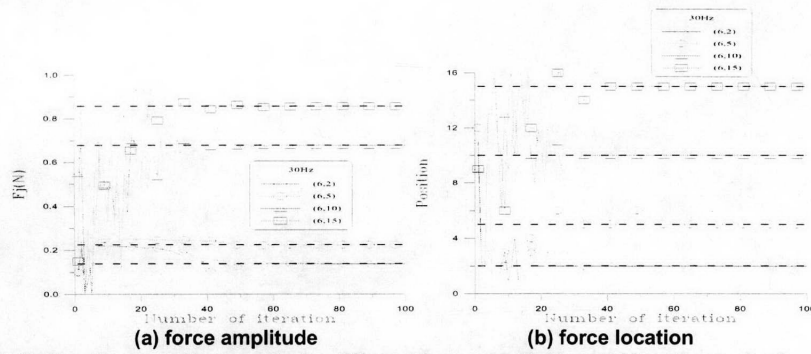


Figure 6. Option (I): prediction results for different force amplitudes and locations,  $f_1 < f_s = 30\text{Hz} < f_2$

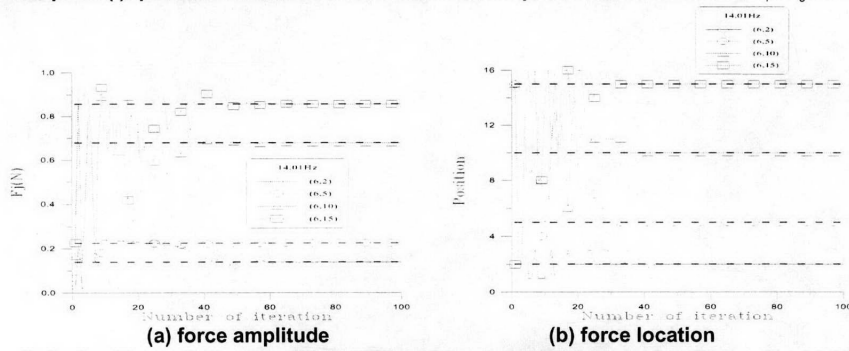


Figure 7. Option (I): prediction results for different force amplitudes and locations,  $f_s = 14.01\text{Hz} \approx f_1$

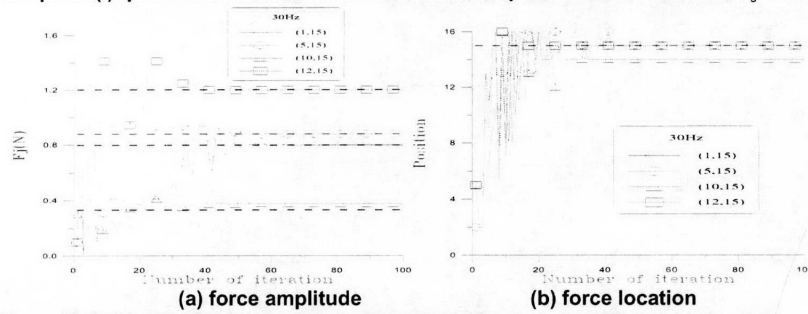


Figure 8. Option (I): prediction results for different sensor locations,  $f_1 < f_s = 30\text{Hz} < f_2$

sensor location  $i=10$  is near the nodal point of the fourth mode shape as shown in Figure 4(d). Similar results can be observed in Figures 9(a) and 9(b) for force amplitude and location prediction, respectively. Only the case  $(i,j)=(12,15)$  with square symbol cannot converge to the correct values. It is the cause that the sensor location  $i=12$  is also near the nodal point of the second mode shape as shown in Figure 4(b). It can be noted that with proper choice of accelerometer location the prediction model can well predict the harmonic force amplitude and location for different excitation frequency conditions.

## 5.2 Experimental prediction results – Option (II)

Previous section theoretically demonstrates the feasibility of the prediction model in determining the unknown



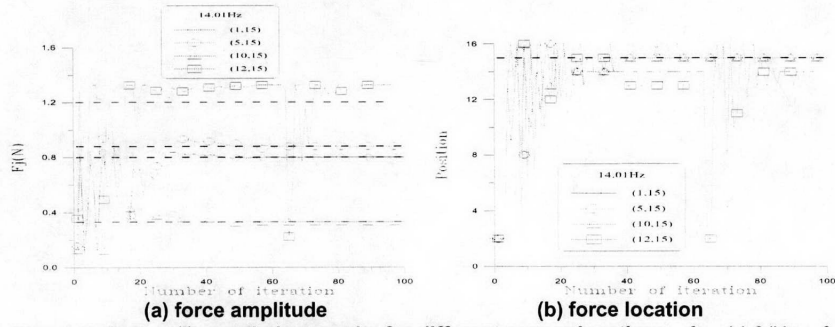


Figure 9. Option (I): prediction results for different sensor locations,  $f_s = 14.01\text{Hz} \approx f_1$

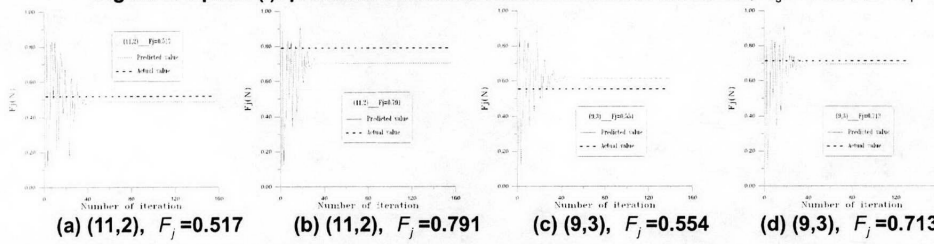


Figure 10. Option (II): force amplitude prediction convergence lines,  $f_s = 14.01\text{Hz} \approx f_1$

Table 3. Option (II): prediction results for different force amplitudes,  $f_s = 14.01\text{Hz} \approx f_1$

(i,j)	Actual Force Amplitude (N)	Predicted Force Amplitude (N)	Error (%)	Predicted Force Location
(11,2)	0.517	0.488	-5.61	2
(11,2)	0.791	0.702	-11.25	2
(9,3)	0.554	0.615	11.01	3
(9,3)	0.713	0.695	-2.52	4

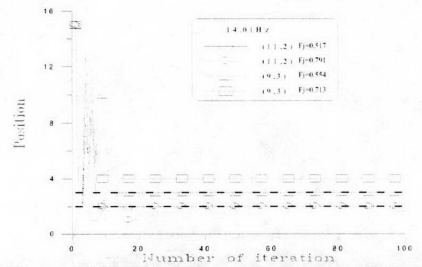


Figure 11. Option (II): prediction results for different force amplitudes and locations,  $f_s = 14.01\text{Hz} \approx f_1$

harmonic force amplitude and its location simultaneously. This section will present the experimental prediction results. The experiments are as detailed in Section IV.

Table 3 shows the prediction results for different force amplitudes and locations, when  $f_s = 14.01\text{Hz} \approx f_1$  near the first resonance excitation. Figure 10 shows the typical convergence curves for the predicted force amplitudes verse the number of iteration, and Figure 11 shows the predicted locations. As seen from Figure 10, the convergence curves vary up and down along the actual value and finally converge close to the actual force amplitudes. The prediction errors of force amplitudes as shown in Table 3 are within  $\pm 12\%$ . From Figure 11 and Table 3, the force location is shown correctly predicted except the case  $(i,j)=(9,3)$  with force amplitude  $F_j=0.713$  N. However, the predicted location  $j=4$  is near to the actual location  $j=3$ .

Table 4 and Figure 12 show more cases for different force locations. The sensor is fixed at position 12, and the harmonic forces are applied at positions 2, 3, 4 and 5, respectively. From Table 4, the maximum prediction error of force amplitude is 13.99% for  $(i,j)=(12,5)$ . The location prediction can be very well as observed in Figure 12. There are about 50 iterations to get convergence solution for the case  $(i,j)=(12,2)$  with diamond symbol in Figure 12. The

**Table 4. Option (II): prediction results for different force locations,  $f_s = 14.01\text{Hz} \approx f_1$**

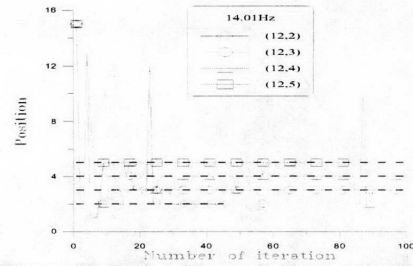
(i,j)	Actual Force Amplitude (N)	Predicted Force Amplitude (N)	Error (%)	Predicted Force Location
(12,2)	4.5	3.998	-11.16	2
(12,3)	0.696	0.718	3.16	3
(12,4)	0.495	0.518	4.65	4
(12,5)	0.0772	0.088	13.99	5

**Table 5. Option (II): prediction results for different sensor locations,  $f_s = 14.01\text{Hz} \approx f_1$**

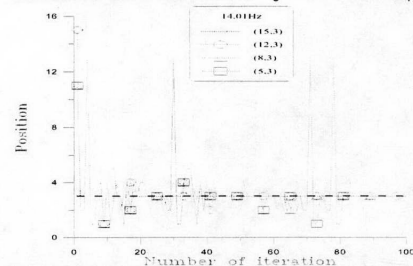
(i,j)	Actual Force Amplitude (N)	Predicted Force Amplitude (N)	Error (%)	Predicted Force Location
(15,3)	0.556	0.398	-28.42	3
(12,3)	0.772	0.9476	22.75	3
(8,3)	0.682	0.915	34.16	3
(5,3)	0.59	0.715	21.19	3

**Table 6. Option (II): prediction results for different excitation frequencies**

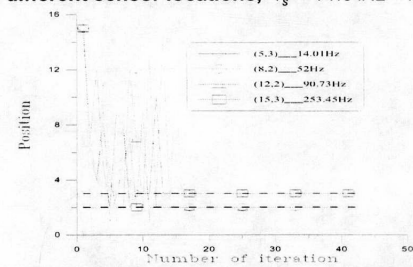
(i,j)	Excitation Frequency (Hz)	Actual Force Amplitude (N)	Predicted Force Amplitude (N)	Error (%)	Predicted Force Location
(5,3)	14.01	0.681	0.7115	4.48	3
(8,2)	52	0.117	0.1215	3.85	2
(12,2)	90.73	0.402	0.488	21.39	2
(15,3)	253.45	0.385	0.415	7.79	3



**Figure 12. Option (II): prediction results for different force locations,  $f_s = 14.01\text{Hz} \approx f_1$**



**Figure 13. Option (II): prediction results for different sensor locations,  $f_s = 14.01\text{Hz} \approx f_1$**



**Figure 14. Option (II): prediction results for different excitation frequencies**

prediction model is, therefore, validated for different force amplitudes and locations.

It is also interesting to study the effect of sensor location on the prediction model. Table 5 and Figure 13 show the predictions results. Although there are about 20-30% errors in the amplitude prediction, the location prediction is exactly correct. The experimental errors are considered in a reasonable range. Finally, Table 6 shows the prediction results for different excitation frequencies. The amplitude prediction errors are within 8% except the case  $(i,j)=(12,2)$  about 21%. Figure 14 shows the convergence curve for the predicted location index verse the number of iteration. One can see that the location prediction is exactly correct. In summary, the prediction model works reasonably well in actual experimental verification for different force amplitudes, locations and excitation frequencies as well as different sensor locations.

## VI. Conclusions

This paper develops the unknown harmonic force prediction algorithm applied to cantilever beam structure. The prediction model can predict the harmonic force amplitude and its location simultaneously. Both theoretical and experimental force prediction results are presented to validate the prediction model. Some conclusions are made as follows:

- 1 The prediction model is well validated through the numerical simulation and successfully predicts the harmonic force amplitude and its location
- 2 In actual experimental prediction, the force amplitude can be reasonably predicted as well as the force location
- 3 The effects of different force amplitudes, locations and excitation frequencies on the prediction model are also studied. With the proper selection of sensor location the prediction model can work reasonably well
- 4 The developed harmonic force prediction methodology can also be extended to other engineering structures as well as employing different sensors

## VII. Acknowledgement

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